

Measurement of the spectral function for the $\tau^- \rightarrow K^- K_S \nu_\tau$ decay in BABAR experiment

Sergey I. Serednyakov^{1*} on behalf of the BABAR collaboration

1 Novosibirsk State University, Budker Institute of Nuclear Physics,
Novosibirsk 630090 Russia

* seredn@inp.nsk.su



Proceedings for the 15th International Workshop on Tau Lepton Physics,
Amsterdam, The Netherlands, 24-28 September 2018
doi:[10.21468/SciPostPhysProc.1](https://doi.org/10.21468/SciPostPhysProc.1)

Abstract

The decay $\tau^- \rightarrow K^- K_S \nu_\tau$ has been studied using $430 \times 10^6 e^+ e^- \rightarrow \tau^+ \tau^-$ events produced at a center-of-mass energy around 10.6 GeV at the PEP-II collider and studied with the BABAR detector. The mass spectrum of the $K^- K_S$ system has been measured and the spectral function has been obtained. The measured branching fraction $\mathcal{B}(\tau^- \rightarrow K^- K_S \nu_\tau) = (0.739 \pm 0.011(\text{stat.}) \pm 0.020(\text{syst.})) \times 10^{-3}$ is found to be in agreement with earlier measurements.



Copyright S. I. Serednyakov.
This work is licensed under the Creative Commons
[Attribution 4.0 International License](https://creativecommons.org/licenses/by/4.0/).
Published by the SciPost Foundation.

Received 16-10-2018

Accepted 17-01-2019

Published 22-02-2019

doi:[10.21468/SciPostPhysProc.1.046](https://doi.org/10.21468/SciPostPhysProc.1.046)



Check for
updates

1 Introduction

The τ lepton provides a remarkable laboratory for studying many open questions in particle physics. With a large statistics of about 10^9 τ_s produced in $e^+ e^-$ annihilation at the BABAR experiment, various aspects can be studied, for example, improving the precision of spectral functions describing the mass distribution of the hadronic decays of the τ . In this work, we analyze the $\tau^- \rightarrow K^- K_S \nu_\tau$ decay¹ and measure the spectral function of this channel defined as [1]

$$V(q) = \frac{m_\tau^8}{12\pi C(q)|V_{ud}|^2} \frac{\mathcal{B}(\tau^- \rightarrow K^- K_S \nu_\tau)}{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{1}{N} \frac{dN}{dq}, \quad (1)$$

where m_τ is the τ mass [2], $q \equiv m_{K^- K_S}$ is the invariant mass of the $K^- K_S$ system, V_{ud} is an element of the CKM (Cabibbo-Kobayashi-Maskawa) matrix [2], $(dN/dq)/N$ is the normalized $K^- K_S$ mass spectrum, and $C(q)$ is the phase space correction factor given by the following formula:

$$C(q) = q(m_\tau^2 - q^2)^2(m_\tau^2 + 2q^2). \quad (2)$$

¹Throughout this paper, inclusion of charge-conjugated channels is implied.

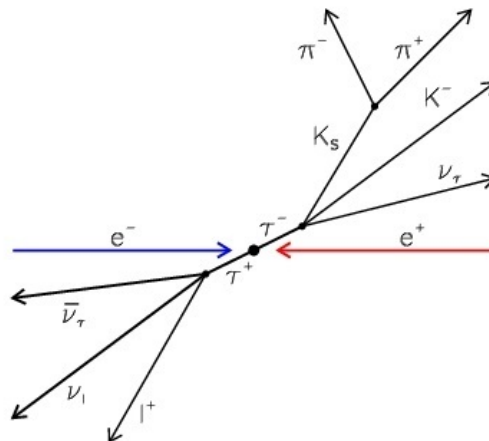


Figure 1: Schematic view of the τ decay chains in $e^+e^- \rightarrow \tau^+\tau^-$ events selected for this analysis. Lepton l^+ can be electron or muon.

The branching fraction for the $\tau^- \rightarrow K^- K_S \nu_\tau$ decay has been measured with relatively high (3%) precision by the Belle experiment [3]. The $K^- K_S$ mass spectrum was measured by the CLEO experiment [4]. In the CLEO analysis, a data set of 2.7×10^6 produced τ pairs was used, and about 100 events in the decay channel $\tau^- \rightarrow K^- K_S \nu_\tau$ were selected. In this work [5], using about $\sim 10^9$ τ leptons, we significantly improve upon the measurement of the spectral function for the $\tau^- \rightarrow K^- K_S \nu_\tau$ decay.

2 Data used in the analysis

We analyze a data sample corresponding to an integrated luminosity of 468 fb^{-1} recorded with the BABAR detector [6], [7] at the SLAC PEP-II asymmetric-energy e^+e^- collider. In the laboratory frame, the energy of electron and positron beams is 9 and 3.1 GeV, respectively.

For simulation of $e^+e^- \rightarrow \tau^+\tau^-$ events the KK2f Monte Carlo generator [8] is used, which includes higher-order radiative corrections to the Born-level process. Decays of τ leptons are simulated using the Tauola package [9]. Two separate samples of simulated $e^+e^- \rightarrow \tau^+\tau^-$ events are used: a generic sample with τ decaying to all significant final states, and the signal channel where $\tau^+ \rightarrow l^+ \nu_l \bar{\nu}_\tau$, $l = e$ or μ and $\tau^- \rightarrow K^- K_S \nu_\tau$. To estimate backgrounds, we use a sample of simulated generic $e^+e^- \rightarrow \tau^+\tau^-$ events after excluding the signal decay channel ($\tau^+\tau^-$ background) and a sample containing all events arising from $e^+e^- \rightarrow q\bar{q}$, $q = u, d, s, c$ and $e^+e^- \rightarrow B\bar{B}$ processes ($q\bar{q}$ background). The $q\bar{q}$ background events with $q = u, d, s, c$ are generated using the JETSET generator [10], while $B\bar{B}$ events are simulated with EVTGEN [11]. The detector response is simulated with GEANT4 [12]. The equivalent luminosity of the simulated sample is 2-3 times higher than the integrated luminosity in data.

3 Event selection

We select $e^+e^- \rightarrow \tau^+\tau^-$ events with the τ^+ decaying leptonically ($\tau^+ \rightarrow l^+ \nu_l \bar{\nu}_\tau$, $l = e$ or μ) and the τ^- decaying to $K^- K_S \nu_\tau$. Such events referred to as signal events below. The K_S candidate is detected in the $K_S \rightarrow \pi^+\pi^-$ decay mode. The topology of events to be selected is

shown in Fig. 1. Unless otherwise stated, all quantities are measured in the laboratory frame. The selected events must satisfy the following requirements:

- The total number of charged tracks, N_{trk} , must be four and the total charge of the event must be zero.
- Among the four charged tracks there must be an identified lepton (electron or muon) and an identified kaon of opposite charge.
- To reject non $\tau^+\tau^-$ signal backgrounds, the lepton candidate must have a momentum above 1.2 GeV/c, the momentum in the center-of-mass frame (c.m. momentum) must be smaller than 4.5 GeV/c, and the cosine of the lepton polar angle $|\cos\theta_l|$ must be below 0.9.
- To suppress background from charged pions, the charged kaon candidate must have a momentum, p_K , above 0.4 GeV/c and below 5 GeV/c, and the cosine of its polar angle must lie between -0.7374 and 0.9005.
- The two remaining tracks, assumed to be pions, form the K_S candidate. The $\pi^+\pi^-$ invariant mass must lie within 25 MeV/c² of the nominal K_S mass, 497.6 MeV/c². The K_S flight length r_{K_S} , measured as the distance between the $\pi^+\pi^-$ vertex and the collision point, must be larger than 1 cm.
- The total energy in neutral clusters, ΣE_γ , must be less than 2 GeV. Here, a neutral cluster is defined as a local energy deposit in the calorimeter with energy above 20 MeV and no associated charged track.
- The magnitude of the thrust [13, 14] for the event, calculated using charged tracks only, must be greater than 0.875.
- The angle between the momentum of the lepton and the direction of the hadronic final state in the c.m. frame should be between 110 and 180 degrees.

The chosen selection requirements are close to those used in previous τ studies in BABAR [15]. As a result of applying these cuts the τ background is suppressed by 3.5 orders of magnitude, and the $q\bar{q}$ background by 5.5 orders.

4 Detection efficiency

The detection efficiency obtained after applying the selection criteria is calculated using signal Monte Carlo simulation as a function of the true $m_{K^-K_S}$ mass. The efficiency is weakly dependent on $m_{K^-K_S}$. The average efficiency over the mass spectrum is about 13%. It should be noted that the K^-K_S mass resolution is about 2-3 MeV/c², significantly smaller than the size of the mass bin (40 MeV/c²) used in our analysis. Therefore, in the following we neglect the effects of the finite K^-K_S mass resolution.

To correct for the imperfect simulation of the kaon identification requirement, the particle identification PID efficiencies have been compared for data and simulation on high purity control samples of kaons from $D^{*+} \rightarrow \pi^+D^0$, $D^0 \rightarrow K^-\pi^+$ decays [16]. We correct the simulated efficiency using the measured ratios of the efficiencies measured in data and Monte Carlo, in bins of the kaon candidate momentum and polar angle. The resulting correction factor is small $\sim 1\%$ and weakly depends on $m_{K^-K_S}$.

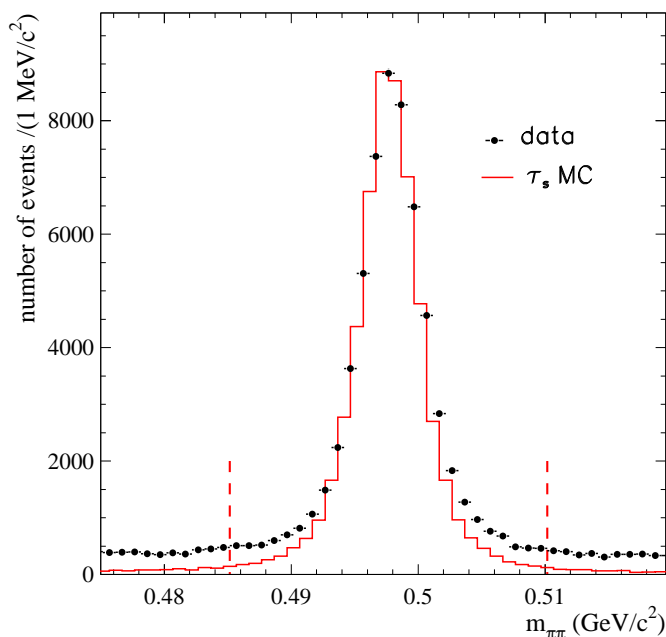


Figure 2: The $\pi^+\pi^-$ mass spectrum for K_S candidates in data (points with errors) and signal simulation (histogram). Between the two vertical lines there is a signal region used in the procedure of non- K_S background subtraction.

5 Subtraction of non- K_S background

The $\pi^+\pi^-$ mass spectra for K_S candidates in data and simulated signal events are shown in Fig. 2. The data spectrum consists of a peak at the K_S mass and a flat background. To subtract the non- K_S background, the following procedure is used. The signal region is set to $\pi^+\pi^-$ masses within $0.0125 \text{ GeV}/c^2$ of the K_S mass (indicated by arrows in Fig. 2), and the sidebands are set to between 0.0125 and $0.0250 \text{ GeV}/c^2$ away from the nominal K_S mass. Let β be the fraction of events with a true K_S that fall in the sidebands, and let α be the fraction of non- K_S events that fall in the sidebands. The total number of events in the signal region plus the sidebands, N , and the number of events in the sidebands, N_{sb} , depend on the number of true K_S , N_{K_S} , and the number of non- K_S background events, N_b according to the following relation :

$$N = N_{K_S} + N_b, \tag{3a}$$

$$N_{sb} = \alpha \cdot N_b + \beta \cdot N_{K_S}. \tag{3b}$$

Therefore:

$$N_{K_S} = (\alpha N - N_{sb})/(\alpha - \beta). \tag{4}$$

The value of β is determined using τ signal simulation. It is found to be nearly independent of the m_{K-K_S} mass and is equal to 0.0315 ± 0.0015 . The value of α is expected to be 0.5 for a uniformly distributed background. This is consistent with the value 0.499 ± 0.005 obtained on simulated $\tau^+\tau^-$ background events. The non- K_S background is subtracted in each m_{K-K_S} bin. Its fraction is found to be about 10% of the selected events with m_{K-K_S} near and below $1.3 \text{ GeV}/c^2$ and increases up to 50% above $1.6 \text{ GeV}/c^2$.

6 Subtraction of τ -background with a π^0

Although the studied process $\tau^- \rightarrow K^- K_S \nu_\tau$ is not supposed to contain a π^0 in the final state, some events from background processes with a π^0 pass the selection criteria. In the following, we describe how the π^0 background contribution is subtracted.

According to the simulation, the number of signal and τ -background events are of the same order of magnitude. The $\tau^+\tau^-$ background consists of events with the decay $\tau^- \rightarrow K^- K_S \pi^0 \nu_\tau$ (79%), events with a misidentified kaon from decays $\tau^- \rightarrow \pi^- K_S \nu_\tau$ (10%) and $\tau^- \rightarrow \pi^- K_S \pi^0 \nu_\tau$ (3%), and events with a misidentified lepton mainly from the decays $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$ and $\tau^+ \rightarrow \pi^+ \pi^0 \bar{\nu}_\tau$ (7%). Thus, more than 80% of the background events contain a π^0 in the final state. The hadronic mass spectra for τ decays with a π^0 are not well known, so we use the experimental data to subtract this background.

The τ background without a π^0 ($\tau^- \rightarrow \pi^- K_S \nu_\tau$, $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$) and $q\bar{q}$ background are simulating well. Therefore, this background is subtracted using Monte Carlo simulation.

To subtract the π^0 background, the selected events are divided into two classes, without and with a π^0 candidate, which is defined as a pair of photons with an invariant mass in the range 100 – 160 MeV/ c^2 .

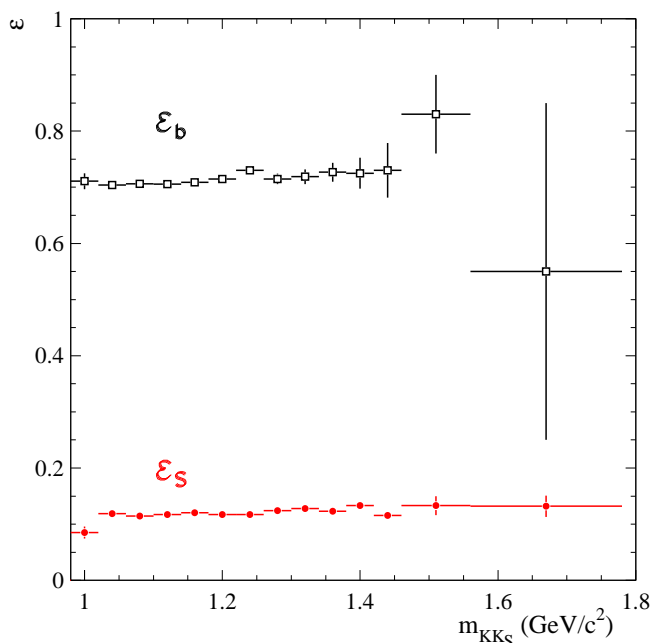


Figure 3: The probabilities ϵ_s and ϵ_b used in Eqs. (5a, 5b) as functions of the K^-K_S mass, measured on simulated events.

On the resulting sample, the numbers of signal (N_s) and background $\tau^+\tau^-$ events containing a π^0 candidate (N_b) are obtained in each $m_{K^-K_S}$ bin:

$$N_{0\pi^0} = (1 - \epsilon_s)N_s + (1 - \epsilon_b)N_b, \quad (5a)$$

$$N_{1\pi^0} = \epsilon_s N_s + \epsilon_b N_b, \quad (5b)$$

where $N_{0\pi^0}$ and $N_{1\pi^0}$ are the numbers of selected data events with zero and at least one π^0 candidate, and ϵ_s (ϵ_b) is the probability for signal (background) $\tau^+\tau^-$ events to be found in events with at least one π^0 candidate calculated using Monte Carlo simulation. The values ϵ_s

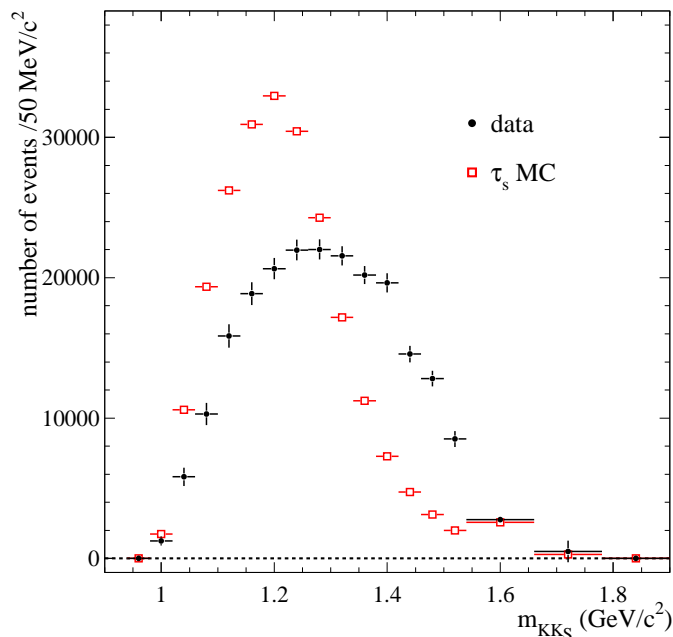


Figure 4: Measured $m_{K^-K_S}$ spectra for signal events in comparison with the Monte Carlo simulation.

and ϵ_b for each bin in $m_{K^-K_S}$ are measured in Monte Carlo by counting how many signal and background event candidates contain a π^0 candidate. Figure 3 shows the ϵ_s and ϵ_b measured in Monte Carlo as a function of $m_{K^-K_S}$. These efficiencies are corrected to take into account the difference between data and Monte Carlo.

With these corrected values for ϵ_s and ϵ_b we solve Eqs. (5a, 5b) for each K^-K_S mass bin and obtain mass spectra for signal (N_s) and background (N_b). The efficiency corrected signal mass spectrum is shown in Fig. 4 in comparison with the simulation. We find a substantial difference between data and simulation for the signal spectrum. The result is not affected by inaccuracies of the simulation since it doesn't depend on the normalization of the simulated $m_{K^-K_S}$ spectrum.

7 Systematic uncertainties

The uncertainty from non- K_S background subtraction (0.4%) is estimated by varying the coefficients of α and β within their uncertainties. This uncertainty is independent on the K^-K_S mass. The PID correction uncertainty due to data-Monte Carlo simulation difference in particle identification is taken to be 0.5%, independent of the K^-K_S mass. The uncertainty on how well the Monte Carlo simulates the tracking efficiency is estimated to be 1%. We take the observed difference between data and Monte Carlo near the end point $M_{K^-K_S} = m_\tau$ as an uncertainty on the $q\bar{q}$ background. This leads to an uncertainty on $\mathcal{B}(\tau^- \rightarrow K^-K_S \nu_\tau)$ of 0.5%. The uncertainty associated with the subtraction of the $\tau^+\tau^-$ background with π_s^0 is estimated to be 2.3%.

The systematic uncertainties from different sources are combined in quadrature. The total systematic uncertainty for the branching fraction $\mathcal{B}(\tau^- \rightarrow K^-K_S \nu_\tau)$ is 2.7%. The systematic uncertainties for the mass spectrum are listed in Table 1. They gradually decrease from $\simeq 9\%$ at $m_{K^-K_S} = 1 \text{ GeV}/c^2$ to 1.5% at $m_{K^-K_S} = m_\tau$. Near the maximum of the mass spectrum (1.3

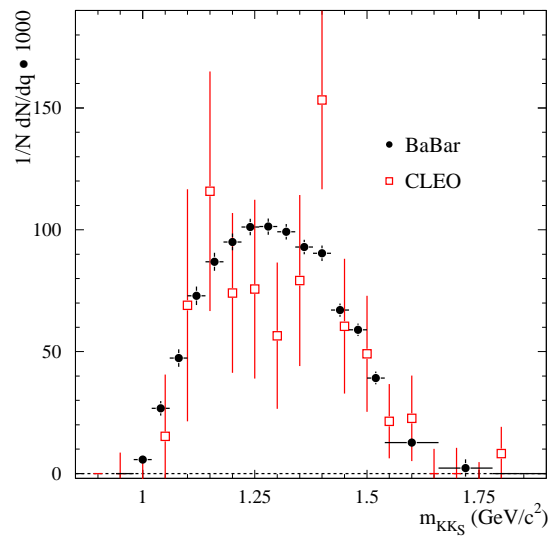


Figure 5: Normalized K^-K_S invariant mass spectrum for the $\tau^- \rightarrow K^-K_S \nu_\tau$ decay measured in this work (filled circles) compared to the CLEO measurement [4] (empty squares). Only statistical uncertainties are shown.

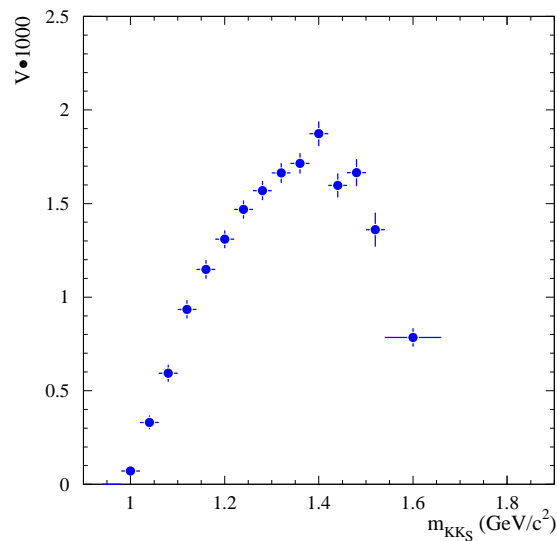


Figure 6: Measured spectral function for the $\tau^- \rightarrow K^-K_S \nu_\tau$ decay. Only statistical uncertainties are shown.

GeV/c^2) the uncertainty is about 2.5%.

8 The results

The branching ratio of the $\tau^- \rightarrow K^-K_S \nu_\tau$ decay is obtained using the following expression:

$$\mathcal{B}(\tau^- \rightarrow K^-K_S \nu_\tau) = \frac{N_{\text{exp}}}{2LB_{\text{lep}}\sigma_{\tau\tau}} = (0.739 \pm 0.011 \pm 0.020) \times 10^{-3}, \quad (6)$$

where $N_{\text{exp}} = 223741 \pm 3461$ (error is statistical) is the total number of signal events

Table 1: Measured spectral function (V) of the $\tau^- \rightarrow K^- K_S \nu_\tau$ decay, in bins of $m_{K^- K_S}$. The columns report: the range of the bins, the normalized number of events, the value of the spectral function. The first error is statistical, the second systematic.

$m_{K^- K_S} (\text{GeV}/c^2)$	$N_s/N_{tot} \times 10^3$	$V \times 10^3$
0.98 – 1.02	5.6 ± 1.4	$0.071 \pm 0.018 \pm 0.006$
1.02 – 1.06	26.0 ± 2.7	$0.331 \pm 0.034 \pm 0.026$
1.06 – 1.10	46.0 ± 3.2	$0.593 \pm 0.042 \pm 0.042$
1.10 – 1.14	70.8 ± 3.5	$0.934 \pm 0.046 \pm 0.056$
1.14 – 1.18	84.4 ± 3.4	$1.148 \pm 0.047 \pm 0.057$
1.18 – 1.22	92.3 ± 3.3	$1.309 \pm 0.046 \pm 0.052$
1.22 – 1.26	98.2 ± 3.2	$1.468 \pm 0.048 \pm 0.044$
1.26 – 1.30	98.4 ± 3.2	$1.569 \pm 0.050 \pm 0.042$
1.30 – 1.34	96.3 ± 3.0	$1.663 \pm 0.052 \pm 0.042$
1.34 – 1.38	90.2 ± 2.9	$1.715 \pm 0.052 \pm 0.039$
1.38 – 1.42	87.8 ± 3.1	$1.873 \pm 0.066 \pm 0.039$
1.42 – 1.46	65.1 ± 2.6	$1.597 \pm 0.064 \pm 0.032$
1.46 – 1.50	57.3 ± 2.5	$1.666 \pm 0.073 \pm 0.032$
1.50 – 1.54	38.1 ± 2.5	$1.361 \pm 0.090 \pm 0.023$
1.54 – 1.66	36.9 ± 2.4	$0.785 \pm 0.049 \pm 0.013$
1.66 – 1.78	6.6 ± 10.2	$0.986 \pm 1.520 \pm 0.014$

in the spectrum in Fig. 5, $L = 468.0 \pm 2.5 \text{ fb}^{-1}$ is the BABAR integrated luminosity [20], $\sigma_{\tau\tau} = 0.919 \pm 0.003 \text{ nb}$ is the $e^+e^- \rightarrow \tau^+\tau^-$ cross section at 10.58 GeV [8] and $B_{\text{lep}} = 0.3521 \pm 0.0006$ is the world average sum of electronic and muonic branching fractions of the τ lepton [2]. The first uncertainty in (6) is the statistical, the second is systematic. Our result agrees well with the Particle Data Group (PDG) value $(0.740 \pm 0.025) \times 10^{-3}$ [2], which is determined mainly by the recent Belle measurement $(0.740 \pm 0.007 \pm 0.027) \times 10^{-3}$ [3].

The measured mass spectrum $m_{K^- K_S}$ for the $\tau^- \rightarrow K^- K_S \nu_\tau$ decay is shown in Fig. 5 and listed in Table 1. Our $m_{K^- K_S}$ spectrum is compared with the CLEO measurement [4]. The BABAR and CLEO spectra are in good agreement. The spectral function $V(q)$ calculated using Eq. (1) is shown in Fig. 6 and listed in Table 1. Due to the large error in the mass interval 1.66-1.78 GeV/c^2 , which exceeds the scale of Fig. 6, the value of $V(q)$ in this interval is not shown in Fig. 6.

9 Conclusions

The $K^- K_S$ mass spectrum and vector spectral function in the $\tau^- \rightarrow K^- K_S \nu_\tau$ decay have been measured by the BABAR experiment. The measured $K^- K_S$ mass spectrum is far more precise than CLEO measurement [4] and the branching fraction $(0.739 \pm 0.011 \pm 0.020) \times 10^{-3}$ is comparable to Belle's measurement [3].

Acknowledgments

The author of this talk is grateful to V. Druzhinin and A. Lusiani for useful discussions. This work in part of data analysis is supported by the Russian Foundation for Basic Researches (grant No. 16-02-00327).

References

- [1] Y. S. Tsai, *Decay correlations of heavy leptons in $e^+e^- \rightarrow l^+l^-$* , Phys. Rev. D **4**, 2821 (1971), doi:[10.1103/PhysRevD.4.2821](https://doi.org/10.1103/PhysRevD.4.2821), [Erratum-ibid. D **13**, 771 (1976), doi:[10.1103/PhysRevD.13.771](https://doi.org/10.1103/PhysRevD.13.771)].
- [2] C. Patrignani et al. [Particle Data Group], *Review of particle physics*, Chin. Phys. C **40**, 100001 (2016), doi:[10.1088/1674-1137/40/10/100001](https://doi.org/10.1088/1674-1137/40/10/100001)
- [3] S. Ryu et al. [Belle Collaboration], *Measurements of branching fractions of τ lepton decays with one or more K_S^0* , Phys. Rev. D **89**, 072009 (2014), doi:[10.1103/PhysRevD.89.072009](https://doi.org/10.1103/PhysRevD.89.072009).
- [4] T. E. Coan et al. [CLEO Collaboration], *Decays of τ leptons to final states containing K_S^0 mesons*, Phys. Rev. D **53**, 6037 (1996), doi:[10.1103/PhysRevD.53.6037](https://doi.org/10.1103/PhysRevD.53.6037).
- [5] J. P. Lees et al. [BABAR Collaboration], *Measurement of the spectral function for the $\tau^- \rightarrow K^- K_S^0 \nu_\tau$ decay*, Phys. Rev. D **98**, 032010 (2018), doi:[10.1103/PhysRevD.98.032010](https://doi.org/10.1103/PhysRevD.98.032010).
- [6] B. Aubert et al. [BABAR Collaboration], *The BaBar detector*, Nucl. Instr. and Meth. A **479**, 1 (2013), doi:[10.1016/S0168-9002\(01\)02012-5](https://doi.org/10.1016/S0168-9002(01)02012-5).
- [7] B. Aubert et al. [BABAR Collaboration], *The BABAR Detector: Upgrades, operation and performance*, Nucl. Instr. and Meth. A **729**, 615 (2013), doi:[10.1016/j.nima.2013.05.107](https://doi.org/10.1016/j.nima.2013.05.107).
- [8] S. Jadach, B. F. Ward, and Z. Wąs, *The precision Monte Carlo event generator KK for two fermion final states in e^+e^- collision*, Comput. Phys. Commun. **130**, 260 (2000), doi:[10.1016/S0010-4655\(00\)00048-5](https://doi.org/10.1016/S0010-4655(00)00048-5).
- [9] S. Jadach, Z. Wąs, R. Decker and J. H. Kühn, *The τ decay library TAUOLA: Version 2.4*, Comput. Phys. Commun. **76**, 361 (1993), doi:[10.1016/0010-4655\(93\)90061-G](https://doi.org/10.1016/0010-4655(93)90061-G).
- [10] T. Sjöstrand, S. Mrenna, and P. Skands, *PYTHIA 6.4 Physics and Manual*, J. High Energy Phys. **05**, 026 (2006), doi:[10.1088/1126-6708/2006/05/026](https://doi.org/10.1088/1126-6708/2006/05/026).
- [11] D. J. Lange, *The EvtGen particle decay simulation package*, Nucl. Instr. and Meth. A **462**, 152 (2001), doi:[10.1016/S0168-9002\(01\)00089-4](https://doi.org/10.1016/S0168-9002(01)00089-4).
- [12] S. Agostinelli et al. *GEANT4: A Simulation toolkit*, Nucl. Instr. and Meth. A **506**, 250 (2003), doi:[10.1016/S0168-9002\(03\)01368-8](https://doi.org/10.1016/S0168-9002(03)01368-8).
- [13] E. Farhi, *Quantum chromodynamics test for jets*, Phys. Rev. Lett. **39**, 1587 (1977), doi:[10.1103/PhysRevLett.39.1587](https://doi.org/10.1103/PhysRevLett.39.1587).
- [14] S. Brandt, C. Peyrou, R. Sosnowski, and A. Wroblewski, *The Principal axis of jets. An Attempt to analyze high-energy collisions as two-body processes*, Phys. Lett. **12**, 57 (1964), doi:[10.1016/0031-9163\(64\)91176-X](https://doi.org/10.1016/0031-9163(64)91176-X)
- [15] J. P. Lees et al. [BABAR Collaboration], *The branching fraction of $\tau^- \rightarrow \pi^- K_S^0 K_S^0 (\pi^0) \nu_\tau$ decays*, Phys. Rev. D **86**, 092013 (2012), doi:[10.1103/PhysRevD.86.092013](https://doi.org/10.1103/PhysRevD.86.092013).
- [16] D. Boutigny et al. [BABAR Collaboration] *The BABAR physics book: Physics at an asymmetric B-factory*, SLAC-R-0504 (2010).

- [17] D. Epifanov et al. [Belle Collaboration], *Study of $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decay at Belle*, Phys. Lett. B **654**, 65 (2007), doi:[10.1016/j.physletb.2007.08.045](https://doi.org/10.1016/j.physletb.2007.08.045).
- [18] M. Fujikawa et al. [Belle Collaboration], *High-statistics study of the $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay*, Phys. Rev. D **78**, 072006 (2008), doi:[10.1103/PhysRevD.78.072006](https://doi.org/10.1103/PhysRevD.78.072006).
- [19] B. Aubert et al. [BABAR Collaboration], *Study of $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ process using initial state radiation with BaBar*, Phys. Rev. D **70**, 072004 (2004), doi:[10.1103/PhysRevD.70.072004](https://doi.org/10.1103/PhysRevD.70.072004).
- [20] J. P. Lees et al. [BABAR Collaboration], *Time-integrated luminosity recorded by the BABAR detector at the PEP-II e^+e^- collider*, Nucl. Instr. and Meth. A **726**, 203 (2013), doi:[10.1016/j.nima.2013.04.029](https://doi.org/10.1016/j.nima.2013.04.029).