T-violating effect in $\nu_\tau(\bar{\nu}_\tau)-$nucleon quasielastic scattering

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Abstract

The production cross sections and polarization observables of the τ leptons produced in the $|\Delta S| = 0$ and 1 induced $\nu_\tau(\bar{\nu}_\tau)-N$ quasielastic scattering have been studied. The effect of T violation, in the case of $\Delta S = 0$ and 1 processes, and the SU(3) symmetry breaking effects, in the case of $\Delta S = 1$ processes, on the total scattering cross sections as well polarization observables are explored. Experimentally, it would be possible to observe these effects in the forthcoming (anti)neutrino experiments like DUNE, SHiP and DsTau.

1   Introduction

In this work, we present a theoretical study of the production cross section as well as the polarization observables of the τ lepton produced in the quasielastic $\nu_\tau(\bar{\nu}_\tau)-N$ scattering in the few GeV energy region relevant to the Deep Underground Neutrino Experiment (DUNE), Search for Hidden Particles (SHiP) and Study of tau neutrino production at the CERN-SPS (DsTau) experiments. The τ lepton produced in $\nu_\tau-N$ scattering decays to leptons and pions through the leptonic and hadronic decay modes. In this energy region, the production cross section of τ, its decay and the characteristics of the decay products depend significantly on the τ polarization. The production cross sections and polarization of τ leptons are calculated using various weak nucleon form factors which are determined using symmetry properties of the weak currents in the vector and axial vector sectors, assuming G and T invariances and SU(3) symmetry. The longitudinal and perpendicular components of polarization lie in the plane while the transverse component of polarization lies perpendicular to the reaction plane and is forbidden by G- and T-invariance. We have earlier studied the effects of G and T violation on the total cross section as well as on the polarization observables of the final nucleon/hyperon and the lepton produced in the quasielastic scattering of (anti)neutrinos with nucleons for both $\Delta S = 0$ and 1 processes [1–3]. In the case of $\Delta S = 1$ reactions, we study the effect of SU(3) symmetry breaking on these observables.

In Section 2, we discuss in brief the formalism for calculating the differential as well as total scattering cross sections for quasielastic scattering of $\nu_\tau(\bar{\nu}_\tau)$ with nucleons. The polarization components of the produced τ lepton are discussed in Section 2.2. The SU(3) symmetry breaking effects, following the works of Faessler *et al.* [4] and Schlumpf [5], are discussed in Section 3. In Section 4, we present and discuss the results for the total cross sections and average polarizations and Section 5 concludes the present work.

2 Quasielastic production of nucleons and hyperons

The $\nu_\tau(\bar{\nu}_\tau)$ induced quasielastic production on the free nucleon target are given by the reactions

$$\nu_\tau(\bar{\nu}_\tau)(k) + N(p) \longrightarrow \tau^\mp(k') + N(p'), \quad N = n, p, \tag{1}$$

$$\bar{\nu}_\tau(k) + N(p) \longrightarrow \tau^+(k') + Y(p'), \quad Y = \Lambda, \Sigma^0, \Sigma^-, \tag{2}$$

for which the transition matrix element is given by

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} a \, l^\mu J_\mu, \tag{3}$$

where $a = \cos\theta_c \, (\sin\theta_c)$ for $\Delta S = 0 \, (1)$ processes and the leptonic (l^μ) and the hadronic (J_μ) currents are defined as

$$l^\mu = \bar{u}(k')\gamma^\mu(1 \mp \gamma_5)u(k), \tag{4}$$

$$J_\mu = \bar{u}(p')\left[\gamma_\mu f_1(Q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M + M_Y}f_2(Q^2) + \frac{2q_\mu}{M + M_Y}f_3(Q^2)\right.$$

$$\left. - \gamma_\mu\gamma_5 g_1(Q^2) - i\sigma_{\mu\nu}\gamma_5\frac{q^\nu}{M + M_Y}g_2(Q^2) - \frac{2q_\mu\gamma_5}{M + M_Y}g_3(Q^2)\right]u(p). \tag{5}$$

In Eq. (4), $+(-)$ stands for $\bar{\nu}_\tau(\nu_\tau)$ induced process. In Eq. (5), $f_1(Q^2)$, $f_2(Q^2)$, $g_1(Q^2)$ and $g_3(Q^2)$ are the form factors associated with the first class currents while $f_3(Q^2)$ and $g_2(Q^2)$ are the form factors associated with the second class currents. These form factors are determined using various symmetry properties of the weak hadronic current like G invariance, T invariance, SU(3) symmetry, etc. The real value of $g_2(0)$ gives G violation while T is conserved whereas the imaginary value of $g_2(0)$ gives G violation as well as T violation.

2.1 Cross sections

The Q^2 distribution is written as

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2 a^2}{8\pi M^2 E_{\nu_\tau(\bar{\nu}_\tau)}^2} N(Q^2), \tag{6}$$

where the expression of $N(Q^2)$ is given in Ref. [3]. The expression for the total cross section is obtained by integrating Eq. (6) over Q^2 as:

$$\sigma = \int \frac{G_F^2 a^2}{8\pi M^2 E_{\nu_\tau(\bar{\nu}_\tau)}^2} N(Q^2) \, dQ^2, \tag{7}$$

with $a = \cos\theta_c \, (\sin\theta_c)$ for $\Delta S = 0 \, (1)$ processes.
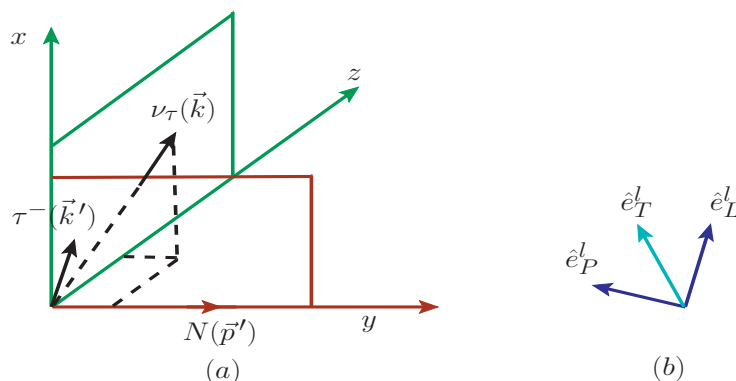
Figure 1: (a) Momentum and polarization directions of the final lepton produced in the reaction $\nu_\tau(k) + n(p) \longrightarrow \tau^-(k') + p(p')$. (b) $\hat{e}_L^l$, $\hat{e}_P^l$ and $\hat{e}_T^l$ represent the orthogonal unit vectors corresponding to the longitudinal, perpendicular and transverse directions with respect to the momentum of the final lepton.

2.2 Polarization observables of the final lepton

If one assumes the final lepton to be polarized, then the polarization 4-vector (ζ^l; $l = \tau$) is written as

$$\zeta^l = \frac{\mathrm{Tr}[\gamma^\tau \gamma_5 \, \rho_f(k')]}{\mathrm{Tr}[\rho_f(k')]}, \tag{8}$$

and the spin density matrix for the final lepton $\rho_f(k')$ is given by

$$\rho_f(k') = \mathcal{J}^{\alpha\beta} \, \mathrm{Tr}[\Lambda(k')\gamma_\alpha(1 \pm \gamma_5)\Lambda(k)\tilde{\gamma}_\beta(1 \pm \tilde{\gamma}_5)\Lambda(k')], \tag{9}$$

with $\tilde{\gamma}_\alpha = \gamma^0 \gamma_\alpha^\dagger \gamma^0$ and $\tilde{\gamma}_5 = \gamma^0 \gamma_5^\dagger \gamma^0$.

With $\rho_f(k')$ defined in Eq. (9), the expression for ζ^l becomes

$$\zeta^l = \left(g^{\tau\sigma} - \frac{k'^\tau k'^\sigma}{m_\tau^2}\right) \frac{\mathcal{J}^{\alpha\beta} \mathrm{Tr}\left[\gamma_\sigma \gamma_5 \Lambda(k')\gamma_\alpha(1 \pm \gamma_5)\Lambda(k)\tilde{\gamma}_\beta(1 \pm \tilde{\gamma}_5)\right]}{\mathcal{J}^{\alpha\beta} \mathrm{Tr}\left[\Lambda(k')\gamma_\alpha(1 \pm \gamma_5)\Lambda(k)\tilde{\gamma}_\beta(1 \pm \tilde{\gamma}_5)\right]}, \tag{10}$$

where m_τ is the mass of the τ lepton, $\Lambda(k) = \slashed{k}$, $\Lambda(k') = \slashed{k} + m_\tau$ and $\mathcal{J}^{\alpha\beta}$ is the hadronic tensor.

In the laboratory frame where the initial nucleon is at rest, the polarization vector $\vec{\zeta}$, is calculated to be a function of 3-momenta of incoming (anti)neutrino ($\vec{k}$) and outgoing lepton ($\vec{k}'$), and is given as

$$\vec{\zeta} = \left[A^l(Q^2)\vec{k} + B^l(Q^2)\vec{k}' + C^l(Q^2)\vec{k} \times \vec{k}'\right], \tag{11}$$

where the expressions of $A^l(Q^2)$, $B^l(Q^2)$ and $C^l(Q^2)$ are given in Ref. [3].

The polarization vector $\vec{\zeta}$, obtained from Eq. (10), can be written as

$$\vec{\zeta} = \zeta_L \hat{e}_L^l + \zeta_P \hat{e}_P^l + \zeta_T \hat{e}_T^l, \tag{12}$$

where $\hat{e}_P^l$, $\hat{e}_L^l$ and $\hat{e}_T^l$ are the unit vectors corresponding to the perpendicular, longitudinal and transverse directions (depicted in Fig. 1), which are given as

$$\hat{e}_L^l = \frac{\vec{k}'}{|\vec{k}'|}, \qquad \hat{e}_P^l = \hat{e}_L^l \times \hat{e}_T^l, \qquad \hat{e}_T^l = \frac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|}, \tag{13}$$

with $\zeta_{L,P,T}(Q^2) = \vec{\zeta} \cdot \hat{e}_{L,P,T}^l$.
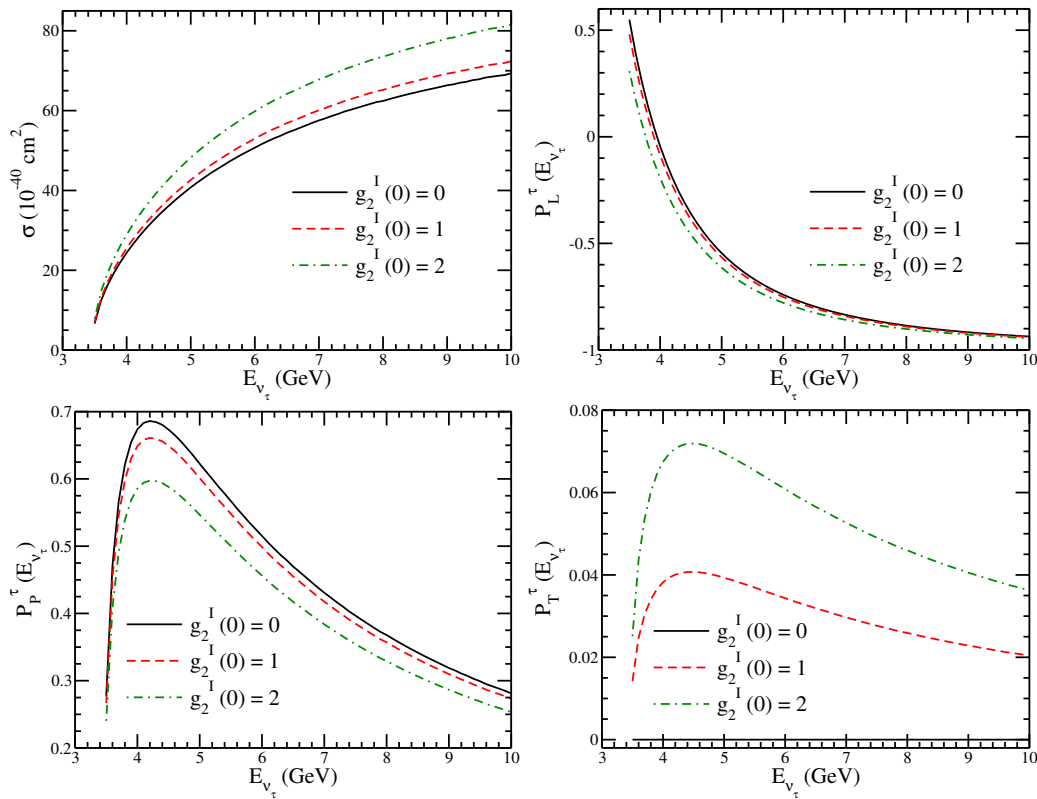
Figure 2: σ (upper left panel), $P_L^\tau(E_{\nu_\tau})$ (upper right panel), $P_P^\tau(E_{\nu_\tau})$ (lower left panel) and $P_T^\tau(E_{\nu_\tau})$ (lower rights panel) vs. E_{ν_τ} for the process $\nu_\tau + n \to \tau^- + p$, using the different values of $g_2^I(0)$ *viz.* $g_2^I(0) = 0$ (solid line), 1 (dashed line) and 2 (double-dotted-dashed line).

The longitudinal $P_L^l(Q^2)$, perpendicular $P_P^l(Q^2)$ and transverse $P_T^l(Q^2)$ components of the polarization vector in the rest frame of the outgoing lepton are obtained as:

$$P_L^l(Q^2) = \frac{m_\tau}{E_{k'}}\zeta_L(Q^2) = \frac{m_\tau}{E_{k'}}\frac{A^l(Q^2)\vec{k}.\vec{k}' + B^l(Q^2)|\vec{k}'|^2}{N(Q^2)\,|\vec{k}'|}\,, \tag{14}$$

$$P_P^l(Q^2) = \zeta_L(Q^2) = \frac{A^l(Q^2)[|\vec{k}|^2|\vec{k}'|^2 - (\vec{k}.\vec{k}')^2]}{N(Q^2)\,|\vec{k}'|\,|\vec{k}\times\vec{k}'|}\,, \tag{15}$$

$$P_T^l(Q^2) = \zeta_T(Q^2) = \frac{C^l(Q^2)M[(\vec{k}.\vec{k}')^2 - |\vec{k}|^2|\vec{k}'|^2]}{N(Q^2)\,|\vec{k}\times\vec{k}'|}\,, \tag{16}$$

where $\frac{m_\tau}{E_{k'}}$ is the Lorentz boost factor, which appears due to the fact that we are measuring the polarization observables in the rest frame of the τ lepton.

3 SU(3) symmetry breaking

The SU(3) symmetry breaking effects are incorporated following the works of Faessler *et al.* [4] and Schlumpf [5]. In the following, we will discuss in brief the main features of these models.

3.1 Model of Faessler et al. [4]

Faessler *et al.* have studied the SU(3) symmetry breaking effects on $f_2(Q^2)$ and $g_1(Q^2)$ form factors, using the constituent quark model. $f_1(Q^2)$ recieves no contribution from the SU(3)
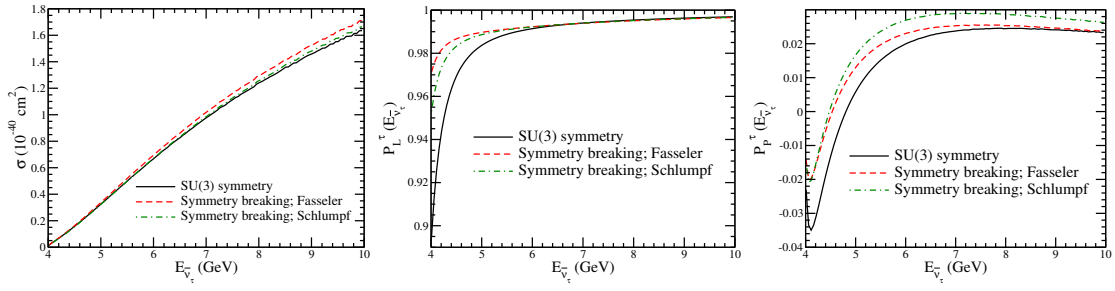
Figure 3: σ (left panel), $P_L(E_{\bar{\nu}_\tau})$ (middle panel), and $P_P(E_{\bar{\nu}_\tau})$ (right panel) vs $E_{\bar{\nu}_\tau}$ for $\bar{\nu}_\tau + p \rightarrow \tau^+ + \Lambda$ process. The calculations have been performed using the SU(3) symmetry (solid line), the SU(3) symmetry breaking effects parameterized by Faessler *et al.* (dashed line) and by Schlumpf (dashed-dotted line).
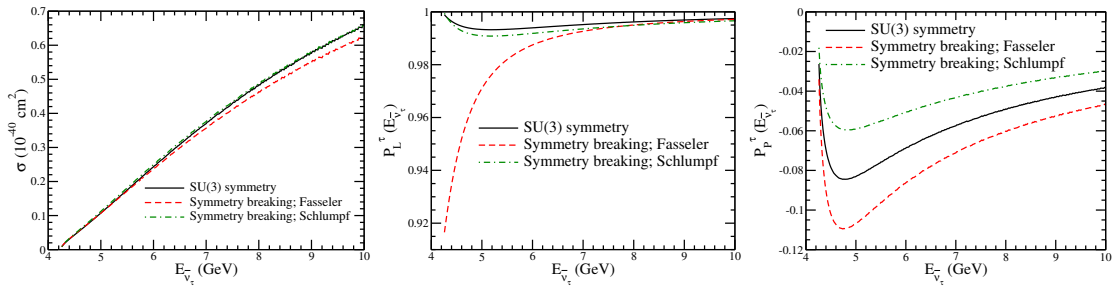
Figure 4: σ (left panel), $P_L(E_{\bar{\nu}_\tau})$ (middle panel), and $P_P(E_{\bar{\nu}_\tau})$ (right panel) vs $E_{\bar{\nu}_\tau}$ for $\bar{\nu}_\tau + n \rightarrow \tau^+ + \Sigma^-$ process. The calculations have been performed using the SU(3) symmetry (solid line), the SU(3) symmetry breaking effects parameterized by Faessler *et al.* (dashed line) and by Schlumpf (dashed-dotted line).

symmetry breaking, at the leading order because of the Ademollo-Gatto theorem. $g_3(Q^2)$ recieves the SU(3) breaking effects via $g_1(Q^2)$. In this model, the form factors $f_2(Q^2)$ and $g_1(Q^2)$ are modified, at $Q^2 = 0$ in a similar manner *i.e.*, $f_2^{SU3} = g_1^{SU3} = \mathcal{F}$, which are given as:

$$\mathcal{F}^{p\Lambda}(0) = -\sqrt{\frac{3}{2}}\left(F + \frac{D}{3} + \frac{1}{9}\left(H_1 - 2H_2 - 3H_3 - 6H_4\right)\right), \qquad (17)$$

$$\mathcal{F}^{n\Sigma^-}(0) = D - F - \frac{1}{3}(H_1 + H_3), \qquad (18)$$

where D and F are the SU(3) symmetric couplings while $H_i, i = 1-4$ are the SU(3) symmetry breaking couplings, and the value of these parameters for $f_2(Q^2)$ and $g_1(Q^2)$ form factors can be found in Refs. [2, 4].

3.2 Model of Schlumpf [5]

Schlumpf [5] has studied SU(3) symmetry breaking effect on the vector $f_1(Q^2)$ and axial vector $g_1(Q^2)$ form factors using relativistic quark model. In this model, the modified $f_1(Q^2)$ and $g_1(Q^2)$ form factors are given by

$$f_1'(Q^2) = \alpha f_1(Q^2), \qquad g_1'(Q^2) = \beta g_1(Q^2), \qquad (19)$$

where α=0.976 and 0.975; β=1.072 and 1.056, respectively, for $p \longrightarrow \Lambda$ and $n \longrightarrow \Sigma^-$ transitions [5].

4 Results and discussions

In Fig. 2, we have studied the effect of T violation on the total scattering cross section $\sigma(E_{\nu_\tau})$ (Eq. (7)), as well as on the polarization observables $P_L^\tau(E_{\nu_\tau})$, $P_P^\tau(E_{\nu_\tau})$ and $P_T^\tau(E_{\nu_\tau})$, for the process $\nu_\tau + n \rightarrow \tau^- + p$, by varying the value of $g_2^I(0)$ in the range $0 - 2$. It may be observed from the figure that the total cross section as well as the polarization observables (P_P^τ and P_T^τ) are quite sensitive to the variation in the value of $g_2^I(0)$, while the effect of $g_2^I(0)$ variation on $P_L^\tau(E_{\nu_\tau})$ is small. The transverse component of polarization is non-zero when we take the T-violating effect ($g_2^I(0) > 0$) into account. Therefore, the experimental measurement of the non-zero value of P_T^τ, if any, would independently determine the degree of T violation in weak interactions.

In Fig. 3, we have presented the results for $\sigma(E_{\bar{\nu}_\tau})$, $P_L(E_{\bar{\nu}_\tau})$ and $P_P(E_{\bar{\nu}_\tau})$ vs $E_{\bar{\nu}_\tau}$ for the charged current process $\bar{\nu}_\tau + p \longrightarrow \tau^+ + \Lambda$ with SU(3) symmetry as well as when the SU(3) symmetry breaking effects are taken into account. We find that the effect of SU(3) symmetry breaking on $P_P(E_{\bar{\nu}_\tau})$ is quite significant, while on $P_L(E_{\bar{\nu}_\tau})$ the effect is small and there is hardly any effect of SU(3) symmetry breaking on $\sigma(E_{\bar{\nu}_\tau})$.

In Fig. 4, we have presented the results for $\sigma(E_{\bar{\nu}_\tau})$, $P_L(E_{\bar{\nu}_\tau})$ and $P_P(E_{\bar{\nu}_\tau})$ vs $E_{\bar{\nu}_\tau}$ for the reaction $\bar{\nu}_\tau + n \longrightarrow \tau^+ + \Sigma^-$ with SU(3) symmetry as well as when the SU(3) symmetry breaking effects are taken into account. We find that in Σ production, there is a large variation in $P_L(E_{\bar{\nu}_\tau})$ and $P_P(E_{\bar{\nu}_\tau})$ due to the SU(3) symmetry breaking while the effect on $\sigma(E_{\bar{\nu}_\tau})$ is small. In the case of $P_L(E_{\bar{\nu}_\tau})$, the variation is more when Faessler *et al.* [4] prescription is used and the nature of dependence on SU(3) breaking effect using Faessler *et al.* [4] and Schlumpf [5] prescriptions are different.

5 Conclusion

The effect of T violation is appreciable for $g_2^I(0) \geq 1$ in the case of σ as well as the polarization observables both for the neutrino as well as antineutrino induced processes. There is a significant variation in P_L and P_P, on the different parameterizations of the SU(3) symmetry breaking, while the effect of SU(3) symmetry breaking is not much in the case of total scattering cross sections. These results are more prominent in the case of $\Delta S = 1$ processes.

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