A model for F_L structure function at low values of Q^2 and x - revisited

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Abstract

A reanalysis of the model for the longitudinal structure function $F_L(x,Q^2)$ at low x and low Q^2 was undertaken, in view of the advent of the EIC. The model includes the kinematic constraint $F_L \sim Q^4$ as $Q^2 \to 0$. It is based on the photon-gluon fusion mechanism suitably extrapolated to the region of low Q^2. Revised model was critically updated, extended to the EIC kinematic region, and e.g. contains new parameterisations of the parton distribution functions.

1 Introduction

The knowledge of both F_2 and F_L structure functions, from the photoproduction to the DIS region is needed in the procedure used to extract polarised and non–polarised structure functions from the experimental data, especially in the calculations of QED radiative corrections, see for example [1]. In the context of radiative corrections, both structure functions need to be known down to $Q^2 \to 0$. Thus both F_2 and F_L will be indispensable in the data analysis, especially in the context of the future Deep Inelastic Facilities like the Electron Ion Collider EIC in the US [2]. Unlike F_2, the experimental data for F_L are rather scarce in the low Q^2 region and thus the extrapolation to low x and low Q^2 region needs to be performed. It is desirable to have a physically motivated model with least number of free parameters. Such model was proposed some time ago in [3]. It was based on the k_T factorization formula with the off-shell gluon which corresponds to the unintegrated gluon density with transverse momentum dependence. Such formulation is especially suitable for the high energy kinematics, and it resums low x contributions. The unintegrated gluon distribution function is constructed from

the collinear integrated gluon density through the logarithmic derivative over the scale dependence. The formula is then extrapolated to the low Q^2 region by introducing the cutoff on the low quark transverse momenta. The region of low quark transverse momenta is responsible for the higher twist contribution which vanishes at large Q^2. It was treated phenomenologically and its normalization was determined from the (nonperturbative) part of the structure function F_2. The model embodied the kinematical constraint $F_L \sim Q^4$ in the limit $Q^2 \to 0$ for fixed Bjorken x. In this paper we revisit a model [3] of the extrapolation of $F_L(x, Q^2)$ to the region of low values of Q^2. We have included latest parametrizations of the PDFs for the gluon densities. The dependence on the gluon PDFs is negligible in the region of interest, i.e. low values of Q^2. We have compared the model to the HERA data and found very good agreement. The comparison with the data from JLAB [5] and SLAC [6,7] show that the model underestimates the data. The region of JLAB and SLAC data however is the regime which is practically outside the region of applicability of the model, since it is rather high values of Bjorken x. Finally we discuss the outlook for possible improvements of the model.

2 Longitudinal structure function

The longitudinal structure function $F_L(x, Q^2)$ corresponds to the interaction of the longitudinally polarized virtual photon in the deep inelastic lepton-nucleon scattering. In the low x region it is dominated by the gluon density. The experimental measurement of F_L is rather challenging since it requires a measurement of the dependence on the inelasticity y of the DIS cross section for fixed values of x and Q^2, where x is Bjorken variable and Q^2 is minus photon virtuality. This requires performing measurements at varying center of mass energies of the collider. Unlike the F_2 structure function, where the experimental data are abundant, the number of F_L data points is rather limited so far.

In the 'naive' quark-parton model the structure function $F_L(x, Q^2)$ vanishes, it is a statement of the Callan-Gross relation. More precisely, the transverse momentum of the quark is limited in the naive parton model in the limit of large Q^2. This remains approximately valid in the leading logarithmic approximation while it no longer holds in the next-to-leading logarithmic approximation when the point-like QCD interactions allow (with probability $\sim \alpha_s(Q^2)$) for the average transverse momentum to grow with increasing Q^2. Thus $F_L(x, Q^2)$ acquires a leading twist contribution, proportional to the strong coupling constant $\alpha_s(Q^2)$ at this order of perturbation theory. In the limit of small x the longitudinal structure function is driven by the gluons through the $g \to q\bar{q}$ transition. Therefore F_L is a very useful quantity for a direct measurement of the gluon density in a nucleon.

In the limit $Q^2 \to 0$ the structure function F_L has to vanish as Q^4. It reflects the simple physical fact that the total cross section $\sigma_L \sim F_L/Q^2$ describing the interaction of the longitudinally polarised virtual photons has to vanish in the real photoproduction limit.

The longitudinal structure function is theoretically fairly well understood at high Q^2, thanks to the framework of perturbative QCD. On the contrary very little is known about its possible extrapolation towards the region of low Q^2 and it is possible that it contains large contributions from higher twists.
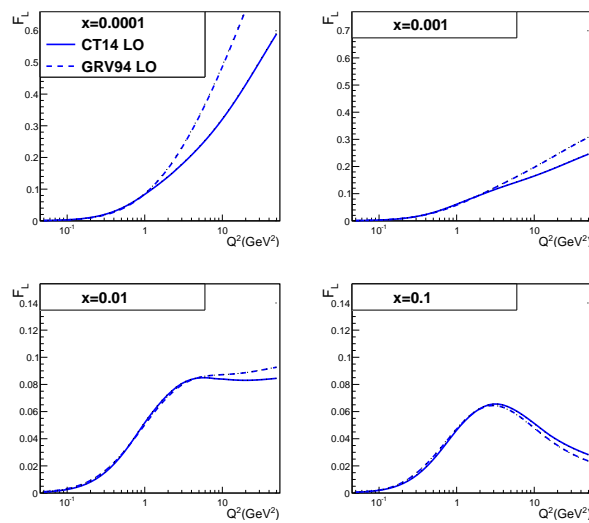
Figure 1: The results from the model for F_L for two different PDFs, CTEQ14LO and GRV94LO as a function of Q^2 for fixed values of x.

3 The model

The model [3] is based on the k_T factorization theorem

$$F_L(x,Q^2) = \int_x^1 \frac{dx'}{x'} \int \frac{dk_T^2}{k_T^2} \Phi_L(x',Q^2,k_T^2) f(\frac{x}{x'},k_T^2), \tag{1}$$

where Φ_L is the photon-gluon impact factor for the longitudinally polarized photon and f is the unintegrated gluon distribution function. The photon-gluon impact factor contains integration over the quark transverse momenta. In the model it is assumed that the k_T factorization formula applies for large transverse momenta, above certain cutoff κ_0. For small quark transverse momenta it is assumed that the gluon is on-shell, thus one makes an approximation $k_T \rightarrow 0$ and in addition one makes an ansatz that $\alpha_s(Q^2) x' g(x',Q^2) \rightarrow A$ where A is a constant. this contribution is a higher twist term. The higher twist contribution behaves as $F_L^{HT} \sim Q^4$ as $Q \rightarrow 0$ and $F_L^{HT} \sim 1/Q^2$ as $Q \rightarrow \infty$. Parameter A is not free and it was determined from the fit of the F_2 structure function, assuming that the non-perturbative contribution comes from the region of the low quark transverse momenta. This non-perturbative (soft) contribution has been set from another analysis to be around $F_2^{\text{soft}} \sim 0.4$.

The unintegrated gluon density has been obtained from the integrated gluon density through the derivative, which is a formula applicable in the region of low x $f(x,k_T^2) = \left. \frac{\partial \, xg(x,Q^2)}{\partial \ln Q^2} \right|_{Q^2=k_T^2}$.

We used standard collinear PDFs from the set CT14LO. The variation of the cutoff was in the range of $0.8-1.5$ GeV^2 and the results were not very sensitive to the cutoff.

4 Results

In Fig. 1 we show the results of the model as a function of Q^2 for fixed values of x. Two choices of the PDFs are shown, the GRV98LO and CT14LO. The earlier one was used in the previous version of the model and the latter one in the current update. We do not observe large sensitivity to the chosen PDFs. The only differences are in the region of large values
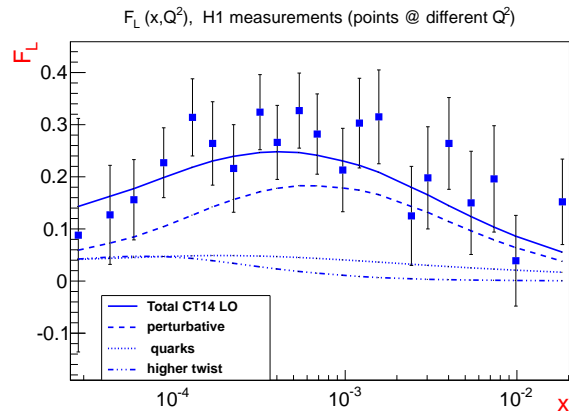
Figure 2: The results from the model for F_L for CTEQ14LO PDF resolved into various contributions and compared to the HERA data from H1 experiment [4].

of Q^2 where the perturbative component dominates. The updated calculations are slightly lower than the original model. The values of F_L are largest for smallest values of x. The longitudinal structure function also increases with Q^2 except at largest values of x where there is a maximum at a few GeV^2. This region is completely dominated by the higher twist contribution which dies off at large values of Q^2 as mentioned before.

In Fig. 2 we show a comparison of the model with the H1 data from HERA [4]. Good description of the data is found with the perturbative component being the dominant one for this kinematic region.

In Fig. 3 we show a comparison of the model with the experimental data from JLAB [5] and SLAC [6,7]. We see that the model underestimates the data in all parameter space for this data. We note however that this is the region of large x is beyond the applicability of this model
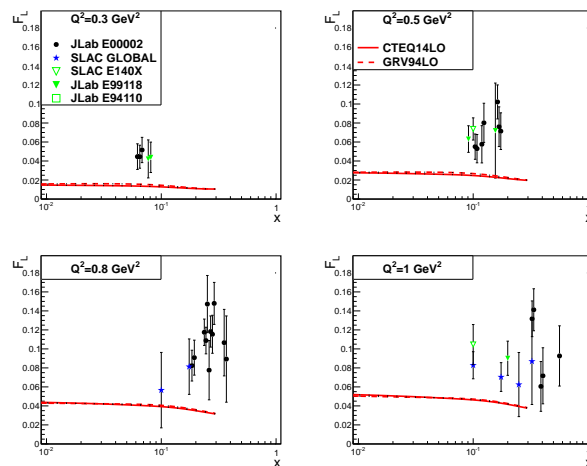
Figure 3: The results from the model for F_L for two different PDFs, CTEQ14LO and GRV94LO compared to the JLAB [5] and SLAC [6,7] data.

5 Conclusion

We have presented an update of the model for F_L at low values of Q^2 and x. The model is based on the k_T factorization with the higher twist. In the kinematic region considered, the model is not very sensitive to the choices of the PDFs. The desription of the HERA data is good. On the other hand the model underestimates the JLAB and SLAC data which are at higher values of x. Possible improvements of the model include the target mass corrections, additional higher twist terms in the large x limit and different forms of the unintegrated parton densities.

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