# Correlations between azimuthal asymmetries and multiplicity and mean transverse momentum in small collisions systems in the CGC

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## Abstract

**Considering a dilute-dense situation suitable for pA collisions, we compute in the Color Glass Condensate the correlation between azimuthal asymmetries, specifically the squared second Fourier coefficient $v_2^2$, and the total multiplicity in the event. We also analyse the correlation between $v_2^2$ and the mean squared transverse momentum of particles in the event. In both cases, we find that the correlations are generally very small, consistent with the observations. We also note an interesting sharp change in the value of $v_2^2$ and its correlations as a function of the width of the transverse momentum bin, related with a change of the dominance of Bose and HBT quantum correlations.**

## 1 Introduction

The measurement at the LHC and RHIC of many observables that behave similarly in small collision systems, pp and pA, and in heavy ion collisions, named the small system problem, is one of the key recent findings on the strong interaction, see [1] and refs. therein. Among these features, the most celebrated one is the ridge: a maximum in two particle correlations elongated along pseudorapidity and peaked at zero and $\pi$ azimuthal angle. While in heavy ion collisions such azimuthal asymmetries are taken as signatures of a collective expansion

describable by relativistic viscous hydrodynamics, in small systems this final state explanation looks tenuous and initial state ones have been essayed, see [2] and refs. therein.

Concerning correlations between azimuthal asymmetries and global characteristics of the event, the ridge in small collision systems seems to be almost independent of the multiplicity in the collision [3]. Furthermore, the correlation of azimuthal asymmetries with the average transverse momentum has been proposed as sensitive to the initial geometry in the collision and to the initial or final dynamics underlying the correlations [4–6], and measured in [7].

The aim of this contribution is to present and discuss the results on such correlations obtained in the framework of the initial state explanations provided by the CGC, see [2]. In the CGC correlations come from the Bose enhancement of gluons in the wave function of the colliding hadrons and the HBT correlations of finally produced gluons [8,9]. The work is based in the formalism developed in [10] to compute two and three gluon correlations in pA collisions. Here we show and discuss the main results, referring the reader to [11] for full details.

## 2 $v_2^2$ and correlations

In the CGC, the number of produced gluons reads

$$\frac{dN}{d^2k\,dy}\bigg|_{\rho_p,\rho_t} = \frac{2g^2}{(2\pi)^3} \int \frac{d^2q}{(2\pi)^2} \frac{d^2q'}{(2\pi)^2} \Gamma(k,q,q') \rho_p^a(-q') \big[U^\dagger(k-q')U(k-q)\big]_{ab} \rho_p^b(q), \quad (1)$$

with

$$L^i(k,q) = \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right], \quad \Gamma(k,q,q') = L(k,k-q) \cdot L(k,k-q'), \quad (2)$$

$\rho_p(p)$ a given configuration of the color charged density in the projectile, and $U(q)$ the eikonal scattering matrix – adjoint Wilson line – for scattering of a single gluon on a fixed configuration of target fields. The target Wilson lines implicitly depend on the target color sources, $\rho_t$.

Single inclusive and double inclusive gluon production are computed as

$$\frac{dN^{(1)}}{d^2k\,dy} = \left\langle\!\left\langle \frac{dN}{d^2k\,dy}\bigg|_{\rho_p,\rho_t} \right\rangle_{\!p}\right\rangle_{\!t}, \quad \frac{dN^{(2)}}{d^2k_1\,dy_1\,d^2k_2\,dy_2} = \left\langle\!\left\langle \frac{dN}{d^2k_1\,dy_1}\bigg|_{\rho_p,\rho_t} \frac{dN}{d^2k_2\,dy_2}\bigg|_{\rho_p,\rho_t} \right\rangle_{\!p}\right\rangle_{\!t}, \quad (3)$$

where the averaging is performed over the projectile and target color charge configurations.

The total multiplicity $N$ per unit of rapidity and mean transverse momentum squared per particle $\overline{k^2}$ are calculated as ($\int_\phi \equiv \int d\phi$, $\int_k \equiv \int d^2k$)

$$N = \int_k \frac{dN^{(1)}}{d^2k\,dy}, \quad \overline{k^2} = \frac{1}{N} \int_k k^2 \frac{dN^{(1)}}{d^2k\,dy}, \quad (4)$$

and the azimuthal flow harmonics $v_n^2$ defined as

$$v_n^2(k_1,k_2) \equiv \int_{\phi_1,\phi_2} e^{in(\phi_1-\phi_2)} \frac{d^2N^{(2)}}{d^2k_1\,d^2k_2} \bigg/ \int_{\phi_1,\phi_2} \frac{d^2N^{(2)}}{d^2k_1\,d^2k_2}, \quad (5)$$

where $\phi_1$, $\phi_2$ are the azimuthal angles of the corresponding transverse momenta. Below, we focus on $v_2^2$ only. In this framework, each collision event corresponds to a fixed configuration of $\rho_p$ and $\rho_t$, with the averaging introduced in (3) being equivalent to averaging over all possible events.

Studying the dependence of any observable on multiplicity would require to select from the total ensemble only events with total multiplicity in some multiplicity bin and calculate the

observable by averaging only over those events. In practice this has not yet been accomplished, so we choose to compute the correlation between $v_2^2$ and $N$, i.e. $\langle\langle v_2^2(k_1,k_2)|_{\rho_p,\rho_t} N|_{\rho_p,\rho_t}\rangle_p\rangle_t$, and similarly between $v_2^2$ and the squared transverse momentum per particle. The averaging in these expressions goes over the whole ensemble of events, and thus there is no need to consider particular sub ensembles. For that we make use of the results for two and three gluon inclusive production in pA collisions obtained in [10].

Our calculation is done using the MV model [12,13] for projectile ensembles that we consider translationally invariant, and taking only leading contributions in the number of sources (i.e., $Q_s^2 S_\perp$, the squared saturation scale times the overlap area of the collision) [14,15], see also the talk by P. Agostini in this workshop. We work at leading order in the number of colours and use the GW model for the target average of two Wilson lines [16]. In order to proceed analytically as far a possible, we compute the results at leading power of $Q_s^2/k_i^2$, with $k_i$ the transverse momenta of the measured final gluons, thus only valid for $Q_s^2/k_i^2 \ll 1$. Besides, we take only the leading correlated pieces in the two and three gluon inclusive cross section, denoted below by $Q$ and $X$ below, respectively. Finally, when required we use an IR cutoff $\lambda \sim 1/(Q_s^2 S_\perp) \sim 1/25$ (the results show small sensitivity to the exact value of this cutoff).

The correlators that we compute are defined:

$$\langle v_2^2(k_2,k_3)N\rangle \tag{6}$$
$$= \int_{\phi_2,\phi_3} e^{i2(\phi_2-\phi_3)} \int_{k_1} \frac{dN^{(3)}}{d^2k_1\,d^2k_2\,d^2k_3}\bigg|_X \bigg/ \int_{\phi_2,\phi_3} e^{i2(\phi_2-\phi_3)} \frac{dN^{(2)}}{d^2k_2 d^2k_3}\bigg|_Q \int_{k_1} \frac{dN^{(1)}}{d^2k_1},$$

$$\langle v_2^2(k_2,k_3)\overline{k^2}\rangle \tag{7}$$
$$= \int_{\phi_2,\phi_3} e^{i2(\phi_2-\phi_3)} \int_{k_1} k_1^2 \frac{dN^{(3)}}{d^2k_1\,d^2k_2\,d^2k_3}\bigg|_X \bigg/ \int_{\phi_2,\phi_3} e^{i2(\phi_2-\phi_3)} \frac{dN^{(2)}}{d^2k_2 d^2k_3}\bigg|_Q Q_s^2 \int_{k_1} \frac{dN^{(1)}}{d^2k_1},$$

with additional integrals over $k_2$ and $k_3$ in regions $[k-\Delta/2, k+\Delta/2]$ and $[k'-\Delta/2, k'+\Delta/2]$, respectively, with $k,k' \gg \Delta \sim Q_s$, both in numerators and denominators. Full explanations and detailed expressions can be found in [11].

## 3 Numerical results

In all our results we plot momentum in units of $Q_s$ and the quantities of interest multiplied by $(N_c^2-1)S_\perp Q_s^2$ ($\sim 200$ for pPb collisions) in order to exhibit the universal features of the results applicable to any target (any $Q_s$) and projectile (any $S_\perp$). We explore the interplay between the relative position of the centres of the two bins, $k$ and $k'$ and the bin width $\Delta$. $v_2^2$ receives contributions from two types of correlations: Bose and HBT. The width of the former in momentum space is naturally of order $Q_s$, while the latter have much shorter range (we took them as delta functions). Thus we expect that when $|k-k'| < \Delta$ both the HBT and Bose effects will contribute to $v_2^2$. However, when there is no overlap between the two bins, the HBT correlation should disappear. We thus expect a steep variation when $k-k' \approx \Delta$.

Fig. 1 shows our results for $v_2^2$. The left panel shows the huge dominance of the HBT contribution, by a factor $\sim 50$, and the different transverse momentum dependence of Bose and HBT, while the right panel demonstrates the expected sharp change in $v_2^2$ at the point when the width of the interval equals the distance between the interval midpoints.

Fig. 2 shows our numerical results for the correlation function between $v_2^2$ and the total multiplicity (a similar behaviour is found for the correlation of $v_2^2$ with average squared transverse momentum). Fig. 2 left shows that the normalised correlation function is strongly
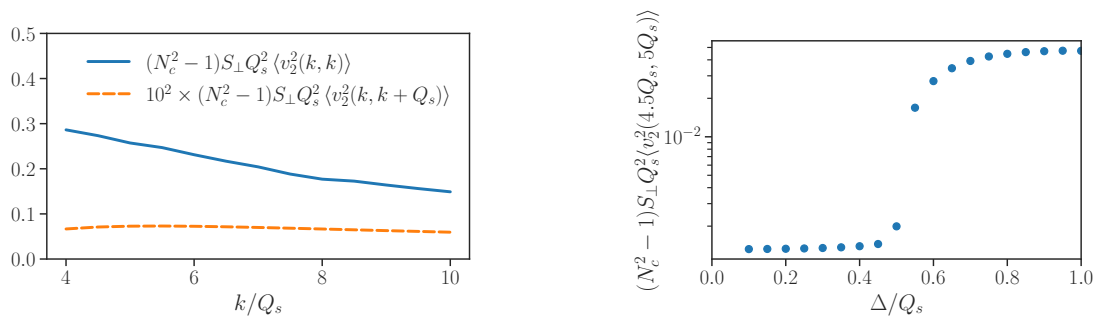
Figure 1: Left panel: The second flow harmonic, $v_2^2$ (rescaled by $(N_c^2-1)S_\perp Q_s^2$), as a function of transverse momentum. The calculation of $v_2^2$ is performed for two cases: a) the same momentum of the pair (solid blue line), b) the momentum of the pair is offset by the saturation momentum of the target in order to avoid the gluon HBT effect (multiplied by 100, dashed orange line). The bin width in both cases is $\Delta = Q_s/2$. Right panel: The second flow harmonic, $v_2^2$ (rescaled by $(N_c^2-1)S_\perp Q_s^2$), as a function of the bin width. The centres of the two bins are chosen at $k = 4.5Q_s$, $k' = 5Q_s$.

suppressed for values of bin width for which $v_2^2$ is sizeable, which is when the HBT effect in $v_2^2$ is dominant. The same effect is also demonstrated in Fig. 2 right, where we show the correlation function as a function of the bin width $\Delta$.
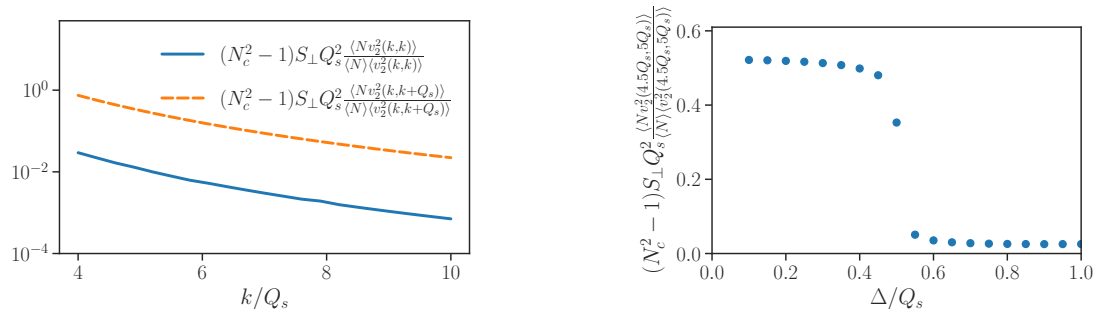
Figure 2: Id. to Fig. 1 but for the three particle correlation function $\langle v_2^2 N \rangle$ defined by the normalised correlations between $v_2^2$ and the total multiplicity of produced particles.

Finally, Fig. 3 shows $R \equiv \langle v_2^2 \overline{k^2} \rangle / \langle v_2^2 N \rangle$ as a function of transverse momentum. The correlation with transverse momentum clearly drops slower than the correlation with multiplicity.
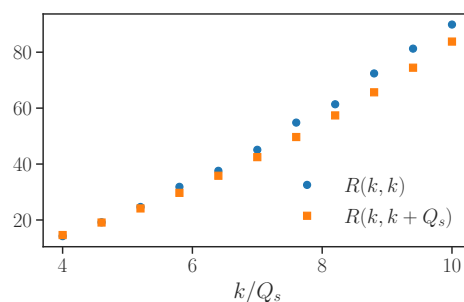
Figure 3: The ratio $R \equiv \langle v_2^2 \overline{k^2} \rangle / \langle v_2^2 N \rangle$ as a function of transverse momentum.

## 4 Conclusion

In this contribution we have examined [11] the correlation between $v_2^2$ and the total multiplicity and the squared transverse momentum per particle, in the CGC approach to multi gluon correlations in pA collisions [10]. We use several approximations, discussed in Sec. 2, to make the problem tractable at an analytical level. By examining $v_2^2$ and the mentioned correlations in different bins of transverse momenta, we find that $v_2^2$ is dominated by the HBT correlation which disappears when the bins are not overlapping and solely Bose correlations contribute. Both contributions show different transverse momentum dependences. The opposite behaviour is found for the correlations, which are suppressed in those regions where the HBT contribution is dominant. The apparent strong dependence of the correlation results on small changes in transverse momenta is due to the assumption on translational invariance of the projectile that we have employed. While our results contain many approximations and we do not aim for their phenomenological application, we observe the smallness of the correlations between $v_2$ and $N$ consistent with experimental data [3].

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