Determination of α_S beyond $NNLO$ using the event shape averages

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Abstract

In this proceedings we discuss a prescription to extract the QCD strong coupling constant at N^3LO precision in perturbative QCD using a combination of $\mathcal{O}(\alpha_S^3)$ calculations in pQCD and estimations of the $\mathcal{O}(\alpha_S^4)$ corrections from the data. The method is applied to a set of event shape averages measured in experiments at the LEP, PETRA, PEP and TRISTAN colliders. In our analysis we account for hadronization effects with models from modern Monte Carlo event generators and analytic hadronization models. We conclude that the precision of the α_S extraction cannot be improved significantly only with pQCD predictions of higher orders, and further progress in these studies requires a significant advances in the studies and modeling of hadronization process.

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1 Introduction

The process of hadroproduction in e^+e^- annihilations is one of the best environments for verification theoretical predictions of Quantum Chromodynamics (QCD). In the past, multiple comparisons of experimental measurements of event shape and jet observables to perturbative QCD (pQCD) predictions were performed. All of these comparisons were done using the data from now retired experiments. Due to an absence of active high-energy e^+e^- experiments new data will not be available in the next decade(s) and the improvements in QCD studies in e^+e^- collisions will depend only on the advances of the theory (and phenomenology). However, most QCD studies of the hadroproduction in e^+e^- with the available data show relatively low impact of the experimental uncertainties on the results in comparison to the pQCD- and modeling-related uncertainties.

In this situation it is interesting if pQCD calculations and/or resummation techniques will be able to improve the precision of the results (e.g. $\alpha_S(M_Z)$) without any new data. And if not, what would be the limiting factors for the precision of QCD studies in the future and what should be done to eliminate them? To answer these questions we perform an extraction of $\alpha_S(M_Z)$ using estimations of higher-order corrections.

As of 2021, the calculations for the $e^+e^- \to Z/\gamma \to jets$ process are available in high precision in pQCD, i.e. the fully differential predictions for the $e^+e^- \to Z/\gamma \to 3jets$ process are available at $\mathcal{O}(\alpha_S^3)$ and the total cross-section $e^+e^- \to Z/\gamma \to jets$ at $\mathcal{O}(\alpha_S^4)$ [1–3]. Obviously, the impact of higher order corrections on the QCD analyses (e.g. extraction of α_S) has a significant interest.

With a sufficient amount of data, proper selection of desired observables it is possible to go beyond the pQCD accuracy of predictions available from the exact calculations. This could be done with simultaneous fits of $\alpha_S(M_Z)$ and the $\mathcal{O}(\alpha_S^4)$ coefficients not available in the exact pQCD predictions. The approach is obviously limited to cases with only a small number of coefficients of the perturbative expansion to be estimated. In these proceeding we describe the results of implementation of this approach using the averages of thrust and C-parameter event shape observables. The argumentation for the choice of observables is given in Ref. [4].

2 Theory predictions and hadronization models

The experimentally measured averages of event shape observables (i.e. first moments) O, $\langle O^1\rangle$ are normalized to the total hadronic cross section, and the perturbative expansion of predictions for these quantities up to $\mathcal{O}(\alpha_S^4)$ in the massless QCD[1] reads:

$$\langle O^1\rangle = \frac{\alpha_S(\mu_0)}{2\pi}\bar{A}_0^{\langle O^1\rangle} + \left(\frac{\alpha_S(\mu_0)}{2\pi}\right)^2 \bar{B}_0^{\langle O^1\rangle} + \left(\frac{\alpha_S(\mu_0)}{2\pi}\right)^3 \bar{C}_0^{\langle O^1\rangle} + \left(\frac{\alpha_S(\mu_0)}{2\pi}\right)^4 \bar{D}_0^{\langle O^1\rangle}.$$

In the later expression the coefficients $\bar{A}_0^{\langle O^1\rangle}$, $\bar{B}_0^{\langle O^1\rangle}$, $\bar{C}_0^{\langle O^1\rangle}$ [4] are known exactly and the coefficient $\bar{D}_0^{\langle O^1\rangle}$ can be extracted from a simultaneous fit of multiple data points at different center-of-mass energies performed with four-loop running of $\alpha_S(\mu)$. In the fit procedure it is also essential to take into account the hadronization effects and we consider two types of models to handle this problem. Namely, we consider the Monte Carlo event generator (MCEG) models generating predictions at NLO accuracy in pQCD and in addition to that we consider analytic hadronization models [4] extended to $\mathcal{O}(\alpha_S^4)$ for the first time.

The hadron level predictions by the MCEGs for the averages of event shape observables [4] reasonably well describe the data for a wide range of center-of-mass energies. Contrary, the corresponding quantities at the parton level are reasonable only for $\sqrt{s} > 29$ GeV. Therefore, as the analysis was aiming also at comparison of the used hadronization models, the fits were performed only to the data with $\sqrt{s} > 29$ GeV. The procedure to correct the pQCD predictions for hadronization effects with the MCEGs models consisted of applying $\sqrt{s}$-depending factor which was the ratio of values of event shape averages on the hadron and parton levels, see Fig. 1.

The models based on the dispersive model of analytic hadronization corrections for event shapes [5] differ in their application from the models based on the MCEG approach. The dispersive model predicts the hadron-level differential distributions of the event shapes can be obtained from the pQCD differential distributions with a simple shift $\frac{d\sigma_{\text{hadrons}}(O)}{dO} = \frac{d\sigma_{\text{partons}}(O-a_O\mathcal{P})}{dO}$, where the power correction $\mathcal{P}$ being universal for all event shapes, and a_O being specific known

[1] The prescription on the treatment of effects related to massive b-quarks up to NLO given in Ref. [4].

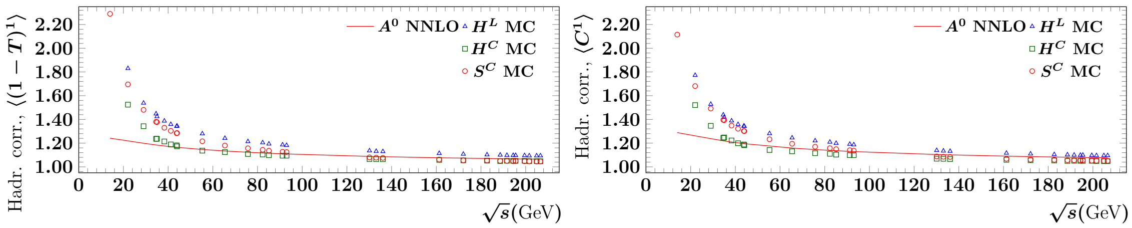

Figure 1: Multiplicative hadronization corrections extracted from MCEGs and analytic ($\mathcal{A}_0$ scheme) hadronization models [4]. Figures from Ref. [4].

constants [4]. As a result, $\langle O^1\rangle_{\text{hadrons}} = \langle O^1\rangle_{\text{partons}} + a_O\mathcal{P}$, with $\langle O^1\rangle_{\text{partons}}$ obtained as described in Sect. 2. The expression $\mathcal{P}$ depends on theoretically calculable constants, namely the so-called "Milan factor" and the value of effective coupling below the low fixed scale $\mu_I = 2\,\text{GeV}$, $\alpha_0(\mu_I)$, which is a non-perturbative parameter of the dispersive model and can be related to the effective soft coupling α_S^{CMW} (Catani-Webber-Marchesini scheme). The relation between the strong coupling defined in the $\overline{\text{MS}}$ scheme and the effective soft coupling α_S^{CMW} is scheme-dependent and complex at higher orders [6,7]. However, in one particular scheme, the relation between α_S and α_S^{CMW} has recently been computed up to $\mathcal{O}(\alpha_S^4)$ accuracy [7] (A^0 scheme), which allow for an implementation of a consistent analytic model of hadronization corrections at order that matches the order of pQCD predictions. For the details of the implementation see Ref. [4]. For the qualitative comparison with the models based on the MCEG approach, the corrections obtained with the A^0-scheme are transformed into multiplicative factors and presented in Fig. 1.

3 Results and discussion

The values of $\alpha_S(M_Z)$ were determined in the optimization procedures as described in Ref. [4] using multiple data sets from e^+e^- collision experiments at LEP, PETRA, PEP and TRISTAN colliders. The multiple numerical results of the NNLO and N^{3}LO fits are presented in Ref. [4], while the predictions of the N^{3}LO fits for individual energy points are shown in Fig. 2. The

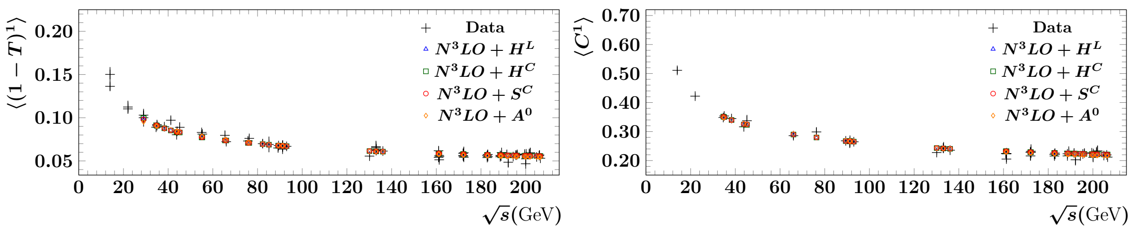

Figure 2: Data and fits to the data obtained with different types of hadronization models. Figures from Ref. [4].

NNLO results for $\alpha_S(M_Z)$ obtained with all MCEG and analytic hadronization models are in good agreement between the fits to $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$, which can be viewed as a check of the consistency of the $\alpha_S(M_Z)$ extraction method at NNLO. Similarly to previous studies [8], a discrepancy between the results obtained with the MCEGs hadronization models and the analytic hadronization models are seen. While the parameters $\alpha_0(2\,\text{GeV})$ in different schemes do not represent the "same" quantity and cannot be compared compared, their numerical values obtained in the fits are numerically very similar.

The obtained results with N^{3}LO predictions have similar patterns to the results obtained with the NNLO predictions. However, as expected, the obtained uncertainties are somehow larger than for the corresponding quantities obtained with NNLO predictions. The values $D^{\langle(1-T)^1\rangle}$ and $D^{\langle C^1\rangle}$ obtained with different hadronization models reasonably agree with each other, which could serve as an indirect evidence that the higher-order coefficients $D^{\langle O^n \rangle}$ can be extracted from the data even with higher precision if more data would be available. In the same time, the differences between the $\alpha_S(M_Z)$ values obtained with different types of hadronization models are preserved even at N^{3}LO accuracy. This suggests that the discrepancy pattern has a fundamental origin and will hold in the future analyses even with more data and exact N^{3}LO predictions available.

Hereby, improvement of the hadronization modeling and a better understanding of hadronization itself is more important for increasing the precision of $\alpha_S(M_Z)$ extractions than the calculation of perturbative corrections beyond NNLO.

4 Conclusions

We discussed the extraction of the $\alpha_S(M_Z)$ from available data on the averages of event shapes $\langle(1-T)^1\rangle$ and $\langle C^1 \rangle$ in N^{3}LO and NNLO accuracy in pQCD using different types of hadronization models. The results obtained using NNLO predictions and analytic hadronization corrections based on the dispersive model are consistent with the recent world average.

The method of extraction of $\alpha_S(M_Z)$ in N^{3}LO precision in pQCD uses a combination of NNLO predictions calculated from the first principles and estimations of the N^{3}LO contributions from the data. The method produced results which are compatible with the current world average within the somewhat large uncertainties, e.g. the result from the fits to the $\langle(1-T)^1\rangle$ data reads $\alpha_S(M_Z)^{N^3LO+A^0} = 0.12911 \pm 0.00177(exp.) \pm 0.0123(scale)$. The obtained precision can be increased with more high-quality data from future experiments.

In the discussed analysis the hadronization corrections were derived from the MCEGs models and analytic hadronization models extended to higher orders for the first time. The results obtained with those two approaches imply that future analyses will be strongly affected by the hadronization effects even if the exact higher -order corrections will be available.

However, in the last decades the developments in the modeling of particle collision by MCEGs were driven by the need to model the processes at high energies of LEP, HERA and LHC colliders and therefore had limited impact on the description of phenomena at lower energies. Similarly, it is expected that rapid developments in the modeling of particle collisions at lower energies and understanding of hadronization can be expected only with the availability of new measurements in the corresponding (lower) energy ranges. In the context of future e^+e^- colliders it can be achieved with a program of measurements of the hadronic final state properties at $\sqrt{s} \approx 20-50$ GeV performed with radiative events or in dedicated collider runs.

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