Improved treatment of dark matter capture in compact stars

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Abstract

Compact stellar objects are promising cosmic laboratories to test the nature of dark matter (DM). DM captured by the strong gravitational field of these stellar remnants transfers kinetic energy to the star during the collision. This can have various effects such as anomalous heating of old compact stars. The proper calculation of the DM capture rate is key to derive bounds on DM interactions in any scenario involving DM accretion in a star. We improve former calculations, which rely on approximations, for both white dwarfs (WDs) and neutron stars (NSs). We account for the stellar structure, gravitational focusing, relativistic kinematics, Pauli blocking, realistic form factors, and strong interactions (NSs). Considering DM capture by scattering off either ions or degenerate electrons in WDs, we show that old WDs in DM-rich environments could probe the elusive sub-GeV mass regime for both DM-nucleon and DM-electron scattering. In NSs, DM can be captured via collisions with strongly interacting baryons or relativistic leptons. We project the NS sensitivity to DM-nucleon and DM-lepton scattering cross sections which greatly exceeds that of direct detection experiments, especially for low mass DM.

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1 Introduction

Direct detection (DD) experiments lead the quest to unveil the particle nature of dark matter (DM). In recent years, they have seen an impressive increase in sensitivity, especially to spin-independent (SI) interactions. However, their reach is limited by the achievable mass of the target material and the recoil energy threshold. In addition, DD experiments are less sensitive to spin-dependent (SD) and DM-electron cross sections. It is then natural to look for alternative systems in which DM interactions lead to observable consequences. In this sense, DM capture in the Sun has long been used as an indirect detection technique. If DM couples to visible matter, it will scatter with the Sun constituents. Provided that DM loses enough energy in the collision, it becomes gravitationally bound to the star. Accreted DM can be detected via its annihilation to neutrinos that escape the Sun [1–5].

Because of their high density that will result in more efficient DM capture, compact stars were identified long ago as potential DM probes [6, 7]. It was recently pointed out that DM capture could transfer enough kinetic energy to heat old, isolated neutron stars (NSs) in the solar neighbourhood to infrared temperatures [8]. In light of this, in a series of papers we improved former calculations of the DM capture rate, which rely on simplifying assumptions, in both white dwarfs (WDs) [9] and NSs [10–14]. We accounted for the stellar structure, gravitational focusing, a fully relativistic treatment of the scattering process, the star opacity, Pauli blocking (for degenerate targets), nuclear (WDs) and nucleon (NSs) form factors, and strong interactions (for baryonic targets in NSs). Using observations of old WDs in the globular cluster Messier 4 (M4) [15], which we assumed to be formed in a DM subhalo, we derive bounds on DM-nucleon and DM-electron scattering cross sections. For NSs, we provide sensitivity projections to DM-nucleon and DM-lepton interactions, which surpass that of DD experiments especially for light DM. This paper is structured as follows. In section 2, we briefly summarise the internal structure of compact stars. In section 3, we outline the capture rate calculation in both WDs and NSs. Our results are presented in section 4 and concluding remarks in section 5.

2 Compact stars

The fate of a star is determined by its mass when it enters the main sequence. Main sequence stars with masses below $\sim 8-10\, M_\odot$ end up their life cycles as WDs. More massive stars have a more spectacular end, a core-collapse supernova explosion that leaves behind a proto NS.

2.1 White Dwarfs

WD progenitors are low and intermediate mass stars, therefore WDs are the most abundant stellar remnants. Moreover, WD physics is far more constrained than that of NSs. E.g., there is much less uncertainty in their equation of state (EoS), and their luminosity-age relation is better understood. WDs are supported against gravitational collapse by electron degeneracy pressure. Most of them are composed mainly of carbon and oxygen. To solve the WD structure equations, we coupled the relativistic Feynman-Metropolis-Teller EoS [16,17] with the Tolman-Oppenheimer-Volkoff (TOV) equations [18, 19] (hydrostatic equilibrium in general relativity), and obtained the WD mass $M_\star$, radius $R_\star$, as well as radial profiles of the ion $n_T(r)$ and electron $n_e(r)$ number densities, electron Fermi energy and escape velocity $v_{esc}(r)$ [9].

2.2 Neutron Stars

NSs are the most compact stars known in the Universe. Neutron degeneracy pressure supports them against collapse. Despite recent breakthroughs in NS physics, their exact composition remains still unknown and the EoS of neutron-rich matter an open problem in nuclear astrophysics. NSs are mainly composed of degenerate neutrons, but inverse beta equilibrium allows the presence of protons and electrons. Muons appear in the NS core when the electron chemical potential reaches the muon mass. We model the NS interior and related microphysics by assuming a relativistic EoS that satisfies current observational constraints and enables the presence of hyperons in the NS inner core, the quark-meson coupling (QMC) model [20, 21], and solving the TOV equations. Radial profiles of the relevant quantities can be found in ref. [14].

3 Capture of dark matter in compact stars

3.1 Capture by scattering off ions

First, we consider DM scattering off the ionic targets in WDs. Since ions are non-relativistic and the WD gravitational potential is sufficiently weak so that Newtonian gravity holds, we use an approach similar to that of the Sun [22, 23] to compute the capture rate [9]

$$C = \frac{16\pi\mu_+^2\rho_\chi}{\mu m_\chi}\int_0^{R_\star} dr\, n_T(r)\eta(r)r^2 \int_0^\infty du_\chi \frac{f_{\rm MB}(u_\chi)}{u_\chi}\int_{w(r)\frac{|\mu_-|}{\mu_+}}^{v_{esc}(r)} dv v \frac{d\sigma_{T\chi}}{d\cos\theta_{cm}}(w,q^2), \tag{1}$$

where $\mu = m_\chi/m_T$, $\mu_\pm = (\mu \pm 1)/2$, ρ_χ is the DM density, m_χ the DM mass and m_T the target mass, respectively; $\eta(r)$ is the optical factor that accounts for the star opacity, defined in refs. [9, 10]; $w^2(r) = u_\chi^2 + v_{esc}^2(r)$ and v are the DM velocity before and after the collision, respectively; q is the momentum transfer. We assumed a Maxwell Boltzmann distribution $f_{\rm MB}(u_\chi)$ for the DM velocity far away from the star u_χ. Note that the differential DM-target cross section is written in the basis of non-relativistic operators [24] and includes the nuclear response function (form factors) as calculated in ref. [25] (see ref. [9] for further details).

3.2 Capture by scattering off a free Fermi gas of degenerate leptons

Degenerate leptons in both, white dwarfs and neutron stars, are relativistic and subject to Pauli blocking. Therefore, Eq. 1 cannot be applied to this case. We re-derived this expression using the TOV equations, the Schwarzschild metric and relativistic kinematics, and found [10]

$$C = 4\pi\frac{\rho_\chi}{m_\chi}\int_0^\infty du_\chi \frac{f_{\rm MB}(u_\chi)}{u_\chi}\int_0^{R_\star} dr\, \eta(r) r^2 \frac{\sqrt{1-B(r)}}{B(r)}\Omega^-(r), \tag{2}$$

$$\Omega^-(r) = \frac{\zeta(r)}{32\pi^3}\int dt\, dE_i\, ds \frac{\left|\overline{M}(s,t,m_i)\right|^2}{s^2-[m_i^2-m_\chi^2]^2}\frac{sE_i}{m_\chi}\sqrt{\frac{B(r)}{1-B(r)}}\frac{f_{\rm FD}(E_i,r)(1-f_{\rm FD}(E_i^{'},r))}{\sqrt{(s-m_i^2-m_\chi^2)^2-4m_i^2m_\chi^2}}, \tag{3}$$

where $f_{\rm FD}$ is the Fermi Dirac distribution, terms containing this function deal with Pauli suppression of the target initial and final states, $B(r)$ is the coefficient of the time part of the Schwarzschild metric and encodes general relativity corrections (very relevant for NSs), $|\overline{M}|^2$, is the squared matrix element, m_i is the mass of the target i, s and t are the Mandelstam variables, E_i and $E_i^{'}$ are the target initial and final energies, respectively. The integration range for s, t and E_i can be found in refs. [9, 10]. $\zeta(r) = n_i(r)/n_{free}(r)$ is a correction factor that accounts for the fact that we are using realistic number density $n_i(r)$ and Fermi energy profiles while assuming a free Fermi gas. The expression for n_{free} is given in ref. [10].

3.3 Capture by scattering off a Fermi sea of interacting baryons

At the extreme densities found in NSs, nucleons, and in general baryons undergo strong interactions. Strong many body forces are described in terms of relativistic scalar and vector mean fields in the QMC EoS. Under the former field, baryons develop an effective mass $m_i^{\rm eff}$, which decreases with increasing density. Thus, $m_i^{\rm eff}$, where i denotes the specific baryon, is lower than the rest mass in vacuum m_i towards the NS centre, and can be as low as $\sim 0.5m_i$ for nucleons [12, 14]. This entails that the ideal Fermi gas is not a good approximation to calculate the DM-baryon interaction rate Eq. 3. Properly incorporating the effect of strong interactions in Eq. 3 implies not only replacing m_i with $m_i^{\rm eff}$, but also calculating the Fermi energy of a single baryon as a function of its number density and $m_i^{\rm eff}$, and thereby $\zeta(r) = 1$ [12, 14].

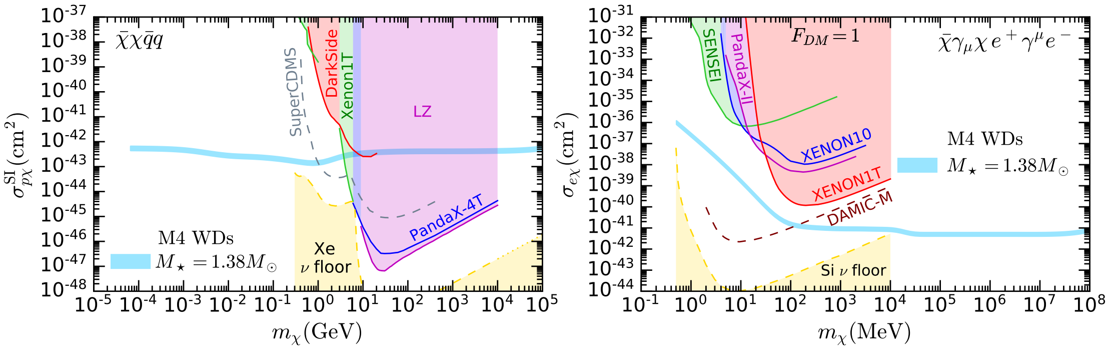

Figure 1: Upper bounds (light blue band) on the DM-proton (left) and DM-electron (right) scattering cross sections from WDs in the globular cluster M4, for the scalar and vector operators, respectively; assuming the existence of DM in M4. The band width depicts the uncertainty in ρ_χ in M4 [26]. The leading DD bounds, sensitivity projections from future experiments, and the neutrino floor [27–42] are also shown.

In addition, since DM is accelerated to quasi-relativistic speeds upon infall to a NS, the momentum transfer in the DM-baryon scattering process is sufficiently large that baryon targets cannot be treated as point-like particles. We take this into account by incorporating the momentum dependence of the hadronic matrix elements. Thus, the squared couplings of the baryon i are $c_i(t) = c_i(0)/(1-t/Q_0^2)^4$, where $Q_0 \simeq 1\,\text{GeV}$ is a scale that depends on the specific interaction and target, and $c_i(0)$ are the squared coefficients at zero momentum transfer which depend on the hadronic matrix elements of the specific interaction and baryon as in DD [12–14]. Note that the t-dependent baryon couplings are embedded in the squared matrix element $|\overline{M}(s,t,m_i^{\text{eff}})|^2$.

4 Results

We consider fermionic DM that scatters off either electron or ion targets in WDs, these interactions are described by the dimension-6 effective field theory (EFT) scattering operators [9]. We compute the capture rate for carbon WDs using Eq. 1 for ions and Eqs. 2 and 3 for electron targets, and the radial profiles obtained in section 2.1. Next, we derive limits on the cutoff scale of these operators by comparing the DM contribution to the WD luminosity due to capture and further annihilation with the observed luminosity of old WDs in the globular cluster M4 [15]. The most constraining WD being the heaviest $M_\star = 1.38M_\odot$ and faintest. Note that we have assumed the existence of DM in M4, which is yet to be proved, and $\rho_\chi \simeq 531.5-798\,\text{GeV}\,\text{cm}^{-3}$ [26]. In Fig. 1, we recast these bounds in terms of the DM-proton (left panel) and DM-electron (right panel) cross sections (light blue band) for the scalar and vector operators, respectively. For DM-nucleon scattering, we find that WDs can probe the sub-GeV mass range, with its reach limited by evaporation [9]. For DM-electron scattering, the WD bound outperforms electron recoil experiments in the full mass range, with its low mass endpoint limited by DM annihilation to neutrinos that escape the WD [9].

To project the NS sensitivity to DM-nucleon and DM-lepton scattering cross sections, we calculate the capture rate in the optically thin limit, $\eta(r) = 1$, for the EFT operators, as outlined in sections 3.2 and 3.3, and the radial profiles from section 2.2 for NSs of mass in the $1-1.9M_\odot$ range. To determine the maximum cross section that can be probed with NSs, the threshold cross section σ_{th}, we equate $C(m_\chi, \sigma_{i\chi})$ with the expression for the geometric limit given in refs. [43,44]. Note that for $\sigma_{i\chi} > \sigma_{th}$, the capture rate saturates the geometric limit. In Fig. 2,

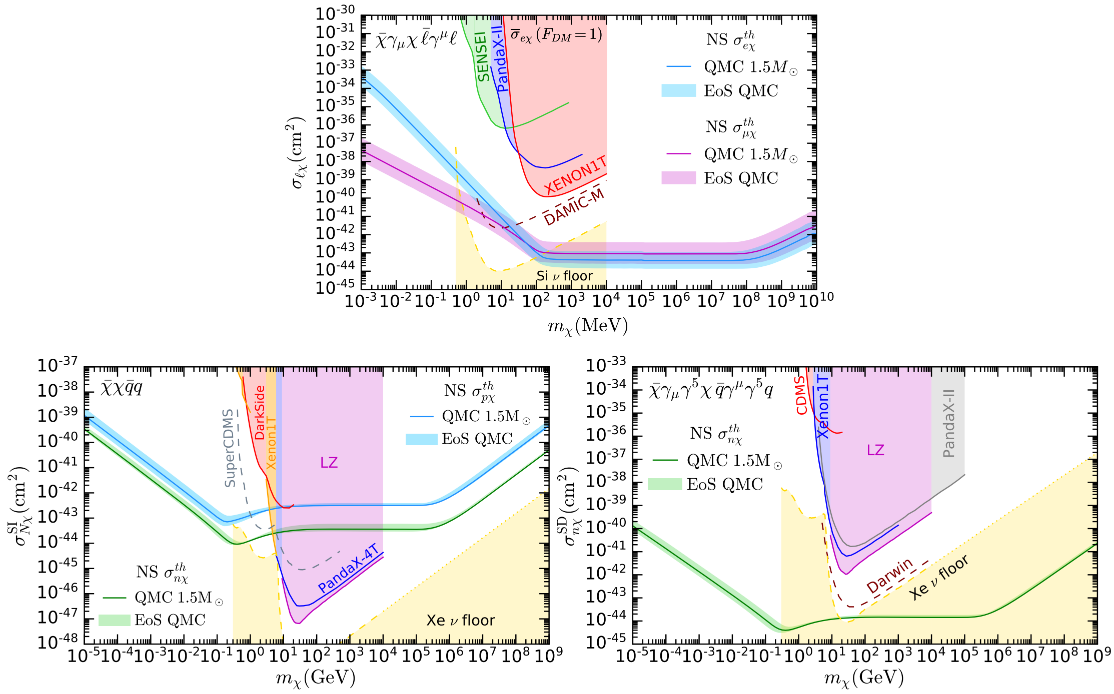

Figure 2: Top: NS sensitivity to DM-electron (light blue) and DM-muon (magenta) scattering cross sections for the vector operator. Bottom: NS sensitivity to DM-neutron (green) and DM-proton (light blue) interactions for the scalar (left panel) and axialvector (right panel) EFT operators. The solid lines represent the threshold cross section for a $1.5\,M_\odot$ NS with a QMC EoS, and the shaded bands the variation in σ_{th} due to the EoS. We also show the leading DD bounds, sensitivity projections from future experiments, as well as the neutrino floor [27–42].

we show σ_{th} for the vector operator and leptonic targets (top panel), as well as nucleon targets for the scalar (bottom left panel) and axialvector (bottom right panel) operators. The decrease in sensitivity below $m_\chi \sim 0.2\,\text{GeV}$ is due to Pauli blocking and that above $m_\chi \simeq 4\times10^5\,\text{GeV}$ (nucleons) and $m_\chi \sim [1,2]\times10^5\,\text{GeV}$ (leptons) to the fact that multiple collisions are required to capture heavy DM. As we can see, the NS sensitivity greatly surpasses that of DD in the whole DM mass range considered for DM-neutron SD and DM-lepton interactions. The leading SI DD bounds are more stringent in the $\sim 10-10^4\,\text{GeV}$ mass range, below which the NS sensitivity outperforms present and future DD experiments.

5 Conclusion

The extreme conditions found in compact stars made them promising dark matter (DM) probes. DM that accumulates and annihilates in the interior of old isolated white dwarfs (WDs), may transfer enough energy to these stars that can prevent them from cooling, provided that they are located in DM-rich environments. Thus, the null detection of anomalously warm old WDs could constrain DM interactions with ordinary matter. In neutron stars (NSs), on the other hand, due to their stronger gravitational field that accelerates DM to quasi-relativistic speeds, only the energy transferred in the capture process would be enough to heat local NSs up to infrared temperatures for maximal capture efficiency. We have shown that the NS sensitivity excels that of direct detection experiments for DM-nucleon spin-dependent and DM-lepton scattering in the full DM mass range, and for the spin-independent scattering of sub-GeV DM.

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