Quantum spin nematic liquid in the low-dimensional anisotropic magnets -S=1/2 delta spin chain with the anisotropic ferromagnetic interaction in magnetic field-

Toru Sakai[1,2⋆], Rito Furuchi[1], Hiroki Nakano[1] and Kiyomi Okamoto[1]

1 Graduate School of Science, University of Hyogo, Hyogo 678-1297, Japan
2 National Institutes for Quantum Science and Technology (QST), SPring-8, Hyogo 679-5148, Japan

⋆ sakai@spring8.or.jp

International Conference on Strongly Correlated Electron Systems (SCES 2022)
Amsterdam, 24-29 July 2022
doi:10.21468/SciPostPhysProc.11

Abstract

The magnetization process of the $S = 1/2$ delta chain with the anisotropic ferromagnetic interaction is investigated using the numerical diagonalization of finite-size clusters. It is found that the spin nematic liquid phase appears in higher magnetization region, as well as the SDW liquid one in lower region.

Received 12-07-2022
Accepted 11-11-2022
Published 05-06-2023
doi:10.21468/SciPostPhysProc.11.001

Contents

1 Introduction

The spin nematic state [1] is one of interesting topics in the field of the strongly correlated electron systems. It is the quadrupole order of spins. Recently many theoretical and numerical studies on the spin nematic state have been reported about various quantum spin systems. In most theoretical works the mechanism of the spin nematic order is based on the spin frustration [1–4] or the biquadratic exchange interaction [5–7]. In this paper we propose a theoretical model without the spin frustrations, or the biquadratic interaction, that exhibits the spin nematic liquid phase in magnetic field. It is the $S = 1/2$ delta spin chain [8] with the anisotropic ferromagnetic interaction. In one-dimensional systems like this model, the nematic order is reduced to the quasi-long-range order characterized by the power-law decay of the spin correlation function, which is called the Tomonaga-Luttinger liquid (TLL). We investigate this model using the numerical diagonalization of finite-size clusters and obtain the phase diagrams with respect to the anisotropy and the magnetization, which include the nematic-correlation dominant TLL phase.

2 Model and Calculation

The magnetization process of the $S = 1/2$ delta chain shown in Fig. 1 is investigated. It is described by the Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_Z \,, \tag{1}$$

$$\mathcal{H}_0 = J_1 \sum_{j=1}^{L} \left[S^x_{2j-1} S^x_{2j} + S^y_{2j-1} S^y_{2j} + \lambda S^z_{2j-1} S^z_{2j} \right] + J_2 \sum_{j=1}^{L} \vec{S}_{2j} \cdot \vec{S}_{2j+1} + J_3 \sum_{j=1}^{L} \vec{S}_{2j-1} \cdot \vec{S}_{2j+1} \,, \tag{2}$$

$$\mathcal{H}_Z = -H \sum_{j=1}^{L} S^z_{2j-1} + S^z_{2j} \,, \tag{3}$$

where λ is the anisotropy and H is the magnetic field. J_1 and J_2 are fixed to -1 and $+1$, respectively. For the length L system, the lowest energy of $\mathcal{H}_0$ in the subspace where $\sum_j S^z_j = M$, is denoted by $E(L, M)$. The reduced magnetization m is defined by $m = M/M_s$, where M_s denotes the saturation of the magnetization, namely $M_s = L$. The energy $E(L, M)$ is calculated by the Lanczos algorithm under the periodic boundary condition ($\vec{S}_{2L+1} = \vec{S}_1$).

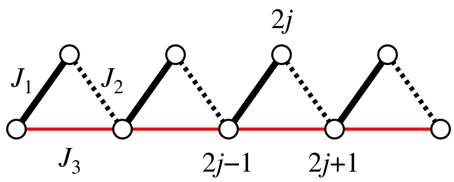

Figure 1: Delta spin chain.

3 Ground State without Magnetic Field

Since the system (1) is not frustrated, the spin pair at the J_1 bond behaves like the $S=1$ object. Thus for $\lambda=1$ and $H=0$ the ground state is expected to be in the Haldane phase [9]. The Néel order would be realized for sufficiently large λ. Using the phenomenological renormalization [10], the phase boundary can be estimated by the fixed point equation $L\Delta_\pi(L,\lambda)=(L+2)\Delta_\pi(L+2,\lambda)$, where Δ_π is the excitation gap with $k=\pi$ in the subspace with $M=0$. The scaled gap $L\Delta_\pi$ for $J_3=0.2$ is plotted versus λ for $L=8$, 10, 12 and 14 in Fig. 2(a). The extrapolation of the size-dependent fixed point for L and $L+2$ assuming the size correction proportional to $1/(L+1)$, as shown in Fig. 2(b), results in $\lambda_c=2.1376\pm0.0001$ in the infinite length limit.

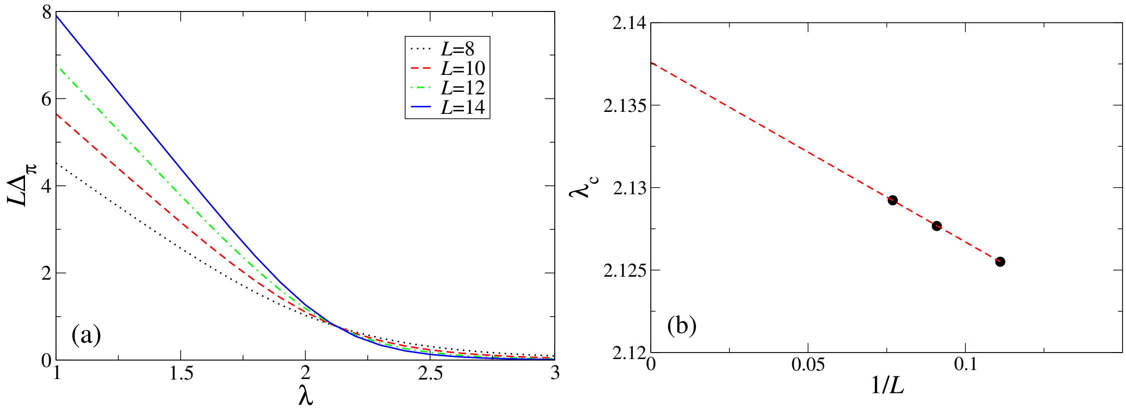

Figure 2: (a) $L\Delta_\pi$ for $J_3=0.2$ is plotted versus λ for $L=8$, 10, 12 and 14. (b) Extrapolation of the size-dependent fixed point for L and $L+2$ assuming the size correction proportional to $1/(L+1)$.

4 Field-Induced TLL Phases

The system (1) is expected to behave as the $S=1$ antiferromagnetic chain with anisotropy. With the Ising-like anisotropy ($\lambda>1$), it would be an effective $S=1$ chain model with the easy-axis single-ion anisotropy. According to the previous numerical diagonalization study on the magnetization process of the $S=1$ antiferromagnetic chain [11], the two-magnon TLL phase, where each magnetization step is $\delta S^z=2$, is realized for the sufficiently large easy-axis anisotropy, while the conventional TLL phase appears near the isotropic case. The single-magnon excitation gap and the $2k_{\rm F}$ excitation gap of the two magnon bound state are defined as Δ_1 and $\Delta_{2k_{\rm F}}$, respectively. The phase boundary between the conventional and two-magnon TLL phases can be estimated as the point of $\Delta_1=\Delta_{2k_{\rm F}}$, because Δ_1 ($\Delta_{2k_{\rm F}}$) is gapless (gapped) in the former phase, while gapped (gapless) in the latter one. The scaled gaps $L\Delta_1$ and $L\Delta_{2k_{\rm F}}$ of the system (1) at $m=1/2$ for $J_3=0.4$ are plotted versus λ for $L=8$ and 12 in Fig. 3. It confirms the gapless and gapped behaviors of Δ_1 and $\Delta_{2k_{\rm F}}$ are switched at the expected phase boundary. Thus we determine the phase boundary λ_c as $\Delta_1=\Delta_{2k_{\rm F}}$ at each magnetization.

5 Critical Exponent Analysis

In the field-induced two-magnon TLL phase, the nematic spin correlation perpendicular to H and the SDW one parallel to H are expected to exhibit the power-law decay. These are

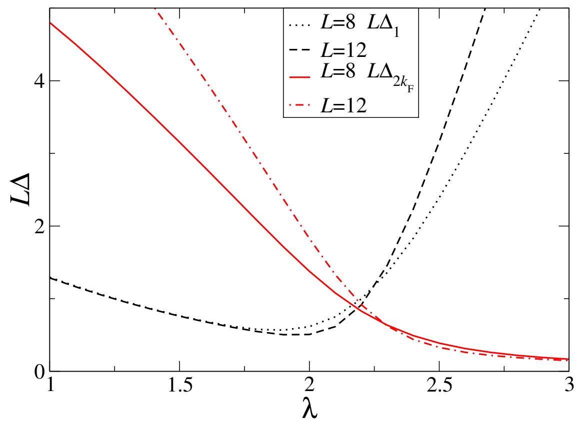

Figure 3: Scaled gaps $L\Delta_1$ and $L\Delta_{2k_\mathrm{F}}$ at $m = 1/2$ for $J_3 = 0.4$ are plotted versus λ for L =8 and 12.

described by the following spin correlation functions

$$\langle S_0^z S_{2r}^z \rangle - \langle S^z \rangle^2 \sim \cos(2k_\mathrm{F} r) r^{-\eta_z} \,, \tag{4}$$

$$\langle S_1^+ S_2^+ S_{2r-1}^- S_{2r}^- \rangle \sim r^{-\eta_2} \,. \tag{5}$$

The correlation with the smaller exponent η is dominant. Namely, the nematic spin correlation dominant TLL phase is realized for $\eta_2 < \eta_z$, while the SDW dominant one for $\eta_2 > \eta_z$. According to the conformal field theory, these critical exponents can be estimated by the forms of several energy gaps

$$\eta_2 = \frac{E(L, M+2) + E(L, M-2) - 2E(L, M)}{E_{k_1}(L, M) - E(L, M)} \,, \tag{6}$$

$$\eta_z = 2\frac{E_{2k_F}(L, M) - E(L, M)}{E_{k_1}(L, M) - E(L, M)} \,, \tag{7}$$

for each magnetization M, where k_1 is defined as k_1=$L/2\pi$. The exponents η_2 and η_z estimated for $L = 12$ and 14 are plotted versus m for $J_3 = 0.4$ and $\lambda = 2.5$ in Fig.4. It indicates that the spin nematic dominant TLL phase ($\eta_2 < \eta_z$) is realized at larger m while the SDW one at smaller m. Since the relation $\eta_2 \eta_z = 1$ should be satisfied in the TLL phase, the crossover between the two dominant spin correlations should occur at the magnetization with $\eta_2 = \eta_z = 1$. Fig. 4 suggests that the system size dependence of η_z is too large to estimate the crossover point in the infinite length limit. Thus we determine the crossover magnetization as $\eta_2 = 1$.

6 Phase Diagrams

Finally the phase diagrams with respect to λ and m is presented for $J_3 = 0.2$, 0.4 and 1.0 in Figs. 5 (a), (b) and (c) respectively. The boundaries between the conventional and two-magnon TLL are given as solid diamonds ($L = 10$), circles ($L = 12$) and squares ($L = 14$), determined as $\Delta_1 = \Delta_{2k_\mathrm{F}}$. The crossover lines between the nematic and the SDW correlation dominant TLL phases are given as stars, determined by $\eta_2 = 1$. The boundaries between the Haldane and Néel ordered phases at $m = 0$ estimated by the phenomenological renormalization are up triangles and the ones between the conventional and two-magnon TLL at $m = 1$ calculated for $L = 14$ are down triangles. CTLL, NTLL and $\mathrm{SDW_2}$TLL correspond to the conventional TLL phase, the nematic correlation dominant two-magnon TLL phase, and the SDW

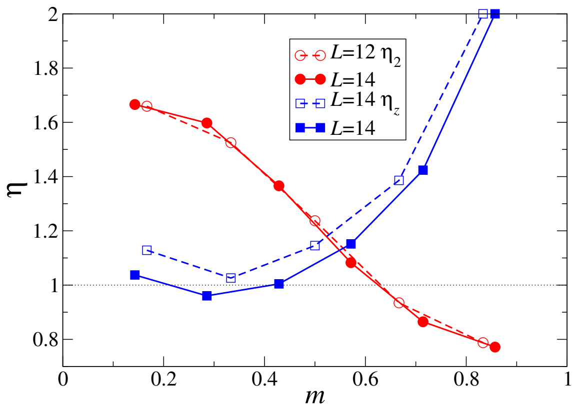

Figure 4: Exponents η_2 and η_z estimated for $L = 12$ and 14 are plotted versus m for J_3=0.4 and λ =2.5

dominant two-magnon TLL phase, respectively. The shape of the phase diagram depends on J_3. For smaller J_3, the magnetization process from the Haldane phase would meet the quantum phase transition from CTLL to NTLL or SDW_2TLL. In contrast, for larger J_3 the one from the Néel ordered phase would meet the quantum phase transition from SDW_2TLL or NTLL to CTLL.

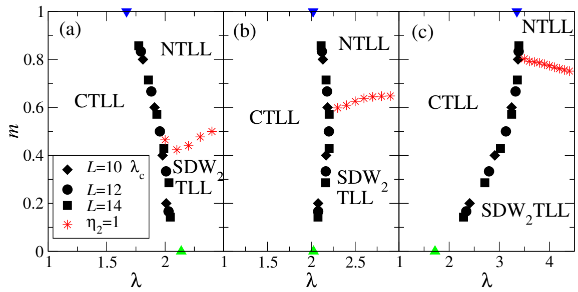

Figure 5: Phase diagrams of the λ-m plane for $J_3 = 0.2$ (a), 0.4 (b) and 1.0 (c). The up triangle is the boundary between the Haldane and Néel ordered phases. The down triangle is determined by $\Delta_1 = \Delta_2$ at $m = 1$ limit, where Δ_2 is the two-magnon excitation gap.

7 Summary

The magnetization process of the $S = 1/2$ delta chain with the anisotropic ferromagnetic interaction is investigated using the numerical diagonalization. It is found that for sufficiently large easy-axis anisotropy the spin nematic correlation dominant TLL phase appears at higher magnetization region, while the SDW dominant one at lower magnetization.

Acknowledgements

This work has been partly supported by JSPS KAKENHI, Grant Numbers 16K05419, 16H01080 (J-Physics), 18H04330 (J-Physics), JP20K03866, and JP20H05274. We also thank the Supercomputer Center, Institute for Solid State Physics, University of Tokyo and the Computer Room, Yukawa Institute for Theoretical Physics, Kyoto University for computational facilities. We have also used the computational resources of the supercomputer Fugaku provided by the RIKEN through the HPCI System Research projects (Project ID: hp200173, hp210068, hp210127, hp210201, and hp220043).

References

[1] N. Shannon, T. Momoi and P. Sindzingre, *Nematic order in square lattice frustrated ferromagnets*, Phys. Rev. Lett. **96**, 027213 (2006), doi:10.1103/PhysRevLett.96.027213.

[2] S. Nakatsuji et al., *Spin disorder on a triangular lattice*, Science **309**, 1697 (2005), doi:10.1126/science.1114727.

[3] T. Hikihara, L. Kecke, T. Momoi and A. Furusaki, *Vector chiral and multipolar orders in the spin-1/2 frustrated ferromagnetic chain in magnetic field*, Phys. Rev. B **78**, 144404 (2008), doi:10.1103/PhysRevB.78.144404.

[4] J. Sudan, A. Lüscher, and A. M. Läuchli, *Emergent multipolar spin correlations in a fluctuating spiral: The frustrated ferromagnetic spin-1/2 Heisenberg chain in a magnetic field*, Phys. Rev. B **80**, 140402(R) (2009), doi:10.1103/PhysRevB.80.140402.

[5] A. V. Chubukov, *Fluctuations in spin nematics*, J. Phys.: Condens. Matter **2**, 1593 (1990), doi:10.1088/0953-8984/2/6/018.

[6] A. Läuchli, G. Schmid and S. Trebst, *Spin nematics correlations in bilinear-biquadratic $S = 1$ spin chains*, Phys. Rev. B **74**, 144426 (2006), doi:10.1103/PhysRevB.74.144426.

[7] S. R. Manmana, A. Läuchli, F. H. Essler and F. Mila, *Phase diagram and continuous pair-unbinding transition of the bilinear-biquadratic $S = 1$ Heisenberg chain in a magnetic field*, Phys. Rev. B **83**, 184433 (2011): doi:10.1103/PhysRevB.83.184433.

[8] T. Nakamura and K. Kubo, *Elementary excitations in the Δ chain*, Phys. Rev. B **53**, 6393 (1996), doi:10.1103/PhysRevB.53.6393.

[9] F. D. M. Haldane, *Nonlinear field theory of large-spin Heisenberg antiferromagnets: Semi-classically quantized solitons of the one-dimensional easy-axis Néel state*, Phys. Rev. Lett. **50**, 1153 (1983), doi:10.1103/PhysRevLett.50.1153.

[10] P. Nightingale, *Finite-size scaling and phenomenological renormalization*, J. Appl. Phys. **53**, 7927 (1982), doi:10.1063/1.330232.

[11] T. Sakai, *Field-induced transition of the $S = 1$ antiferromagnetic chain with anisotropy*, Phys. Rev. B **58**, 6268 (1998), doi:10.1103/PhysRevB.58.6268.