Hadron production in terms of Green's functions in non-equilibrium matter

Andrew Koshelkin*

National Research Nuclear University (MEPhI), Moscow, Russia

$\star$ and.kosh59@gmail.com

Abstract

Following the quark-hadron duality concept, we show that the number of hadrons generated in the deconfinement matter is entirely determined by the exact non-equilibrium Green's functions of partons in the medium and the vertex function governing the probability of the confinement-deconfinement phase transition. In such an approach, compactifying the standard (3+1) chromodynamics into $QCD_{xy} + QCD_{zt}$, the rate of the hadrons produced in particle collisions is derived in the explicit form provided that the hadronization is the first order phase transition. The pion production is found to be in good agreement to the experimental results on the pion yield in pp collisions.

1 Introduction

The hadronization of the deconfined matter arising in high-energy particle collisions plays a key role in hadron production in *pp, pA* and *AA* collisions. Such a problem is however an extremely complicate to be solved in the unified approach, starting from the fundamental theory of QCD that leads to development of various models describing the hadronization of the deconfined matter, whose applicability depends essentially on the energies of colliding particles. One of them is the color flux tube approach based on the longitudinal dominance and transverse confinement. The hadron production is found to be the leading process in the hadron generation at the low and intermediate-p_T region in high-energy e^+e^- annihilations and pp collisions [1-7], where the tube arises between a quark and antiquark in e^+e^- annihilation reaction, and between the valence quarks and antiquarks in nucleon-nucleon collisions, respectively.

2 Hadron production in quark-hadron duality concept

The key assumption we follow in the derivation of hadron rate is the concept of the quark-hadron duality [8]. The central point of this concept is that partons are the same particles in the confinement and deconfinement phases of the strong interaction matter. Then, the probability of the hadronization is proportional to the projection of the state vector of partons $|q_{deconf} >$ in the deconfinement matter on the such a vector determining the parton states $|q_{conf} >$ in the confinement medium

$$\mathcal{M} = < out|in > = < \bar{q}_{deconf}|q_{conf} >, \tag{1}$$

where $|q_{deconf} >$ and $|q_{conf} >$ mean the exact dressed quark states in the corresponding matters. When the transition of the quark states is governed by some unitarian operator U

$$|q_{conf} > = U|q_{deconf} >, \tag{2}$$

then, the matrix element given by Eq.(1) is

$$\mathcal{M} = < \bar{q}_{deconf}|U|q_{deconf} > . \tag{3}$$

When $|q_{deconf} >$ describes a single quark state the squared matrix element is expressed in terms of the single particle Green's function in a non-equilibrium medium $G^{-+}(x_1, x_2)$ [9]

$$< |\mathcal{M}|^2 > = -Tr\left\{ (U_2^\dagger \gamma^0 G_{21}^{-+})(\gamma^0 U_1 G_{12}^{-+}) \right\} = Tr\left\{ |\gamma^0 U_1 G_{12}^{-+})|^2 \right\}, \tag{4}$$

where the trace symbol means summing with respect to all quantum number, including integration over x_1 and x_2, and averaging with respect to the deconfinement quark vacuum; the indexes denote the coordinate x and spin σ variables, $1 = (\sigma, x)$. The subscribe at U implies acting on the corresponding variable.

In the momentum representation we have the following

$$\frac{d < |\mathcal{M}|^2 >}{dp} = -Tr\left\{ \int \frac{dq}{(2\pi)^8} \bar{G}^{-+}(p+q/2) \, \varrho(p+q/2, p-q/2) \, G^{-+}(p-q/2) \right\}, \tag{5}$$

where $\varrho(p+q/2, p-q/2) \, G^{-+}(p-q/2)$ is the probability to couple a quark-antiquark pair into a hadron whose 4-momentum is p.

Provided that the created hadron is on-shell , we obtain for the number of hadrons

$$\frac{E(\boldsymbol{p})dN_h}{d^3p} = \int dp^0 (E(\boldsymbol{p})) \frac{d < |\mathcal{M}|^2 >}{dp}, \tag{6}$$

where $E(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 + m_h^2}$ and $\boldsymbol{p}$ are the hadron energy and momentum, respectively, m_h is its mass.

3 Hadron rate in longitudinal dominance approach

The longitudinal dominance and the transverse confinement in terms of the $\text{QCD}_{xy}+\text{QCD}_{zt}$ dynamics [14] means the factorization in Eq.(5) with respect to the coordinate x, y and z, t, so that the combined p_T and rapidity distribution can be derived by overlapping the Wigner functions corresponding to the parton motion in the transverse and longitudinal direction with the probability of hadron production. Then, going to wave function in Eqs.(5), (6), assuming the standard relation between them [9] for both quarks and antiquarks, we obtain

$$\frac{dN_h}{d^3p} = \int \frac{d^3q}{(2\pi)^6} \varrho(\boldsymbol{p};\boldsymbol{q}) \left| \Psi_{\bar{q}}\left(\boldsymbol{p}_\perp + \frac{\boldsymbol{q}_\perp}{2}\right) \Psi_q\left(\boldsymbol{p}_\perp - \frac{\boldsymbol{q}_\perp}{2}\right) \Psi_{\bar{q}}\left(p_z + \frac{q_z}{2}\right) \Psi_q\left(p_z - \frac{q_z}{2}\right) \right|^2, \quad (7)$$

where $\Psi_{q,\bar{q}}(\boldsymbol{p})$ is the wave function of a quark (antiquark) in the momentum representation.

The convolution with respect to $\vec{q}_\perp$ in Eq.(7) is some function of $\boldsymbol{p}$ and q_z which can be interpreted as the probability to create a hadron with the momentum $\boldsymbol{p}$, so that the difference in the momenta quark and antiquark, constituating this hadron, is q_z. Since a boson is created due coupling quarks with the same q_z, such a convolution is

$$\rho(\boldsymbol{p}, q_z) = \int \frac{d^2q_\perp}{(2\pi)^6} E(\boldsymbol{p}) \varrho(\boldsymbol{p};\boldsymbol{q}) |\Psi_{\bar{q}}(\boldsymbol{p}_\perp + \boldsymbol{q}_\perp/2) \Psi_q(\boldsymbol{p}_\perp - \boldsymbol{q}_\perp/2)|^2 \equiv \mathcal{P}(\boldsymbol{p}) \delta(q_z), \quad (8)$$

where $\mathcal{P}_a(\boldsymbol{p})$ is a probability to create a boson with the momentum $\boldsymbol{p}$ which is assumed to be governed by the first order phase transition, so that

$$\mathcal{P}(\boldsymbol{p}, y) \propto \exp\left(-\frac{\Delta G(E_t) - \Delta g(T)}{T}\right), \quad \Delta g(T) = \alpha \ln\left(\frac{C_{q_h}(T)}{C_{q_q}(T)}\right), \quad (9)$$

where ΔG is the change of the Gibbs thermodynamic potential due the transition of particles from one phase to another, which is equal to the transverse hadron energy E_t. The term $\Delta g(T)$, where $C_{q_h}(T)$ and $C_{q_q}(T)$ are the quark concentrations in the deconfinement and confinement phases, guarantees correct behavior of the probability of the phase transition with respect to the transition temperature T_c. When temperature $T > T_c$ the concentration of quark in the hadronic phase is equal to zero that leads to $\mathcal{P}_a(\boldsymbol{p}_\perp, y) \equiv 0$ due to the logarithm. In the opposite case, when $T \leq T_c$ the number of quarks in the hadronic and deconfined phases are the same, so that the probability of the phase transition has the finite, well-defined magnitude.

As for the functions $\Psi_{q,\bar{q}}(p_z)$, they are governed by the $(1+1)$ Dirac equation [14] which fundamental solution, having been written in terms of the kinematic rapidity η and the proper time τ [7], are in the massless case

$$\psi_\pm(\tau, \eta) = \frac{\exp\left(-\frac{(\eta \mp \ln(\tau/\tau_0))^2}{2\sigma^2}\right)}{\sqrt{\sigma \pi^{1/2}}}, \quad \psi(\tau = \tau_0, \eta) = \frac{\exp\left(-\frac{\eta^2}{2\sigma^2}\right)}{\sqrt{\sigma \pi^{1/2}}}, \quad (10)$$

where is assumed that at the initial time $\tau = \tau_0 + 0$ the parton has the rapidity localized near $\eta = 0$, mainly inside the interval of the order of σ. Substituting Eqs.(8)-(10) into Eq.(7) we obtain the hadron distribution $dN_h/dy\,d^2p_\perp$ with respect to the transverse momentum $p_\perp$ and rapidity y

$$\frac{dN_h}{dy\,d^2 p_\perp} = \frac{<N_{ch}(s)> \exp(m_h/T)\theta(T_c - T)}{(2\pi)^{3/2}(1+(T/m_h))\sum\limits_{a=1}^{N}\sigma_a^{-1}} \sum_{a=1}^{N} \frac{\exp\left(-\frac{E_t}{T}\right)}{m_h T \sigma_a^2}$$

$$\left\{ \exp\left(-\frac{2(y-y_a)^2}{\sigma_a^2}\right) + \exp\left(-\frac{2(y+y_a)^2}{\sigma_a^2}\right) \right\}, \tag{11}$$

where m_h is a hadron mass, T and T_c are the matter temperature and the phase transition temperature, respectively, $\theta(z)$ is the unit step function, σ_a and y_a are the parameter related to the initial beams of particles so that, $y_a = y_{beam}/2$ where y_{beam} is the rapidity of the initial beam which is incident on a rest target. In this way $\sigma_a = \sigma$ are fitted by a formula

$$\frac{y_b}{2} = \left(\int\limits_{-\infty}^{+\infty} dy |\psi_{a\pm}(y)|^2 |y^2\right)^{1/2} = \frac{\sigma}{\sqrt{2}}, \tag{12}$$

whereas the obtained rate is assumed to be normalized by the total hadron multiplicity $< N_{ch}(s) >$.

The summation with respect to a in the above formula arises since in the LUND [11], which we follow here, the creation of hadron occurs due the fragmentation of the initial tube into secondary ones which are virtual hadron in the deconfinement matter, and hereby [11] $y = \eta$. The results of the comparison of the derived hadron distribution with the experimental results on pion production in pp collisions are presented in Figs.1,2. In Fig.1 the lines of various types
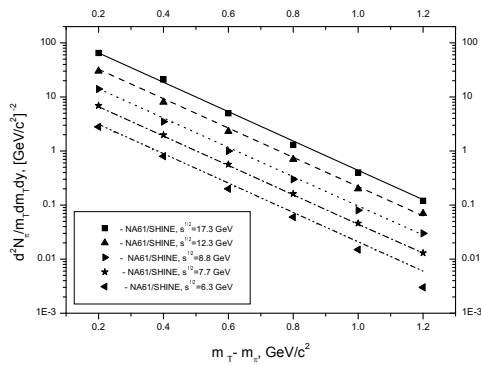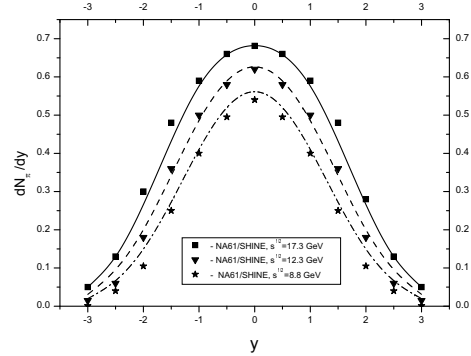

Figure 1

Figure 2

are the p_T and rapidity distributions of pions coming from Eq.(11) at $T_c = 160 MeV$, and are normalized by the experimental value of the pion rate at $(m_T - m_\pi) = 0.2 GeV/c$, v.s. the pion rate in pp collisions [6](the scattered symbols) at the same projectile energies, where $m_T = \sqrt{p_T^2 + m_\pi^2}$, whereas m_π is the pion mass. In Fig2. the rapidity distributions coming from Eq.(11) at $\sqrt{s} = 17,3 GeV$ (solid lines), $\sqrt{s} = 12,3 GeV$ (dashed line), $\sqrt{s} = 8,8 GeV$ (dot-dashed line), and at $T_c = 160 MeV$ v.s. the rapidity distributions in pp collisions [6].

4 Conclusion

Based on the quark-hadron duality concept [8] we study the hadronization of the deconfinement matter in the case of a single quark-antiquark coupling. The hadron rate is found to be

expressed in terms of the exact quark Green's functions in non-equilibrium matter and of the probability of the first-order equilibrium phase transition. Based on the $QCD_{xy}+QCD_{zt}$ compactification [14] we derived both the p_T and rapidity distributions of hadrons in the explicit form, provided that the hadronization is the equilibrium first-order phase transition. When hadrons are the pions generated in the proton collisions of intermediate energies, we have compared the hadron rate with the experimental results [6], and have gotten to a good relation to the experimental data for all proton energies used in the experiment. We should note that the developed approach should be improved to get to a good relation to experimental data in the cases of p-A and A-A collisions that is in the first turn connected with necessary to take into account of both multiple collisions of partons and collective modes excited in nuclei.

References

[1] B. I. Abelev et al., *Strange particle production in pp collisions at $\sqrt{s} = 200 GeV$*, Phys. Rev. C **75**, 064901 (2007), doi:10.1103/PhysRevC.75.064901.

[2] A. Adare et al., *Nuclear modification factors of ϕ mesons in d+Au,Cu+Cu, and Au+Au collisions at $\sqrt{s} = 200 GeV$*, Phys. Rev. C **83**, 024909 (2011), doi:10.1103/PhysRevC.83.024909.

[3] G. Aad et al., *Charged-particle multiplicities in pp interactions measured with the ATLAS detector at the LHC*, New J. Phys. **13**, 053033 (2011), doi:10.1088/1367-2630/13/5/053033.

[4] V. Khachatryan et al., *Charged particle transverse momentum spectra in pp collisions at $\sqrt{s} = 0.9$ and $7 TeV$*, J. High Energy Phys. **08**, 086 (2011), doi:10.1007/JHEP08(2011)086.

[5] K. Aamodt et al., *Transverse momentum spectra of charged particles in proton-proton collisions at $\sqrt{s} = 900 GeV$ with ALICE at the LHC*, Phys. Lett. B **693**, 53 (2010), doi:10.1016/j.physletb.2010.08.026.

[6] N. Abgrall et al., *Measurement of negatively charged pion spectra in inelastic p+p interactions at $p_{lab} = 20, 31, 40, 80$ and 158 GeV/c*. Eur. Phys. J. C. **74** , 2794 (2014), doi:10.1140/epjc/s10052-014-2794-6.

[7] C.-Y. Wong, *Introduction to High-Energy Heavy-Ion Collisions*, World Scientific, ISBN 9789810202637 (1994), doi:10.1142/1128.

[8] W. Melnitchouk, R. Ent and C. E. Keppel, *Quark–hadron duality in electron scattering*, Phys. Rep. **406**, 127 (2005), doi:10.1016/j.physrep.2004.10.004.

[9] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics*, Pergamon Press (1980).

[10] S. Dalley, *Adjoint QCD_2 in Two-Dimensions and Non-Abelian Schwinger Mechanism*, Phys. Lett. B **418**,160 (1998), doi:10.1016/S0370-2693(97)01474-3.

[11] B. Andersson, G. Gustafson, G. Ingelman and T. Sjöstrand, *Parton fragmentation and string dynamics*, Phys. Rep. **97**, 31 (1983), doi:10.1016/0370-1573(83)90080-7.

[12] L. McLerran, *The Color Glass Condensate and Small-x Physics*, Lect. Notes Phys. **583**, 291 (2002), doi:10.1007/3-540-45792-5-8.

[13] L. McLerran, *The CGC and the Glasma: Two Lectures at the Yukawa Institute*, Prog. Theor. Phys. Suppl. **187**, 17 (2011), doi:10.1143/PTPS.187.17.

[14] A. V. Koshelkin and C.-Y. Wong, *The compactification of QCD_4 to QCD_2 in a flux tube*, Phys. Rev. D **86**, 125026 (2012), doi:10.1103/PhysRevD.86.125026.