A no-go theorem for non-standard explanations of the $\tau \to K_S \pi \nu_\tau$ CP asymmetry

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Abstract

A new tensor interaction is the only possibility to explain the 2.8σ tension observed by the BaBar collaboration in the CP asymmetry in $\tau \to K_S \pi \nu_\tau$ with physics beyond the Standard Model (BSM) realized above the electroweak scale. However, the strong phase generated by the interference between vector and tensor phases is suppressed by at least two orders of magnitude due to Watson's final-state-interaction theorem, and the strength of the CP-violating tensor interaction is strongly constrained by bounds from the neutron electric dipole moment and D–$\bar{D}$ mixing. As a result, a confirmation of the tension at Belle II would point to light BSM physics.

1 Introduction

CP-violating observables are particularly interesting because of their potential connections to baryogenesis mechanisms. Here, we consider the asymmetry of the decay width for $\tau \to K_S \pi \nu_\tau$

$$A_{\text{CP}}^\tau = \frac{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau) - \Gamma(\tau^- \to \pi^- K_S \nu_\tau)}{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau) + \Gamma(\tau^- \to \pi^- K_S \nu_\tau)}. \tag{1}$$

This asymmetry is non-vanishing already in the Standard Model (SM), driven by indirect CP violation in K^0–$\bar{K}^0$ mixing [1, 2], and can be predicted accurately from the CP violation as measured in semileptonic kaon decays. Including corrections from the experimental conditions and time-dependent efficiencies, the corresponding prediction $A_{\text{CP}}^{\tau,\text{SM}} = 3.6(1) \times 10^{-3}$ [2] disagrees with the measurement by the BaBar collaboration [3]

$$A_{\text{CP}}^{\tau,\text{exp}} = -3.6(2.3)(1.1) \times 10^{-3} \tag{2}$$

at the level of 2.8σ. In this note we summarize our arguments why this tension cannot be resolved by BSM physics above the electroweak scale [4].

2 Effective Lagrangian and decay rate

BSM physics if entering above the electroweak scale is described by an effective Lagrangian whose relevant terms at the hadronic scale read

$$\mathcal{L}_{su}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{us} \Big[c_V(\bar{s}\gamma^\mu u)(\bar{\nu}\gamma_\mu \ell) + c_A(\bar{s}\gamma^\mu u)(\bar{\nu}\gamma_\mu \gamma_5 \ell) +$$

$$+ c_S(\bar{s}u)(\bar{\nu}\ell) + ic_P(\bar{s}u)(\bar{\nu}\gamma_5 \ell) + c_T(\bar{s}\sigma^{\mu\nu}u)(\bar{\nu}\sigma_{\mu\nu}(1+\gamma_5)\ell) \Big] + \text{h.c.}, \qquad (3)$$

where $c_V = -c_A = 1$ and all other coefficients are equal to zero in the SM. The interference between the in general complex Wilson coefficients can then produce a weak phase that could generate a direct CP-violating contribution to the decay rate. However, for a non-vanishing CP asymmetry one needs the interference of two amplitudes

$$\mathcal{A}_j = |\mathcal{A}_j| e^{i\delta_j^s} e^{i\delta_j^w}, \qquad j \in \{1, 2\}, \qquad (4)$$

with relative strong and weak phases $\delta^s = \delta_1^s - \delta_2^s$ and $\delta^w = \delta_1^w - \delta_2^w$ and both phases have to be non-vanishing, i.e.

$$A_{\text{CP}} \propto |\mathcal{A}_1 + \mathcal{A}_2|^2 - |\bar{\mathcal{A}}_1 + \bar{\mathcal{A}}_2|^2 = -4|\mathcal{A}_1||\mathcal{A}_2| \sin \delta^s \sin \delta^w. \qquad (5)$$

For $\tau \to K_S \pi \nu_\tau$ the decay rate takes the form [4]

$$\frac{d\Gamma}{ds} = G_F^2 |V_{us}|^2 S_{\text{EW}} \frac{\lambda_{\pi K}^{1/2}(s)(m_\tau^2 - s)^2 (M_K^2 - M_\pi^2)^2}{1024\pi^3 m_\tau s^3} \times$$

$$\times \left[\xi(s) \left(|V(s)|^2 + |A(s)|^2 + \frac{4(m_\tau^2 - s)^2}{9sm_\tau^2} |T(s)|^2 \right) + |S(s)|^2 + |P(s)|^2 \right], \qquad (6)$$

where

$$V(s) = f_+(s)c_V - T(s), \qquad S(s) = f_0(s)\left(c_V + \frac{s}{m_\tau(m_s - m_u)}c_S \right),$$

$$T(s) = \frac{3s}{m_\tau^2 + 2s}\frac{m_\tau}{M_K}c_T B_T(s), \qquad (7)$$

and similarly for the axial-vector and pseudoscalar terms. This structure implies that there cannot be a contribution from the vector–scalar interference because they involve the same hadronic form factor $f_0(s)$, leaving the vector–tensor interference as the only possibility [5]. The corresponding asymmetry

$$A_{\text{CP}}^{\tau,\text{BSM}} = \frac{\text{Im} c_T}{\Gamma_\tau \text{BR}(\tau \to K_S \pi \nu_\tau)} \int_{s_{\pi K}}^{m_\tau^2} ds' \kappa(s')|f_+(s')||B_T(s')| \sin\big(\delta_+(s') - \delta_T(s')\big) \qquad (8)$$

depends on the imaginary part of the tensor Wilson coefficient $\text{Im} c_T$ and the weighted integral of the phase difference of vector and tensor form factors $\delta_+(s) - \delta_T(s)$.

3 Hadronic form factors

While the normalization is known from lattice QCD [6], it had been assumed in previous work that the tensor form factor stays constant as a function of s [5]. However, elastic unitarity

$$\text{Im} f_+(s) = \frac{\lambda_{\pi K}^{1/2}(s)}{s} f_+(s)\big(f_1^{1/2}(s)\big)^*, \qquad \text{Im} B_T(s) = \frac{\lambda_{\pi K}^{1/2}(s)}{s} B_T(s)\big(f_1^{1/2}(s)\big)^*, \qquad (9)$$
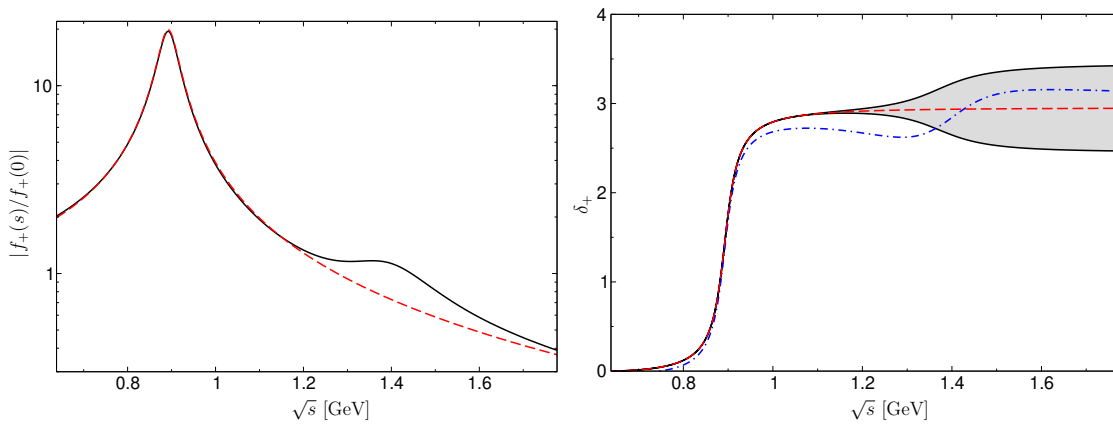
Figure 1: Left: $|f_+(s)/f_+(0)|$ from [11] (black solid line) in comparison to the Omnès factor (10) (red dashed line). Right: δ_+ from a Breit–Wigner approximation for the $K^*(892)$ (red dashed line) in comparison to the phase from the experimental fit [11] (blue dot-dashed line). The gray band gives our estimate of inelastic effects.

implies that both phases are equal to the πK isospin-1/2, P-wave phase shift $\delta_1^{1/2}(s)$ up to inelastic corrections, a manifestation of Watson's final-state theorem [7]. In both cases, the energy dependence is therefore dominated by the Omnès factor [8]

$$\Omega(s) = \exp\left\{\frac{s}{\pi}\int_{s_{\pi K}}^\infty \frac{\delta_1^{1/2}(s')}{s'(s'-s)}\right\}, \tag{10}$$

which implements in a model-independent way the dominance by the $K^*(892)$ resonance. Phenomenologically, this result follows from the observation that spin-1 resonances can be described equivalently by vector or antisymmetric tensor fields [9, 10], so that the same resonances that contribute to $f_+(s)$ will appear in $B_T(s)$ as well, most notably the $K^*(892)$. The first inelastic effects arise around the $K^*(1410)$, see Fig. 1, and assuming that $\delta_+(s) - \delta_T(s) \sim 2\delta_+^{\rm inel}(s)$ we estimate

$$\left|A_{\rm CP}^{\tau,\rm BSM}\right| \lesssim 0.03|{\rm Im}\, c_T|, \tag{11}$$

suppressing the integral in (8) by about two orders of magnitude compared to $\delta_T(s) = 0$ [5].

4 Limits on $\mathrm{Im}\, c_T$

At the high scale, the tensor current originates from the $SU(2) \times U(1)$ gauge-invariant Lagrangian

$$\begin{aligned}
\mathcal{L}_T &= C_{abcd}\, \bar{L}_{La}^i \sigma_{\mu\nu} e_{Rb}\, \epsilon^{ij}\, \bar{q}_{Lc}^j \sigma^{\mu\nu} u_{Rd}\ +\ \text{h.c.}\\
&= C_{3321}\Big[(\bar{\nu}_\tau \sigma_{\mu\nu} R\tau)(\bar{s}\sigma^{\mu\nu} Ru) - V_{us}(\bar{\tau}\sigma_{\mu\nu} R\tau)(\bar{u}\sigma^{\mu\nu} Ru)\Big]\ +\ \text{h.c.}, \tag{12}
\end{aligned}$$

where $R = (\mathbb{1} + \gamma_5)/2$. While the first term indeed contributes to the tensor current in $\tau \to K_S \pi \nu_\tau$, the second induces a u-quark electric dipole moment (EDM) via renormalization group evolution [12], see Fig. 2 (left), and thus a contribution to the stringently constrained EDM of the neutron d_n. Using the 90% C.L. bound $d_n = g_T^u(\mu)d_u(\mu) < 2.9\times10^{-26}\, e\,\text{cm}$ [13,14] together with the tensor charge [15] $g_T^u(\mu = 2\,\text{GeV}) = -0.233(28)$ we obtain

$$|{\rm Im}\, c_T(\mu_\tau)| \le \frac{4.4\times10^{-5}}{\log\frac{\Lambda}{\mu_\tau}} \lesssim 10^{-5}, \tag{13}$$
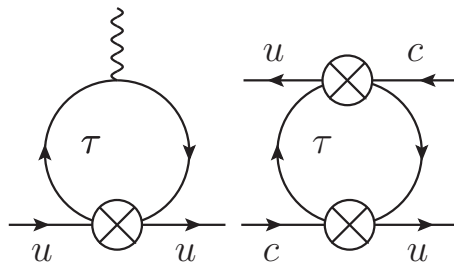
Figure 2: Diagrammatic representation of the u-quark EDM (left) and the contribution to D–$\bar{D}$ mixing (right) originating from single and double insertions of the tensor operators, respectively.
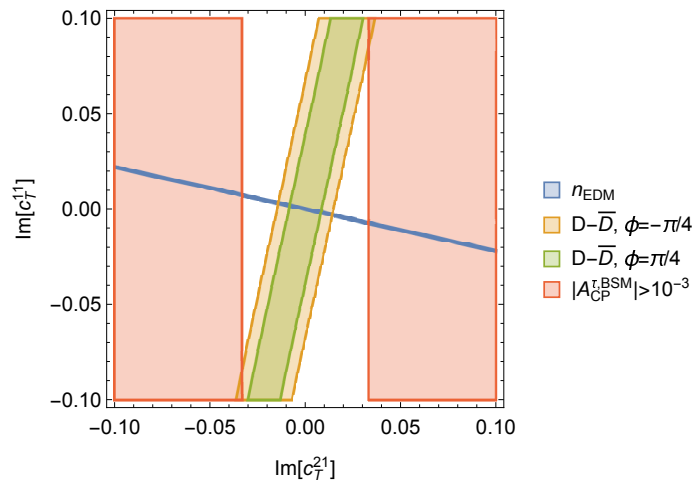
Figure 3: Allowed regions in the $\operatorname{Im} c_T^{21}$–$\operatorname{Im} c_T^{11}$ plane from the neutron EDM and D–$\bar{D}$ mixing (for $\phi = \pm\pi/4$ and $\Lambda = 1\,\mathrm{TeV}$), compared to the favored region from the $\tau \to K_S \pi \nu_\tau$ CP asymmetry.

where the last bound holds for $\Lambda \gtrsim 100\,\mathrm{GeV}$. Since an explanation of the $\tau \to K_S \pi \nu_\tau$ CP asymmetry requires $\operatorname{Im} c_T \sim 0.1$, one therefore needs cancellations in the neutron EDM of one part in 10^4. In principle, such a cancellation is possible from an operator with a different flavor structure C_{3311}.

The neutron EDM then probes the combination $V_{ud}\operatorname{Im} c_T^{11} + V_{us}\operatorname{Im} c_T^{21}$, where $c_T^{21} = c_T$ and c_T^{11} derives from C_{3311}, so that, in principle, some symmetry might be conceivable that enforces this cancellation exactly. However, an orthogonal constraint follows from D–$\bar{D}$ mixing, which is sensitive to $(V_{cd} c_T^{11} + V_{cs} c_T^{21})^2$ by a double insertion of the operators, see Fig. 2 (right), so that in addition to tuning the neutron EDM constraint to 10^{-4}, this second combination has to be close to purely imaginary to evade the constraint from D–$\bar{D}$ mixing. Using the global fit from [16], we find the combined exclusion regions as given in Fig. 3. We note that due to the double insertion required in effective field theory, the leading D–$\bar{D}$ effect only enters at dimension 8, but in an ultraviolet complete model there are in general already dimension-6 contributions, to the effect that the corresponding bound will become much stronger. In this way, an explanation of the $\tau \to K_S \pi \nu_\tau$ CP asymmetry would require an intricate conspiracy of BSM couplings, which for all practical purposes excludes such a scenario.

5 Conclusions

The BaBar measurement of the $\tau \to K_S \pi \nu_\tau$ CP asymmetry differs from the SM by 2.8σ. We have shown that based on very general arguments a non-standard explanation from heavy BSM physics, realized above the electroweak scale, is exceedingly unlikely: such a direct CP violation could only come from a tensor–vector interference, but the strong phase is greatly suppressed by Watson's theorem, and a large BSM Wilson coefficient required to compensate for this suppression is in conflict with limits on the neutron EDM and D–$\bar{D}$ mixing. If confirmed at Belle II [17], this would point to some exotic light BSM physics.

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