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Probing new physics with the leptonic g-2

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Abstract

We present a concise review of the new physics sensitivity of leptonic dipole moments and their interrelationship. In particular, focusing on the current muon g-2 anomaly, we analyse both high-energy and low-energy tests to confirm or to falsify it.

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1 Introduction

The anomalous magnetic moment of the muon, $a_\mu \equiv (g_\mu - 2)/2$, has provided an enduring hint of new physics (NP) for many years. The recent a_μ measurement by the Muon $g-2$ collaboration at Fermilab [1] has confirmed the earlier result by the E821 experiment at Brookhaven [2], yielding the average $a_\mu^{\text{EXP}} = 116592061(41) \times 10^{-11}$. The comparison of this result with the Standard Model (SM) prediction $a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11}$ of the Muon $g-2$ Theory Initiative [3] leads to an intriguing 4.2σ discrepancy [1]

$$\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = 251\,(59) \times 10^{-11}\,. \tag{1}$$

On the theory side, the only source of sizable uncertainties in a_μ^{SM} stems from the non-perturbative contributions of the hadronic sector, which have been under close scrutiny for several years. The SM prediction a_μ^{SM} in Eq. (1) has been derived using $(a_\mu^{\text{HVP}})^{\text{TI}}_{e^+e^-}$, the leading hadronic vacuum polarization (HVP) contribution to the muon $g-2$ based on low-energy $e^+e^- \to$ hadrons data obtained by the Muon $g-2$ Theory Initiative [3]. Alternatively, the HVP contribution has been computed using a first-principle lattice QCD approach [3]. Recently, the BMW lattice QCD collaboration (BMWc) computed the leading HVP contribution to the muon $g-2$ with sub per-cent precision, finding a value, $(a_\mu^{\text{HVP}})_{\text{BMW}}$, larger than $(a_\mu^{\text{HVP}})^{\text{TI}}_{e^+e^-}$ [4]. If $(a_\mu^{\text{HVP}})_{\text{BMW}}$ is used to obtain a_μ^{SM} instead of $(a_\mu^{\text{HVP}})^{\text{TI}}_{e^+e^-}$, the discrepancy with the experimental result is reduced to 1.6σ only. The above results are respectively

$$(a_\mu^{\text{HVP}})^{\text{TI}}_{e^+e^-} = 6931\,(40) \times 10^{-11}\,, \qquad (a_\mu^{\text{HVP}})_{\text{BMW}} = 7075\,(55) \times 10^{-11}\,. \tag{2}$$

The difference between these two values has been referred to as the *new muon* $g-2$ *puzzle* [5]. In [5], it was investigated the possibility to solve this tension invoking NP in the hadronic cross-section. It was argued that the most plausible scenario requires the presence of a light NP mediator that modifies the experimental cross-section σ_{had}. However, this non-trivial setup, where NP hides in $e^+e^- \to$ hadrons data, is excluded by a number of experimental constraints [5]. Alternative confirmations of the e^+e^- determinations of the HVP contribution to the muon $g-2$, based on either additional lattice QCD calculations or direct experimental measurements, as proposed by the MUonE experiment [6–8], will be crucial to solve this intriguing puzzle. Interestingly, the muon $g-2$ discrepancy of eq. (1) can be solved by a NP effect of the same order as the SM weak contribution $\approx 2\times10^{-9}$ [3]. In principle, NP scenarios entailing weakly coupled particles at the electroweak scale could provide a natural explanation of eq. (1), see e.g. [9]. In practice, however, the experimental bounds by LEP and LHC highly disfavours this possibility. Therefore, the scenarios preferred by data include either very light and feebly coupled particles, see e.g. [12], or very heavy and strongly coupled particles.

Heavy NP contributions to the muon $g-2$ stem from the dipole operator $\left(\bar{\mu}_L\sigma_{\mu\nu}\mu_R\right)HF^{\mu\nu}$ where $H=v+h/\sqrt{2}$ contains both the Higgs boson field h and its vacuum expectation value $v=174$ GeV while $F^{\mu\nu}$ is the electromagnetic field strenght tensor. After electroweak symmetry breaking, $\Delta a_\mu^{\text{NP}} \sim (g_{\text{NP}}^2/16\pi^2)\times(m_\mu v/\Lambda^2)$, where g_{NP} is a representative NP coupling. Therefore, the chiral enhancement $v/m_\mu \sim 10^3$, together with the assumption of a new strong dynamics ($g_{\text{NP}}\sim 4\pi$), bring the sensitivity of the muon $g-2$ to NP scales of order $\Lambda\sim 100$ TeV.

A direct detection of new particles at a so high scales is beyond the capabilities of any foreseen collider. Furthermore, the discovery of new particles by their direct production [14] couldn't be unambiguously associated to Δa_μ. In other words, we need to test the muon $g-2$ anomaly model-independently. The goal of this work is to outline possible directions for such a model-independent test.

2 High-energy tests of the muon g-2 anomaly

In ref. [16], it was argued that a muon collider (MC) running at energies E of several TeV would represent the only machine able to probe NP in the muon $g-2$ model-independently. In fact, the same dipole operator generating Δa_μ unavoidably induces also a NP contribution to the process $\mu^+\mu^- \to h\gamma$. Focusing on the leptonic $g-2$, the relevant effective Lagrangian reads

$$\mathcal{L} = \frac{C^\ell_{eB}}{\Lambda^2}\left(\bar{\ell}_L\sigma^{\mu\nu}e_R\right)HB_{\mu\nu} + \frac{C^\ell_{eW}}{\Lambda^2}\left(\bar{\ell}_L\sigma^{\mu\nu}e_R\right)\tau^I HW^I_{\mu\nu} + \frac{C^\ell_T}{\Lambda^2}(\bar{\ell}^a_L\sigma_{\mu\nu}e_R)\varepsilon_{ab}(\bar{Q}^b_L\sigma^{\mu\nu}u_R) + h.c.\,, \quad (3)$$

where $\Lambda \gtrsim 1$ TeV is assumed. In figure 1, we show the Feynman diagrams contributing to the leptonic $g-2$ as well as to correlated high-energy processes. An explicit one-loop calculation of Δa_ℓ provides the following result

$$\Delta a_\ell \simeq \frac{4m_\ell v}{e\Lambda^2}\left(C^\ell_{e\gamma} - \frac{3\alpha}{2\pi}\frac{c_W^2-s_W^2}{s_W c_W}C^\ell_{eZ}\log\frac{\Lambda}{m_Z}\right) - \sum_{q=c,t}\frac{4m_\ell m_q}{\pi^2}\frac{C^{\ell q}_T}{\Lambda^2}\log\frac{\Lambda}{m_q}\,, \quad (4)$$

where s_W (c_W) is the sine (cosine) of the Weinberg angle while $C_{e\gamma}$ and C_{eZ} are linear combinations of C_{eB} and C_{eW}. From eq. (4), one can find [16]

$$\Delta a_\mu \approx 3\times10^{-9}\left(\frac{250\,\text{TeV}}{\Lambda}\right)^2\left(C^\mu_{e\gamma} - 0.2C^{\mu t}_T - 0.001C^{\mu c}_T - 0.05C^\mu_{eZ}\right).$$

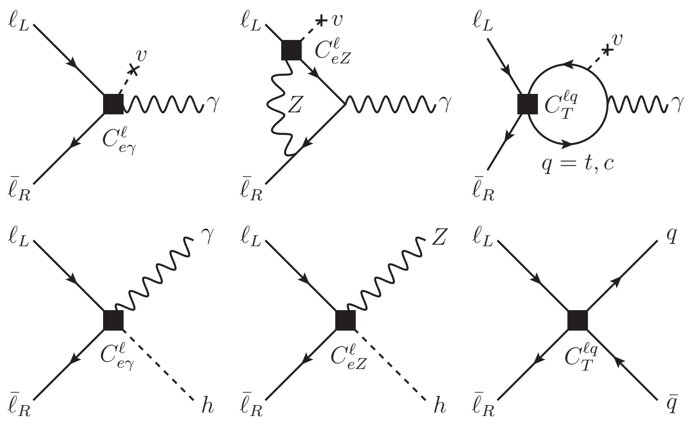

Figure 1: *Upper row:* Feynman diagrams contributing to the leptonic $g-2$ up to one-loop order in the Standard Model EFT. *Lower row:* Feynman diagrams of the corresponding high-energy processes. Dimension-6 effective interaction vertices are denoted by a square (from [16]).

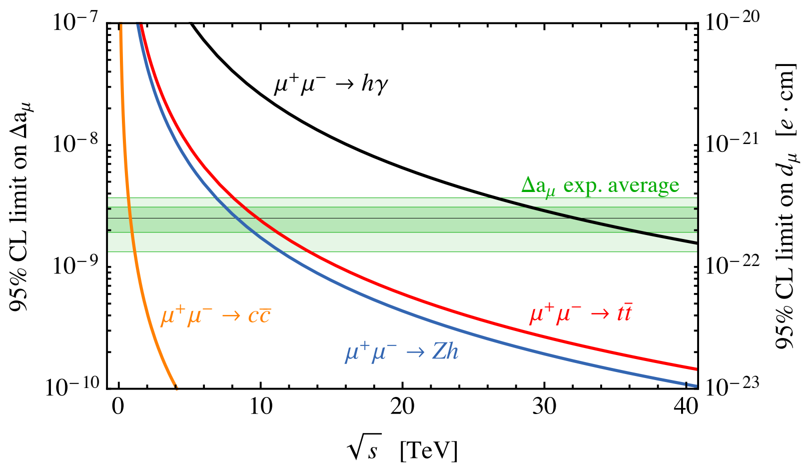

Figure 2: 95% C.L. reach on Δa_μ as a function of $\sqrt{s}$, from the processes $\mu^+\mu^- \to h\gamma$ (black), $\mu^+\mu^- \to hZ$ (blue), $\mu^+\mu^- \to t\bar{t}$ (red), and $\mu^+\mu^- \to c\bar{c}$ (orange) from [16].

The main contribution to Δa_μ comes from the coefficient $C_{e\gamma}$ related to the photonic dipole operator which also induces a contribution to the process $\mu^+\mu^- \to h\gamma$ (see figure 1). In particular, the total cross-section of $\mu^+\mu^- \to h\gamma$ is given by [16]

$$\sigma_{h\gamma} = \frac{s}{48\pi}\frac{|C^{\mu}_{e\gamma}|^2}{\Lambda^4} \approx 0.7\,\text{ab}\left(\frac{\sqrt{s}}{30\,\text{TeV}}\right)^2\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2 , \tag{5}$$

where we kept only the dominant $C^{\mu}_{e\gamma}$ contribution to Δa_μ. In figure 2, we show as a black line the 95% C.L. reach from $\mu^+\mu^- \to h\gamma$ on Δa_μ as a function of the collider energy.

Thanks to the growth with energy of $\sigma_{h\gamma}$ as well as of the reference integrated luminosity $\mathcal{L} = (\sqrt{s}/30\,\text{TeV})^2 \times 10\,\text{ab}^{-1}$, we see that a muon collider with $\sqrt{s} \gtrsim 30$ TeV would have the sufficient sensitivity to test the muon $g-2$ anomaly.

3 Low-energy tests of the muon g-2 anomaly

The dipole operators of eq. (3) generally have a non-trivial flavour and CP structure. As a result, a NP contribution to Δa_μ is typically accompanied by lepton flavor violating (LFV) and CP violating effects [18]. Below the electroweak scale dipole transitions $\ell \to \ell'\gamma$ in the leptonic sector are described by the effective Lagrangian

$$\mathcal{L} = e\frac{m_\ell}{2}\left(\bar{\ell}_R \sigma_{\mu\nu} A_{\ell\ell'} \ell'_L + \bar{\ell}'_L \sigma_{\mu\nu} A^\star_{\ell\ell'} \ell_R\right) F^{\mu\nu}, \tag{6}$$

where $\ell, \ell' = e, \mu, \tau$. Starting from eq. (6), we can evaluate LFV processes, such as $\mu \to e\gamma$,

$$\frac{\mathrm{BR}(\ell \to \ell'\gamma)}{\mathrm{BR}(\ell \to \ell'\nu_\ell \bar{\nu}_{\ell'})} = \frac{48\pi^3\alpha}{G_F^2}\left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2\right). \tag{7}$$

The effective Lagrangian of eq. (6) generates also flavor conserving processes such as the anomalous magnetic moments of leptons, Δa_ℓ, as well as the leptonic electric dipole moments (EDMs, d_ℓ) which read

$$\Delta a_\ell = 2m_\ell^2 \,\mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell \,\mathrm{Im}(A_{\ell\ell}). \tag{8}$$

In concrete NP scenarios, one would generally expect that Δa_ℓ, d_ℓ and BR($\ell \to \ell'\gamma$) are correlated. However, these connections depend on the unknown flavor and CP structures of the underlying NP sector and therefore are model-dependent.

Parametrizing the amplitude $A_{\ell\ell'}$ as $A_{\ell\ell'} = c_{\ell\ell'}/\Lambda^2$, where Λ refers to the NP scale, we can evaluate which are the values of Λ probed by $\mu \to e\gamma$. We find that

$$\mathrm{BR}(\mu \to e\gamma) \approx 10^{-12}\left(\frac{500\ \mathrm{TeV}}{\Lambda}\right)^4 \left(|c_{\mu e}|^2 + |c_{e\mu}|^2\right). \tag{9}$$

Combining Δa_ℓ and BR($\ell \to \ell'\gamma$), one can find that

$$\begin{aligned}
\mathrm{BR}(\mu \to e\gamma) &\approx 10^{-12}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2 \left(\frac{\theta_{e\mu}}{2\times 10^{-5}}\right)^2, \\
\mathrm{BR}(\tau \to \ell\gamma) &\approx 10^{-8}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2 \left(\frac{\theta_{\ell\tau}}{5\times 10^{-3}}\right)^2,
\end{aligned} \tag{10}$$

where $\theta_{\ell\ell'} = \sqrt{|c_{\ell\ell'}|^2 + |c_{\ell'\ell}|^2}/c_{\mu\mu}$. As a result, it is found that the solution of the muon $g-2$ anomaly requires highly suppressed flavor mixing angles $\theta_{e\mu}$ [20]. We also find that

$$d_e \approx 10^{-24}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) \varphi^e_{\mathrm{CP}} \quad e\,\mathrm{cm}, \tag{11}$$

and therefore also the electron EDM exceeds the current experimental bound by several orders of magnitudes unless the CP violating phase $\varphi^e_{\mathrm{CP}} = [\mathrm{Im}(c_{ee})/\mathrm{Re}(c_{\mu\mu})] \lesssim 10^{-5}$ [20].

4 Conclusion

The muon $g-2$ discrepancy is one of most interesting hints of new physics emerged so far in particle physics, which has recently been reinforced by the E989 experiment at Fermilab. However, the low-energy determination of Δa_μ requires that systematic and hadronic uncertainties are under control at the level of $\Delta a_\mu \sim 10^{-9}$. Needless to say, an independent test of Δa_μ, not contaminated by the above uncertainties, would be very desirable. Interestingly, a multi-TeV muon collider can achieve this goal, providing a model-independent test of new physics in the muon $g-2$ through the high-energy processes $\mu^+\mu^- \to h\gamma, hZ, q\bar{q}$. These results rely on measurements with $\mathcal{O}(1)$ accuracy, therefore not requiring a precise control of systematic or theoretical uncertainties. These findings are model-independent, as they are formulated in terms of the same effective operators controlling the lepton dipole moments. Should the muon $g-2$ anomaly be confirmed in the future, this would constitute a *no-lose* theorem for a multi-TeV muon collider, guaranteeing the discovery of new physics in high-energy collisions.

From the low-energy side, the same dipole operator generating a new physics contribution to Δa_μ is expected to generate also other low-energy processes including lepton flavour violating (LFV) decays such as $\mu \to e\gamma$ and CP violating processes like the electron EDM.

We hope that, with the expected sensitivities of next-generation experiments, NP will show up in some of the processes analysed in this contribution. In this case, the interrelationship among leptonic $g-2$, EDMs and LFV will be of outmost importance to disentangle among different NP scenarios.

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