Towards two- and three-loop QCD corrections to the width difference in $B_s - \bar{B}_s$ mixing

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Abstract

In this work we address the issue of large perturbative uncertainties in the theory prediction for $\Delta\Gamma_s$, the width difference in the $B_s^0 - \bar{B}_s^0$ mixing process. To this aim we complete important steps towards the full analytic result for the previously unknown Wilson coefficients from the matching between $|\Delta B| = 1$ and $|\Delta B| = 2$ effective Hamiltonians at next-to-next-leading order (NNLO) in the strong coupling constant. We provide a thorough discussion of technical and conceptual difficulties behind this computation and give an outlook regarding the availability of the new NNLO theory prediction for $\Delta\Gamma_s$.

1 Introduction

Oscillations of neutral meson systems such as $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$ and $B_q^0 - \bar{B}_q^0$ with $q = s, d$ between their matter and antimatter states are an exciting manifestation of a genuine quantum mechanical phenomenon that can be experimentally observed in collider experiments. In the case of the $B_s^0 - \bar{B}_s^0$ mixing there are three relevant physical observables for which theoretical predictions can be confronted with experimental measurements. These are ΔM_s, $\Delta\Gamma_s$ and $a_{\rm fs}^q$, which denote the oscillation frequency, the width difference and the flavor-specific CP-asymmetry respectively. In this work our main interest is devoted to $\Delta\Gamma_s$, which is related to the imaginary part of the box diagrams describing the process $b\bar{s} \to \bar{b}s$. Owing to the absence of tree-level flavor changing neutral currents (FCNCs) in the Standard Model (SM), we are dealing with a loop-induced process, where the leading-order (LO) contribution starts at one loop. One of the motivations to consider $\Delta\Gamma_s$ in more details concerns its role as a precision probe of the SM. In the Feynman diagrams relevant for this quantity possible new physics contributions may arise only in form of light beyond Standard Model (BSM) particles

that are weakly coupled to the SM sector. This is very different from ΔM_s which is on the contrary extremely sensitive to hypothetical heavy particles contributing through loops. For this reason $\Delta\Gamma_s$ is universally regarded as an important indicator for our understanding of the SM flavor sector. As such, it has attracted wide interest from theory and experiment, where the latter [1–3] has already achieved the per-cent level accuracy [4] with

$$\Delta\Gamma_s^{\text{exp}} = (0.085 \pm 0.004)\ \text{ps}^{-1}. \tag{1}$$

Unfortunately, the present day situation on the theory side is less favorable. The current most up-to-date theoretical predictions [5–10]

$$\Delta\Gamma_s^{\overline{\text{MS}}} = (0.088 \pm 0.011_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.014_{\Lambda_{\text{QCD}}/m_b})\ \text{ps}^{-1}, \tag{2}$$

$$\Delta\Gamma_s^{\text{pole}} = (0.077 \pm 0.015_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.017_{\Lambda_{\text{QCD}}/m_b})\ \text{ps}^{-1} \tag{3}$$

exhibit large uncertainties from uncalculated QCD corrections to the Wilson coefficients (denoted as "pert.") that arise in the perturbative matching between $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$ and $\mathcal{H}_{\text{eff}}^{|\Delta B|=2}$ effective weak Hamiltonians at two and three loops. The determination of these corrections by means of an explicit analytic calculation is the main goal of this work.

2 Calculation

On the $|\Delta B| = 1$ side of the matching we employ the operator basis from [11] given by

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \left[-\lambda_t^s \Big(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \Big) - \lambda_u^s \sum_{i=1}^{2} C_i (Q_i - Q_i^u) \right. \tag{4}$$
$$\left. + V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \right] + \text{h.c.},$$

with $\lambda_a^s = V_{as}^* V_{ab}$, where V_{ij} denote CKM matrix elements and G_F stands for the Fermi constant. The Wilson coefficients C_i arise from the matching between SM diagrams and the $|\Delta B| = 1$ effective theory, where all energy modes heavier than the b quark mass m_b are integrated out. We refer to [12] for the explicit definitions of the dimension-six operators Q_i. Of these, $Q_{1,2}$ are usually denoted as current operators, while $Q_{3-6,8}$ belong to the penguin category. In addition to that, the operator basis also contains evanescent operators $E[Q_i]$ [13, 14] whose matrix elements are of $\mathcal{O}(\varepsilon)$ (from $d = 4 - 2\varepsilon$) reflecting the fact that certain Dirac algebra relations such as Fierz identities are ambiguous in dimensional regularization.

To obtain $\Delta\Gamma_s \approx 2|\Gamma_{12}^s|$ we need to evaluate

$$\Gamma_{12}^s = \frac{1}{2M_{B_s}} \text{Abs} \langle B_s | i \int d^4x\ \mathcal{T}\ \mathcal{H}_{\text{eff}}^{\Delta B=1}(x) \mathcal{H}_{\text{eff}}^{\Delta B=1}(0) | \bar{B}_s \rangle, \tag{5}$$

where $\mathcal{T}$ denotes time-ordering, M_{B_s} stands for the meson mass and "Abs" specifies the absorptive part of the bilocal matrix element. The quantity Γ_{12}^s can be decomposed into [5]

$$\Gamma_{12}^s = -(\lambda_c^s)^2 \Gamma_{12}^{cc} - 2\lambda_c^s \lambda_u^s \Gamma_{12}^{uc} - (\lambda_u^s)^2 \Gamma_{12}^{uu}. \tag{6}$$

Applying Heavy Quark Expansion (HQE) [15–23] one can express Γ_{12}^{ab} as

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z)\langle B_s | Q | \bar{B}_s \rangle + \tilde{H}_S^{ab}(z)\langle B_s | \tilde{Q}_S | \bar{B}_s \rangle \right] + \mathcal{O}(\Lambda_{\text{QCD}}/m_b), \tag{7}$$
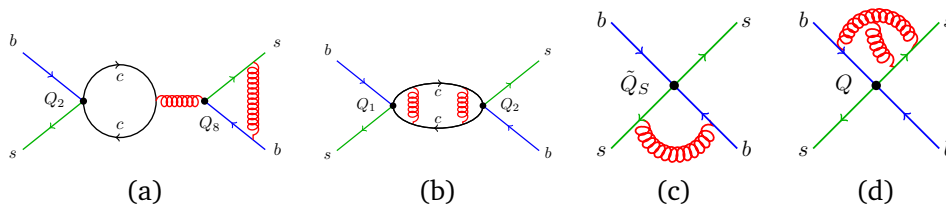
Figure 1: Sample $\Delta B = 1$ and $\Delta B = 2$ Feynman diagrams contributing to the process $b\bar{s} \to \bar{b}s$. Here (a) and (b) represent $\Delta B = 1$ contributions from $Q_2 \times Q_8$ (two loops) and $Q_1 \times Q_2$ (three loops), while (c) and (d) show one- and two-loop matrix elements of $\Delta B = 2$ operators $\tilde{Q}_s$ and Q.

with $z \equiv m_c^2/m_b^2$ and $|\Delta B = 2|$ operators

$$Q = \bar{s}_i \gamma^\mu (1 - \gamma^5) b_i \, \bar{s}_j \gamma_\mu (1 - \gamma^5) b_j, \quad \widetilde{Q}_S = \bar{s}_i (1 - \gamma^5) b_j \, \bar{s}_j (1 - \gamma^5) b_i, \tag{8}$$

where i, j stand for the color indices of the fermions. In addition to that, in intermediate expressions one encounters color-switched versions of the operators given in Eq. (8) denoted as $\widetilde{Q}$ and Q_S. Notice that the NLO $|\Delta B| = 2$ operator basis also includes evanescent operators. Last but not least, a particular linear combination of the operators Q, Q_S and $\tilde{Q}_S$ yields a $1/m_b$-suppressed operator R_0 [5, 24] given by

$$R_0 = \frac{1}{2}Q + Q_S + \widetilde{Q}_S + \mathcal{O}\left(\frac{1}{m_b}\right). \tag{9}$$

Beyond LO the matrix element of R_0 develops corrections in α_s that would not be $1/m_b$-suppressed unless R_0 is correctly renormalized. Further details regarding the $|\Delta B| = 2$ effective operators are provided in [12] and [25].

In order to achieve the NNLO accuracy in the theory prediction for $\Delta\Gamma_s$ we need to extend the knowledge of the Wilson coefficients $H^{ab}(z)$ and $\widetilde{H}_S^{ab}(z)$ to $\mathcal{O}(\alpha_s^2)$. This implies the evaluation of all two-loop diagrams with two insertions of $|\Delta B| = 1$ operators $Q_{1-6,8}$ and the computation of the current-current correlator $Q_{1-2} \times Q_{1-2}$ at 3-loops. Notice that the two-loop contribution to $Q_8 \times Q_8$ is actually $\mathcal{O}(\alpha_s^3)$, but in our framework it does not pose any additional difficulties to obtain this result as a byproduct. All the above-mentioned diagrams on the $|\Delta B| = 1$ side are then matched to one- and two-loop diagrams in the $|\Delta B| = 2$ effective theory (cf. Fig. 1). In the matching we put the b quark external momenta on-shell, while setting the masses and external momenta of the s quarks to zero. The two-loop diagrams are expanded in the charm mass up to $\mathcal{O}(z)$, while at 3-loops we content ourselves with the $z = 0$ limit.

In general, we employ dimensional regularization to handle both UV and IR divergences. However, as a cross check at two loops we also introduce a finite gluon mass m_g as an IR regulator. This is needed to ensure that all subtleties [6] related to the treatment of evanescent operators with $\varepsilon_{\text{UV}} = \varepsilon_{\text{IR}}$ (e.g. keeping $\mathcal{O}(\varepsilon^2)$ and $\mathcal{O}(\varepsilon)$ contributions at LO and NLO for the NNLO matching) are properly taken into account. The full agreement between the two-loop matching coefficients obtained from $m_g \neq 0$ and $m_g = 0$ calculations provides a nontrivial consistency check of our computations.

3 Technical details

Let us elaborate on the technical aspects behind this calculation. The generation of diagrams with genuine 4-fermion operators always requires additional care due to the relative signs

arising from the corresponding vertices. These signs are usually not handled by the diagram generator itself and need to be fixed by the code evaluating the corresponding amplitudes. A popular way to sidestep this issue consists of introducing an auxiliary field (often called σ-particle cf. e. g. [26]) that connects two fermion-fermion-σ vertices with each other. The Feynman rule for the propagator of this particle contains no denominator, while the numerator must be chosen such, that it reproduces the original color structure of the corresponding 4-fermion operator. In this case the relative sings between diagrams containing the so prepared 4-fermion operators automatically come out right. In our calculation we decided to employ both methods which serves as a cross check for the correctness of the generated amplitudes.

For the purpose of creating diagrams with explicit 4-fermion vertices we implemented suitable models in FEYNRULES [27] and exported them to FEYNARTS [28]. Notice that the inclusion of operators containing chains with more than 3 Dirac matrices requires some modifications of the FEYNRULES source code. By default, FEYNRULES chooses to simplify such chains using the Chisholm identity for 3 Dirac matrices, which is highly undesirable in our calculation. This minor technical issue can be circumvented by commenting out the corresponding routine in the `Processing` section of the file `FeynArtsInterface.m`. FEYNARTS does not fix the relative signs of 4-fermion operators on its own, so in our case this task is handled by FEYN-CALC [29–31]. This allows us to generate representative diagrams at tree- and 1-loop level and compare the resulting amplitudes to the expressions obtained using the σ-particle trick.

While FEYNARTS and FEYNCALC are mainly employed for doing cross checks and reproducing selected 1-loop results from the literature, the actual calculation relies on a FORM-based [32] framework. In the first stage we generate the required diagrams with QGRAF [33] (using σ-particles) and process them via the C++ programs Q2E/EXP [34, 35]. Here Q2E is responsible for inserting Feynman rules into the output of QGRAF and extracting the form of the graph representing each of the generated diagrams. The latter information is needed for the graph-based topology identification implemented in EXP. As an alternative to Q2E and the topology identification routines of EXP we also employ a new PYTHON-based framework called TAPIR [36]. The resulting amplitudes are processed using the FORM-based in-house CALC setup. The presence of products of two Dirac chains resulting from the 4-fermion operators necessitates a dedicated treatment of these algebraic structures. Here again we follow two different approaches that consist of constructing a set of suitable Dirac and color projectors or performing tensor reductions. More details on the construction of the projectors can be found in the appendix of [12]. For the generation of tensor reduction identities applicable to 2- and 3-loop integrals with one external momentum and tensor ranks up to 10, we employ the `Tdec` function available in FEYNCALC. Motivated by the performance bottlenecks encountered during this project, the symmetrization procedure for tensor indices implemented in `Tdec` has been rewritten from scratch using the algorithm from [37]. Furthermore, to speed up the process of solving the resulting symbolic linear equations, we augmented the public collection of interfaces between FEYNCALC and other HEP tools, known as FEYNHELPERS [38], with a link to FERMAT [39], implemented as a new function called `FerSolve`. Using these publicly available tools we were able to generate and validate all the required tensor reduction rules and export them to FORM. For the purpose of the IBP reduction [40,41] we make use of FIRE 6 [42] and LITERED [43]. Owing to the excellent performance of FIRE 6, the reduction itself does not pose any difficulties neither at two nor at three loops. For example, the reduction of about 59,000 single scale on-shell three-loop integrals appearing in our calculation can be performed in less than an hour on a laptop equipped with an 8-core CPU (AMD Ryzen 4750U) and 32 GB of RAM. All master integrals encountered in this calculation are on-shell two-point functions or tadpoles. The set of the master integrals at one- and two-loops resulting from diagrams with massless gluons does not contain anything that is not already available in the literature (cf. e. g. [46]). In particular, analytic results for all propagator-type two-loop on-shell
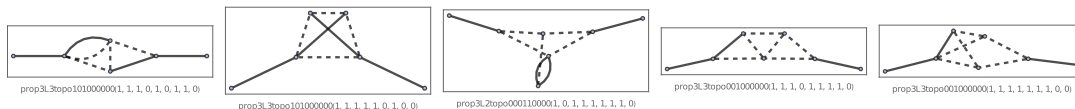
Figure 2: Some of the nonfactorizing on-shell 3-loop master integrals occurring in our matching calculation. A solid line denotes a massive propagator with m_b, while a dashed line corresponds to a massless propagator.

integrals with one mass scale can be found in [47].

The three-loop on-shell integrals with one mass scale (cf. Fig. 2) turn out to be sufficiently simple to be integrated directly from the Feynman parametric form using HYPERINT [48]. To visualize the relevant integrals and derive their Feynman parametrizations we made use of the new FEYNCALC routines FCLoopIntegralToGraph and FCFeynmanParametrize that were specifically developed for this project. A more thorough description of these functions will be provided with the official release of FEYNCALC 10 [49], although the routines themselves are already publicly available. For most of our integrals the analytic results obtained with HYPER-INT involve complicated Goncharov Polylogarithms (GPLs) [50] containing 6th root of unity that require further simplification. To this aim we employed the packages HYPERLOGPROCE-DURES [51] and POLYLOGTOOLS [52], which allowed us to arrive at very simple and compact results, at least for the imaginary parts relevant for our computation. The constants entering our expressions for the imaginary parts of the integrals are

$$\pi,\ \ln(2),\ \zeta_2,\ \zeta_3,\ \zeta_4,\ \mathrm{Cl}_2\left(\frac{\pi}{3}\right),\ \sqrt{3},\ \mathrm{Li}_4\left(\frac{1}{2}\right),\ \ln\left(\frac{1+\sqrt{5}}{2}\right), \tag{10}$$

with $\mathrm{Cl}_2(x) = \frac{i}{2}\left(\mathrm{Li}_2(e^{-ix}) - \mathrm{Li}_2(e^{ix})\right)$. Explicit analytic results for all three-loop integrals entering the NNLO prediction for $\Delta\Gamma_s$ will be given in [25]. All the obtains results for master integrals have been validated numerically using PYSECDEC [53–55] and FIESTA [56].

4 Renormalization and matching

The renormalization of the bare $|\Delta B| = 1$ and $|\Delta B| = 2$ amplitudes is done in the $\overline{\mathrm{MS}}$ scheme. However, the QCD renormalization constants alone are not sufficient to render the amplitudes UV finite. In addition to that, one must also include the operator renormalization. In general, when renormalizing Wilson coefficients W_i of our operators we have

$$(\vec{W}^{\mathrm{bare}}, \vec{W}_E^{\mathrm{bare}}) = (\vec{W}^{\mathrm{ren}}, \vec{W}_E^{\mathrm{ren}})Z \equiv (\vec{W}^{\mathrm{ren}}, \vec{W}_E^{\mathrm{ren}})\begin{pmatrix} Z_{QQ} & Z_{QE} \\ Z_{EQ} & Z_{EE} \end{pmatrix}. \tag{11}$$

Here the submatrices Z_{QQ} and Z_{EE} describe the mixing of physical and evanescent operators among themselves, while the mixing of evanescent operators into the physical ones and vice versa is governed by Z_{QE} and Z_{EQ} respectively. For the $|\Delta B| = 1$ theory in the CMM basis, the NNLO renormalization matrix Z has been computed in [57]. As far as the $|\Delta B| = 2$ theory is concerned, we are not aware of a reference that provides full Z including all four submatrices. For this reason we computed the required $|\Delta B| = 2$ renormalization matrix in a separate calculation, where the external momenta of b and s quarks were put off-shell and set to zero, while both quarks were given the same mass to regulate IR divergences. The so-obtained matrices for NLO and NNLO will be given in [58] and [25].

For calculations performed with a finite gluon mass, the UV-renormalization renders the $|\Delta B| = 1$ and $|\Delta B| = 2$ amplitudes manifestly finite i.e. free of ε-poles. In the case of massless

gluons the renormalized amplitudes still contain IR poles which, however, cancel out in the matching as they should.

Upon putting everything together we obtain final contributions to the matching coefficients for all operator insertions at two loops at $\mathcal{O}(z)$ and preliminary (up to the finite renormalization of R_0) results for the three-loop correlator $Q_{1,2} \times Q_{1,2}$ at $\mathcal{O}(z^0)$. Regarding the two-loop results we observe full agreement between our $m_g = 0$ and $m_g \neq 0$ calculations. To put our work into perspective, let us briefly enumerate the existing results and explain how they are related to the corrections obtained here.

The two-loop $Q_{1,2} \times Q_{1,2}$ contribution with full z-dependence is known since many years [5] and was initially computed in the historical $|\Delta B| = 1$ operator basis and the old $|\Delta B| = 2$ basis made of Q and Q_S operators. We reproduce this result at $\mathcal{O}(z)$ and explicitly verify the correctness of the basis change formulas given in [11] and [5] for $|\Delta B| = 1$ and $|\Delta B| = 2$ theories respectively. The same also applies to the one-loop $Q_{1,2} \times Q_{3-6}$ and $Q_{1,2} \times Q_8$ contributions from [5]. We also reproduce the one-loop result for $Q_{3-6} \times Q_{3-6}$ from [24]. The fermionic parts of the two-loop contributions $Q_{1,2} \times Q_{3-6}$ and $Q_{1,2} \times Q_8$ and of the one-loop contributions $Q_{3-6} \times Q_8$ and $Q_8 \times Q_8$ with full z-dependence were computed in [10]. The N_f-parts of our full matching coefficients agree with those results at $\mathcal{O}(z)$. Finally, we also reproduce the N_f-piece of the three-loop $Q_{1,2} \times Q_{1-2}$ correlator given in [9] (at $\mathcal{O}(\sqrt{z})$) in the $z = 0$ limit.

Therefore, apart from extending the existing incomplete results for $Q_{1,2} \times Q_{1-2}$ at three loops, $Q_{1,2} \times Q_{3-6}$ and $Q_{1,2} \times Q_8$ at two loops and $Q_{3-6} \times Q_8$ as well as $Q_8 \times Q_8$ at one loop, we also obtain new results for $Q_{3-6} \times Q_8$ and $Q_8 \times Q_8$ at two loops. Since the explicit values of the matching coefficients are too lengthy to be presented here even for $N_c = 3$, we refer to [12] for the already published results for the two-loop contribution to $Q_{1,2} \times Q_{3-6}$ and to [25, 58] for the remaining operator insertions.

In view of the still ongoing cross checks for the nonfermionic part of our three-loop result we are not yet in the position to provide an updated prediction for $\Delta\Gamma_s$. However, we observe that as far as the two-loop contributions are concerned, the largest relative shift of $\Delta\Gamma_s$ is generated by the $Q_{1,2} \times Q_{3-6}$ combination. Taking into account all current-current and current-penguin corrections up to $\mathcal{O}(\alpha_s)$ and penguin-penguin contributions up to $\mathcal{O}(\alpha_s^0)$, the ratio of the current-penguin contribution to $\Delta\Gamma_s$ evaluates to

$$\frac{\Delta\Gamma_s^{p,12\times36,\alpha_s^0}}{\Delta\Gamma_s} = 7.6\% \quad \text{(pole)}, \quad \frac{\Delta\Gamma_s^{p,12\times36,\alpha_s^0}}{\Delta\Gamma_s} = 6.1\% \quad (\overline{\text{MS}}), \tag{12}$$

Including our new two-loop current-penguin correction both in the numerator and denominator of Eq. (12) we find

$$\frac{\Delta\Gamma_s^{p,12\times36,\alpha_s}}{\Delta\Gamma_s} = 0.3\% \quad \text{(pole)}, \quad \frac{\Delta\Gamma_s^{p,12\times36,\alpha_s}}{\Delta\Gamma_s} = 1.4\% \quad (\overline{\text{MS}}). \tag{13}$$

which implies a noticeable reduction of the existing perturbative uncertainties. With "$\overline{\text{MS}}$" and "pole" we denote different ways to treat the m_b^2 prefactor in Eq. (7). We can either evaluate it in the $\overline{\text{MS}}$ or in the on-shell scheme. Notice, however, that even in the pole scheme all quantities except for the m_b^2 prefactor are handled in the $\overline{\text{MS}}$ scheme. More details on the numerical input parameters entering these comparisons can be found in [12].

5 Summary

In order to reduce perturbative uncertainties entering theory predictions for the width difference $\Delta\Gamma_s$ in the $B_s^0 - \bar{B}_s^0$ mixing, we have calculated the previously unknown QCD corrections

to this quantity at two- and three-loop accuracy for all relevant combinations of $|\Delta B| = 1$ operator insertions. Our calculation is done in a fully analytic fashion including the evaluation of new three-loop on-shell master integrals. For simplicity, we expand in the ratio m_c^2/m_b^2 up to first order at two loops and set $m_c = 0$ at three loops. In this limit we reproduce all already existing results from the literature. Since many of the literature results contain only fermionic contributions, while our computation includes both n_f- and non-n_f pieces, this work is obviously instrumental to the task of improving theory predictions for $\Delta\Gamma_s$. A part of our new matching coefficients has already been published in [12] while the remaining two- and three-loop contributions will appear in [58] and [25] respectively. For the future we also plan to increase the accuracy in m_c^2/m_b^2 by including higher orders in this parameter or possibly by calculating the relevant master integrals with full m_c dependence.

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