Correlations among 3N observables

Yuriy Volkotrub[1]*, Jacek Golak[1], Roman Skibiński[1],
Kacper Topolnicki[1] and Henryk Witała[1]

1 M. Smoluchowski Institute of Physics, Jagiellonian University, PL-30348 Kraków, Poland

* yuriy.volkotrub@doctoral.uj.edu.pl

Abstract

We use a correlation matrix of two-nucleon (2N) potential parameters to investigate correlations among three-nucleon (3N) observables for the elastic nucleon-deuteron (Nd) scattering process. We employ the up-to-date 2N semilocal momentum-space regularized chiral interactions and the phenomenological OPE-Gaussian potential and using the Faddeev formalism we calculate 3N observables and their correlations. As an example, we present the correlation between the differential cross section and the deuteron analyzing power T_{20}.

1 Introduction

The question about the existence of correlations among nuclear physics observables is closely linked to constructing models of nuclear forces. Namely, the knowledge of the correlations among the various 3N observables will allow the determination of the set of observables, which should be used to fit the parameters of three-body force models. Such information will also be a valuable guide for planning future experiments in low-energy nuclear physics. To the best of our knowledge, the correlations between two- and three-nucleon observables (2N and 3N respectively) have not been investigated yet. The aim of this paper is to start such a study focusing on 3N observables.

Recently R. Navarro Peréz and his collaborators from the Granada group carefully revised the existing 2N data and prepared a new database [1], removing from the data these points for which uncertainties were unknown or poorly defined. They excluded also data sets inconsistent with other data, which led to a self-consistent database at the 3σ level. Currently, this database (Granada-2013) is a standard set of data used for fixing parameters of the 2N forces. In addition, R. Navarro Pérez et al. have derived new models of the 2N interaction, for example the One-Pion-Exchange-Gaussian (OPE-Gaussian) potential [2, 3]. In 2018 the Bochum-Bonn group introduced sophisticated 2N potentials up to the fifth-order in the chiral

expansion with a semilocal regularization in momentum space for the pion exchange contributions [4]. They used the self-consistent Granada-2013 database to fix the free parameters of the model. For both potentials [2] and [4] a detailed error analysis was performed. In particular, the authors quantified for the first time the statistical uncertainties of the potential parameters and obtained not only the values of 2N potential parameters but also their covariance matrices. The knowledge of a correlation matrix of the 2N potential parameters allowed us to perform a statistical analysis aimed at finding theoretical uncertainties for the elastic nucleon-deuteron observables (3N observables). As a result in [5,6], we successfully analyzed the propagation of uncertainties of 2N potential parameters to a given 3N observable. It was found that this uncertainty is much smaller than the uncertainty arising from using various models of the nuclear interaction.

2   Methods and theoretical formalism

Given the mean values of the 2N potential parameters and their correlation matrix, we sampled 50 sets of the potential parameters. Next, for each set of potential parameters, we apply the Faddeev framework to calculate the elastic Nd scattering observables [7]. This means that we first calculate the deuteron wave function by solving the Schrödinger equation and obtain the t-matrix elements from the Lippman-Schwinger equation for a given 2N interaction in momentum space. Next, we solve the Faddeev equation in momentum space to compute the transition amplitude from which any 3N observable can be obtained. In this study, we neglect 3N interactions and use only the 2N force, which enters the Faddeev equation via the precalculated t-matrix operator. The Faddeev equation for the transition amplitude $T|\phi\rangle$ reads:

$$T|\phi\rangle = tP|\phi\rangle + tPG_0 T|\phi\rangle \,, \tag{1}$$

where the initial state $|\phi\rangle$ is composed of a deuteron and a relative momentum eigenstate of the projectile nucleon, P is a permutation operator, G_0 is the free 3N propagator and t is a solution of the Lippmann-Schwinger equation with an interaction V (given by the models of 2N interaction from the Granada or Bochum-Bonn groups). We solve Eq. 1 numerically, in momentum space, using partial wave decomposition and by generating the Neumann series and summing it by the Padé method. More information can be found in Refs. [7] and [8].

3   Results

We analyzed many pairs of 3N observables for which we computed correlation coefficients at selected incoming nucleon energies and in the whole range of scattering angles. As an example in Fig. 1, we present the angular dependence of the correlation coefficient for a chosen pair of 3N observables as functions of the center-of-mass scattering angle in the range $\theta_{c.m.} \in [0°, 180°]$ at two laboratory energies of the incident neutron 13 and 65 MeV. Correlations between two selected 3N observables, i.e. the differential cross section and the deuteron tensor analyzing power T_{20} were investigated with the semilocal momentum-space regularized (SMS) chiral N^2LO, N^4LO and N^4LO+ potentials [4] with the value of the regulator parameter $\Lambda = 450$ MeV and with the OPE-Gaussian potential [2]. In general, as shown in Fig. 1, the differential cross section, $d\sigma/d\theta_{c.m.}$, appears weakly correlated with the deuteron tensor analyzing power, T_{20}, for the two employed potentials and for both energies. However, it is observed that for some intervals of the scattering angle, a relatively strong correlation for this particular pair of 3N observables exists. In the case of the SMS chiral N^2LO potential at scattering energy $E = 13$ MeV the absolute value of the correlation coefficient does not exceed
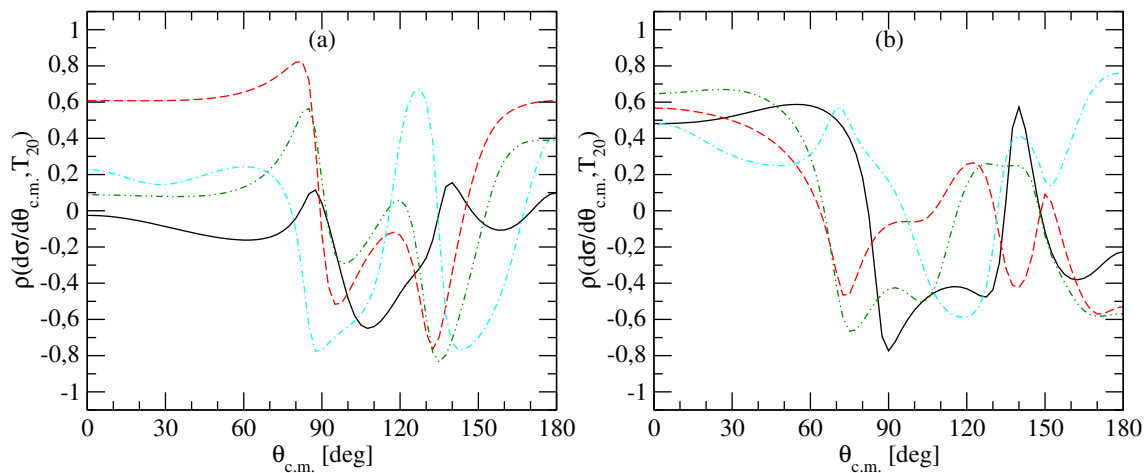
Figure 1: The angular dependence of the correlations coefficient between the differential cross section $d\sigma/d\theta_{c.m.}$ and the deuteron analyzing power T_{20} for the incoming neutron laboratory energies $E = 13$ MeV (a) and $E = 65$ MeV (b) in the elastic neutron-deuteron scattering. The black solid, red dashed, cyan dot-dash and green dash-dot-dotted lines represent predictions of the SMS chiral N^2LO, N^4LO and N^4LO+ forces with $\Lambda = 450$ MeV and the OPE-Gaussian potential, respectively.

$|\rho| = 0.15$ for $0° \leq \theta_{c.m.} \lesssim 70°$, but there is a region for which the value of the correlation coefficient reaches about $|\rho| = 0.6$. For the higher energy $E = 65$ MeV, the correlation coefficient amounts to 0.5 at small scattering angles. With the increasing angle, ρ gradually increases and at $\theta_{c.m.} = 57.5°$ reaches its maximum and then drops sharply reaching a minimum at $\theta_{c.m.} = 90°$ which points to strong anti-correlation ($|\rho|$ is already 0.77). Further growth of ρ is observed with increasing values of the angle and at $\theta_{c.m.} = 140°$ again practically reaches the same value as at $\theta_{c.m.} = 57.5°$. With a growing order of the chiral expansion, one reaches a more accurate version of the chiral 2N force. One example is the SMS chiral N^4LO. Using it we observe the same behavior, but the magnitude of the correlation coefficient at E=13 MeV is different. For the SMS chiral N^4LO+ predictions one can see again more or less the same behavior as for the SMS chiral N^2LO, especially for small scattering angles for both energies. For the phenomenological OPE-Gaussian potential, we see that the behavior of predictions is similar to the one for results based on the SMS chiral N^4LO force for both energies.

4 Conclusion

We showed that it is possible to analyze the correlation among various 3N observables resulting from the correlation matrices provided with the models of 2N interactions from the Granada and Bochum-Bonn groups. It was also demonstrated that when using the chiral interaction the angular dependence of the correlation coefficients for the differential cross section and the deuteron analyzing power depends on the order of chiral expansion as well as on the scattering energy. It is interesting that a strong correlation appears at specific intervals of scattering angles. This could indicate a dependence of the transition amplitude on specific partial waves.

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References

[1] R. Navarro Pérez, J. E. Amaro and E. Ruiz Arriola, *Coarse-grained potential analysis of neutron-proton and proton-proton scattering below the pion production threshold*, Phys. Rev. C **88**, 064002 (2013), doi:10.1103/PhysRevC.88.064002.

[2] R. Navarro Pérez, J. E. Amaro and E. Ruiz Arriola, *Statistical error analysis for phenomenological nucleon-nucleon potentials*, Phys. Rev. C **89**, 064006 (2014), doi:10.1103/PhysRevC.89.064006.

[3] R. Navarro Pérez, J. E. Amaro and E. Ruiz Arriola, *Error analysis of nuclear forces and effective interactions*, J. Phys. G: Nucl. Part. Phys. **42**, 034013 (2015), doi:10.1088/0954-3899/42/3/034013.

[4] P. Reinert, H. Krebs and E. Epelbaum, *Semilocal momentum-space regularized chiral two-nucleon potentials up to fifth order*, Eur. Phys. J. A **54**, 86 (2018), doi:10.1140/epja/i2018-12516-4.

[5] R. Skibiński, Yu. Volkotrub, J. Golak, K. Topolnicki and H. Witała, *Theoretical uncertainties of the elastic nucleon-deuteron scattering observables*, Phys. Rev. C **98**, 014001 (2018), doi:10.1103/PhysRevC.98.014001.

[6] Y. Volkotrub, R. Skibiński, J. Golak, K. Topolnicki and H. Witała, *Theoretical uncertainties in the description of the nucleon–deuteron elastic scattering at E = 135 MeV*, Acta Phys. Pol. B **50**, 367 (2019), doi:10.5506/APhysPolB.50.367.

[7] W. Glöckle, H. Witała, D. Hüber, H. Kamada and J. Golak, *The three-nucleon continuum: achievements, challenges and applications*, Phys. Rep. **274**, 107 (1996), doi:10.1016/0370-1573(95)00085-2.

[8] W. Glöckle, *The quantum mechanical few-body problem*, Springer Berlin Heidelberg, ISBN 9783642820830 (1983), doi:10.1007/978-3-642-82081-6.