SciPost Phys. Proc. 7, 036 (2022)

Two-Loop renormalization of High-Dimensional QCD operators and Higgs EFT amplitudes

Qingjun Jin[1], Ke Ren[2], Gang Yang[2⋆] and Rui Yu[2]

1 Graduate School of China Academy of Engineering Physics, No. 10 Xibeiwang East Road, Haidian District, Beijing, 100193, China
2 CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

⋆ yangg@itp.ac.cn

15th International Symposium on Radiative Corrections: Applications of Quantum Field Theory to Phenomenology, FSU, Tallahasse, FL, USA, 17-21 May 2021
doi:10.21468/SciPostPhysProc.7

Abstract

We consider the two-loop renormalization of high-dimensional operators in QCD and the relevant Higgs EFT amplitudes. Efficient unitarity-IBP strategy is used to compute the two-loop minimal form factors of length-3 operators up to dimension sixteen. From the ultraviolet divergences of form factor results, we extract the renormalization matrices and compute anomalous dimensions. We also obtain the analytic finite remainder functions which exhibit several universal transcendentality structures. The form factors we compute are equivalent to Higgs plus three-gluon amplitudes which capture high-order top mass corrections in Higgs effective field theory.

Received 14-10-2021
Accepted 28-02-2022
Published 21-06-2022
doi:10.21468/ciPostPhysProc.7.036

1 Introduction

Gauge invariant operators play important roles in QFT. For example, they correspond to color-singlet states in QCD and also appear as effective interaction vertices in effective field theories (EFT). At classical level, an important problem is to find a set of independent basis for the operators of a certain canonical dimension. At quantum level, the operators receive quantum loop corrections and it is important to perform renormalization and study the operator mixing effects. Further problems include computing scattering amplitudes in EFTs where the high-dimensional operators need to be taken into account as well.

In this report, we will address all these problems by considering gauge invariant local operators which are composed of field strength $F_{\mu\nu}$ and covariant derivatives D_μ. The field strength carries a color index as $F_{\mu\nu} = F^a_{\mu\nu} T^a$, where T^a are the generators of gauge group and satisfy

$$[T^a, T^b] = i f^{abc} T^c . \tag{1}$$

The covariant derivative acts in the standard way as

$$D_\mu \star = \partial_\mu \star - ig[A_\mu, \star], \qquad [D_\mu, D_\nu] \star = -ig[F_{\mu\nu}, \star]. \tag{2}$$

A gauge invariant Lorentz scalar operator can be written in the following general form:

$$\mathcal{O}(x) \sim c(a_1, ..., a_n) X(\eta) \big(D_{\mu_{11}} ... D_{\mu_{1m_1}} F_{\nu_1 \rho_1}\big)^{a_1} \cdots \big(D_{\mu_{n1}} ... D_{\mu_{nm_n}} F_{\nu_n \rho_n}\big)^{a_n}(x), \tag{3}$$

where $c(a_1, ..., a_n)$ are color factors (*e.g.* given in terms of $\mathrm{tr}(T^{a_1} ... T^{a_n})$). All Lorentz indices $\{\mu_i, \nu_i, \rho_i\}$ are contracted in pairs by metric $\eta^{\mu\nu}$ contained in the function $X(\eta)$; here we consider only parity-even operators. These operators form composite color-singlet states in QCD, and they are also related to the Higgs EFT, which is obtained in the gluon fusion process by integrating out the heavy top quark [1–4].

To study the aforementioned problems associated to these operators, a useful observable to consider is the form factor defined as (see *e.g.* [5] for an introduction):

$$\mathcal{F}_{\mathcal{O},n}(1, \ldots, n; q) \equiv \int d^D x e^{-iq\cdot x} \langle p_1 \ldots p_n | \mathcal{O}(x) | 0 \rangle , \tag{4}$$

where p_i are momenta for external on-shell states and $q = \sum_i p_i$ is an off-shell momentum associated to the operator. The form factors allow to apply modern on-shell amplitude techniques to study “off-shell" operators, for both constructing (classical) operator bases and for computing quantum loop corrections. In the remaining sections, we would like to report some recent progress on these problems, mainly based on [6, 7].

2 Operator basis

The operators at a given canonical dimension are in general not independent with each other, because they can be related to each other through equations of motion (EoM) or Bianchi identities (BI):

$$\text{EoM}: \quad D_\mu F^{\mu\nu} = 0, \qquad\qquad \text{BI}: \quad D_\mu F_{\nu\rho} + D_\nu F_{\rho\mu} + D_\rho F_{\mu\nu} = 0, \tag{5}$$

where we focus only on the pure YM sector in QCD. Our goal in this section is to find a set of independent operators such that there are no above relations among the operators.

For simplicity, we will focus on length-2 and length-3 operators, in which there are only 2 and 3 $F_{\mu\nu}$ fields in the operators, plus arbitrary insertion of covariant derivatives. Two operators will be set equivalent if their difference can be written in terms of high length operators. For example, one can show that $\mathrm{tr}(D_{\mu\nu..\rho} F_{\sigma\lambda}\, D^{\mu\nu..\rho} F^{\sigma\lambda})$ and $\frac{1}{2}\partial_1^2 \mathrm{tr}(D_{\nu..\rho} F_{34} D^{\nu..\rho} F_{34})$ are equivalent up to length-3 operators. Given this criterion, there is only one independent length-2 operator at each even dimension Δ_0: $(\partial)^{\Delta_0 - 4} \mathrm{tr}(F^{\mu\nu} F_{\mu\nu})$.

To construct the basis of length-3 operators, one can first introduce two *primitive operators*:

$$\begin{aligned} \mathcal{O}^{\mathrm{P1}} &= \mathrm{tr}(D_1 F_{23} D_4 F_{23} F_{14}) = \frac{1}{2} f^{abc} (D_1 F_{23})^a (D_4 F_{23})^b (F_{14})^c , \\ \mathcal{O}^{\mathrm{P2}} &= \mathrm{tr}(F_{12} F_{13} F_{23}) = \frac{1}{2} f^{abc} (F^a_{12} F^b_{13} F^c_{23}), \end{aligned} \tag{6}$$

where for short notations, we use integer numbers to represent Lorentz indices and also often abbreviate $D_i D_j ....$ as $D_{ij...}$, for example, $F^{\mu_1\mu_2} D_{\mu_1} D_{\mu_5} F^{\mu_3\mu_4} D_{\mu_2} D^{\mu_5} F_{\mu_3\mu_4} \to F_{12} D_{15} F_{34} D_{25} F_{34}$.

Table 1: Dictionary between operators and form factors in both D and 4 dimensions.

(a) D-dimensional dictionary

operator	D_μ	$F_{\mu\nu}$
kinematics	p_μ	$p_\mu \varepsilon_\nu - p_\nu \varepsilon_\mu$

(b) Four-dimensional dictionary

operator	$D_{\dot{\alpha}\alpha}$	$f_{\alpha\beta}$	$\bar{f}_{\dot{\alpha}\dot{\beta}}$
spinor	$\tilde{\lambda}_{\dot{\alpha}}\lambda_\alpha$	$\lambda_\alpha\lambda_\beta$	$-\tilde{\lambda}_{\dot{\alpha}}\tilde{\lambda}_{\dot{\beta}}$

The "primitive" is in the following sense that: higher dimensional operators can be constructed by inserting pairs of covariant derivatives $\{D^\mu, D_\mu\}$ in the primitive operators. (Note that for the length-2 case there is a single primitive operator $\mathrm{tr}(F^{\mu\nu}F_{\mu\nu})$.)

As a concrete example, let us consider the operators of dimension-10, which will serve as major examples in the following discussion. One can construct basis operators by inserting DD pair in the primitive operators (6). There are five inequivalent ways of adding DD pairs which provide 5 independent length-3 operators:

Example: Dim-10 basis (Form I)

$$\begin{aligned}&\mathcal{O}''_{10;1} = \mathrm{tr}(D_{12}F_{34}D_{15}F_{34}F_{25}),\ \mathcal{O}''_{10;2} = \mathrm{tr}(D_{12}F_{34}D_5F_{34}D_1F_{25}),\ \mathcal{O}''_{10;3} = \mathrm{tr}(D_2F_{34}D_{15}F_{34}D_1F_{25}),\\ &\mathcal{O}''_{10;4} = \mathrm{tr}(D_{12}F_{34}D_1F_{35}D_2F_{45}),\ \mathcal{O}''_{10;5} = \mathrm{tr}(D_{12}F_{34}D_{12}F_{35}F_{45}),\end{aligned} \tag{7}$$

where $\mathcal{O}''_{10;1}$ to $\mathcal{O}''_{10;3}$ are obtained by adding a single DD pair to $\mathcal{O}^{\mathrm{P1}}$, and $\mathcal{O}''_{10;4}, \mathcal{O}''_{10;5}$ are obtained by inserting two DD pairs to $\mathcal{O}^{\mathrm{P2}}$.

For the convenience of loop computations, it is important to further simplify the operator basis such that they manifest certain symmetry properties. This can be achieved by considering the so-called *minimal* tree-level form factors, for which the number of external gluons is equal to the length of the operator. In particular, one can establish a dictionary from an operator to its tree-level minimal form factor [8–10]:

$$\mathcal{O}_L \ \Leftrightarrow\ \mathcal{F}_{\mathcal{O}_L,L}(1,\ldots,L), \tag{8}$$

where each D and F in the operator are mapped to the kinematics in form factor via Table 1.

With the help of minimal form factors, we can decompose operators according to the color structure and helicity structure. There are two color factors for length-3 minimal form factors:

$$\tilde{f}^{abc} = \mathrm{tr}(T^aT^bT^c) - \mathrm{tr}(T^aT^cT^b), \qquad \tilde{d}^{abc} = \mathrm{tr}(T^aT^bT^c) + \mathrm{tr}(T^aT^cT^b), \tag{9}$$

and the corresponding operators associated to these two factors will be said in f-sector and d-sector respectively. Furthermore, the minimal form factors can be divided into two helicity sectors:

$$\begin{aligned}\alpha\text{-sector}: &\quad \mathcal{F}^{(0),\mathrm{min}}_{\mathcal{O}} \neq 0 \text{ only for } (-,-,+),(+,+,-),\\ \beta\text{-sector}: &\quad \mathcal{F}^{(0),\mathrm{min}}_{\mathcal{O}} \neq 0 \text{ only for } (-,-,-),(+,+,+),\end{aligned} \tag{10}$$

where $\pm$ indicates the helicity of external gluons. We summarize the operators in different sectors according the form factor structure as

Operator	$\mathcal{F}^{(0)}(-,-,+)$	$\mathcal{F}^{(0)}(-,-,-)$	color factor
$\mathcal{O}_{\alpha;f}$	$\propto A_1$	0	f^{abc}
$\mathcal{O}_{\alpha;d}$	$\propto A_1$	0	d^{abc}
$\mathcal{O}_{\beta;f}$	0	$\propto A_2$	f^{abc}
$\mathcal{O}_{\beta;d}$	0	$\propto A_2$	d^{abc}

(11)

where the two helicity sectors are associated to two spinor factors:

$$A_1 = \langle 12\rangle^3[13][23], \qquad A_2 = \langle 12\rangle\langle 13\rangle\langle 23\rangle\,. \tag{12}$$

Let us consider again the dim-10 case to illustrate how to choose operators in these sectors. First, by properly (anti)symmetrizing the basis (7), one can write the operators in f and d color sectors as

Example: Dim-10 basis (Form II)

$$\begin{aligned}
&\mathcal{O}'_{10;1} = \frac{1}{2} f^{abc}(D_{12}F_{34})^a(D_{15}F_{34})^b(F_{25})^c, \quad \mathcal{O}'_{10;2} = f^{abc}(D_{12}F_{34})^a(D_5F_{34})^b(D_1F_{25})^c,\\
&\mathcal{O}'_{10;3} = d^{abc}(D_{12}F_{34})^a(D_5F_{34})^b(D_1F_{25})^c,\\
&\mathcal{O}'_{10;4} = \frac{1}{2} f^{abc}(D_{12}F_{34})^a(D_1F_{35})^b(D_2F_{45})^c, \quad \mathcal{O}'_{10;5} = \frac{1}{2} f^{abc}(D_{12}F_{34})^a(D_{12}F_{35})^b(F_{45})^c.
\end{aligned} \tag{13}$$

Next, by investigating the spinor factors of minimal form factors, one can construct operators in different helicity sectors via certain linear combinations. We summarize the final dimension-10 length-3 basis operators that will be used for loop computation as

Example: Dim-10 basis (Form III)

Basis operator	$\mathcal{F}^{(0)}(-,-,+)$	$\mathcal{F}^{(0)}(-,-,-)$	color factor
$\mathcal{O}_{10;\alpha;f;1} = \frac{1}{2}\partial^2\mathcal{O}_{\rm P1} - \frac{1}{12}\partial^4\mathcal{O}_{\rm P2}$	$\frac{1}{2}s_{123}A_1$	0	f^{abc}
$\mathcal{O}_{10;\alpha;f;2} = \mathcal{O}'_{10;1} - \mathcal{O}'_{10;5}$	$\frac{1}{2}s_{12}A_1$	0	f^{abc}
$\mathcal{O}_{10;\alpha;d;1} = \mathcal{O}'_{10;3}$	$\frac{1}{2}(s_{13}-s_{23})A_1$	0	d^{abc}
$\mathcal{O}_{10;\beta;f;1} = \frac{1}{12}\partial^4\mathcal{O}_{\rm P2}$	0	$\frac{1}{4}s_{123}^2A_2$	f^{abc}
$\mathcal{O}_{10;\beta;f;2} = \mathcal{O}'_{10;5}$	0	$\frac{1}{4}(s_{12}^2+s_{23}^2+s_{13}^2)A_2$	f^{abc}

(14)

3 Loop computation via unitarity-IBP

To compute the loop form factors of basis operators, we apply the "unitarity-IBP" strategy that combines unitarity cut [11–13] and integration by parts (IBP) methods [14,15] (using public packages *e.g.* [16–19]). The work flow of our strategy can be illustrated as follows:

$$\mathcal{F}^{(l)}\Big|_{\rm cut} = \prod(\text{Tree blocks}) = \text{Cut integrand} \xrightarrow{\text{IBP with cuts}} \sum_i c_i\left(I_i\big|_{\rm cut}\right). \tag{15}$$

This strategy has been used to study form factors and Higgs amplitudes in [6,7,20,21] and for pure gluon amplitudes in [22,23]. Similar strategy has been used in the numerical unitarity approach [24,25], and the idea of applying cuts to simplify IBP has also been used in [26–29].

The complete set of two-loop master integrals for minimal length-3 form factors are given in Fig. 1. The two-loop color-ordered form factors, associated with color factor $\mathrm{tr}(T^{a_1}T^{a_2}T^{a_3})$ via color decomposition, can be written as a sum of master integrals I_i as

$$F^{(2)}_{\mathcal{O}} = \Big[c_1I_1 + c_2I_2 + c_3I_3 + c_4I_4 + \big(c_5I_5 + c_6I_6\big) + \big(c_7I_7 + c_8I_8\big) + c_9I_9\Big] + \text{cyc.perm.}\,, \tag{16}$$

where master integrals I_i correspond to the topology and labeling given in Fig. 1. The main goal is to compute the master coefficients c_i. The spanning set of unitarity cuts to fix all coefficients are given in Fig. 2. Note that the two-loop minimal form factors of length-3 operators have no sub-leading-color contribution, thus the set of planar cuts are enough to fix the full results. More details can be found in [6].

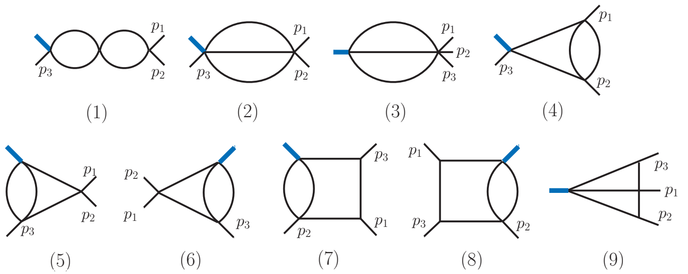

Figure 1: Master integrals for the planar two-loop minimal form factors.

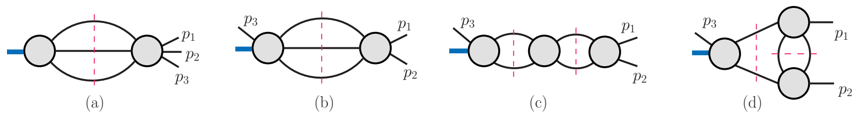

Figure 2: Complete set of unitarity cuts for the planar two-loop form factors.

4 Results and analysis

The master integrals in Fig. 1 are known in terms of 2d harmonic polylogarithms [30, 31]. Together with master coefficients, the form factors can be written in explicit functional form, from which one can extract the wanted physical information. The bare form factors contain divergences and can be schematically expanded as:

$$\text{Loop form factor} = (\text{Universal IR div.}) + (\text{UV div.}) + (\text{Finite part})\,, \tag{17}$$

where the infrared (IR) divergences depend only on the configuration of external on-shell states, while the UV divergences are related to the operator and coupling renormalization.

Operator renormalization

In dimensional regularization, both IR and UV divergences are regularized by $\epsilon = (4-D)/2$, and it may seem non-trivial to disentangle the two divergences. Fortunately, this problem can be easily solved, thanks to the universal structure of IR divergences. In particular, the two-loop IR can be obtained by the Catani form [32], which is determined by the one-loop form factor together with some universal functions (independent of operators).

After subtracting IR divergences, the remaining UV divergences can be eliminated by performing operator renormalization. The renormalization constant Z in general takes a *matrix* form as (the subscripts R or B indicates renormalized or bare operators):

$$\mathcal{O}_{R,i} = Z_i^{\ j}\mathcal{O}_{B,j}\,, \tag{18}$$

since different basis operators generally mix with each other under renormalization. From the renormalization constant, one can further define the dilation operator as

$$\mathbb{D} = -\frac{d\log Z}{d\log\mu}\,. \tag{19}$$

Finally, the eigenvalues of the dilatation operator give the anomalous dimensions.

Below we present the results for the dimension-10 operators in (14). Separated in f and

d sectors, the renormalization matrices at one loop are

$$Z^{(1)}_{\mathcal{O}_{10,f}} = \frac{N_c}{\epsilon}\left(\begin{array}{c|cc|cc} -\frac{11}{3} & 0 & 0 & 0 & 0 \\ \hline 0 & \frac{7}{6} & 0 & 0 & 0 \\ 0 & -\frac{3}{5} & \frac{71}{30} & 0 & 0 \\ \hline 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -1 & \frac{17}{6} \end{array}\right), \qquad Z^{(1)}_{\mathcal{O}_{10;d}} = \frac{N_c}{\epsilon}\frac{13}{6}, \tag{20}$$

where we have included the length-2 operator in the f-sector and the basis is chosen as $\{\partial^6 \mathrm{tr}(F^2), \mathcal{O}_{10;\alpha;f;1}, \mathcal{O}_{10;\alpha;f;2}, \mathcal{O}_{10;\beta;f;1}, \mathcal{O}_{10;\beta;f;2}\}$. For the two-loop renormalization matrices, the $1/\epsilon^2$ part is determined by one-loop results, and the intrinsic new information is contained in the $1/\epsilon$ part, given as

$$Z^{(2)}_{\mathcal{O}_{10,f}}\Big|_{\frac{1}{\epsilon}\text{-part.}} = \frac{N_c^2}{\epsilon}\left(\begin{array}{c|cc|cc} -\frac{34}{3} & 0 & 0 & 0 & 0 \\ \hline -\frac{1}{3} & \frac{269}{72} & 0 & \frac{5}{2} & 0 \\ -\frac{209}{900} & -\frac{5579}{18000} & \frac{712}{125} & \frac{1493}{1200} & \frac{5}{36} \\ \hline -1 & 0 & 0 & \frac{25}{12} & 0 \\ -\frac{19}{36} & \frac{139}{2400} & \frac{499}{800} & -\frac{143}{288} & \frac{2195}{288} \end{array}\right), \quad Z^{(2)}_{\mathcal{O}_{10;d}}\Big|_{\frac{1}{\epsilon}\text{-part.}} = \frac{575}{144}\frac{N_c^2}{\epsilon}. \tag{21}$$

From them it is straightforward to compute the dilation operator as well as anomalous dimensions, see [6] for more detail and further results up to dimension 16. (See also some previous one-loop results on high dimensional operators in [33–37].)

Finite remainder

The finite remainder parts of the form factors capture important information for the Higgs-plus-three-gluon amplitudes. The two-loop form factor with the leading operator $\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$ was computed in [38], and the results with high dimension operators correspond to Higgs amplitudes with high order of top mass corrections [6,7] can be used to improved the precision for the cross section of Higgs plus a jet production. Below we discuss interesting features about their analytic structure.

One can decompose the two-loop remainder functions according to their trancendentality degree as:

$$\mathcal{R}^{(2)}_{\mathcal{O}} = \sum_{n=0}^{4} \mathcal{R}^{(2)}_{\mathcal{O}}\Big|_{\text{deg-}n}, \tag{22}$$

where the maximal transcendentality degree is 4 at two loops. Here "transcendental degree" is a mathematical notation characterizing the algebraic complexity of functions and numbers. For example, the degree of rational functions is zero, π or $\log(x)$ has degree 1, and the Riemann zeta value ζ_n or polylogrithm $\mathrm{Li}_n(x)$ has degree n.

The maximally transcendental part turns out to be universally given by the function (see also [20,21,39–41]):

$$\begin{aligned} \mathcal{R}^{(2)}_{\mathcal{O}}\Big|_{\text{deg-4}} = & -\frac{3}{2}\mathrm{Li}_4(u) + \frac{3}{4}\mathrm{Li}_4\left(-\frac{uv}{w}\right) - \frac{3}{4}\log(w)\left[\mathrm{Li}_3\left(-\frac{u}{v}\right) + \mathrm{Li}_3\left(-\frac{v}{u}\right)\right] \\ & + \frac{\log^2(u)}{32}\left[\log^2(u) + \log^2(v) + \log^2(w) - 4\log(v)\log(w)\right] \\ & + \frac{\zeta_2}{8}\left[5\log^2(u) - 2\log(v)\log(w)\right] - \frac{1}{4}\zeta_4 + \text{perms}(u,v,w), \end{aligned} \tag{23}$$

where

$$u = \frac{s_{12}}{s_{123}}, \qquad v = \frac{s_{23}}{s_{123}}, \qquad w = \frac{s_{13}}{s_{123}}. \tag{24}$$

This suggests that the two-loop minimal form factors of general high-dimensional length-3 operators obey the maximal transcendentality principle [42–44].

Moreover, lower transcendentality degree parts also present universal structures. The degree-3 part is determined by the function $T_3(u,v,w)$ given as

$$
\begin{aligned}
T_3(u,v,w) :=& \Big[-\mathrm{Li}_3\left(-\frac{u}{w}\right) + \log(u)\mathrm{Li}_2\left(\frac{v}{1-u}\right) - \frac{1}{2}\log(u)\log(1-u)\log\left(\frac{w^2}{1-u}\right) \\
&+ \frac{1}{2}\mathrm{Li}_3\left(-\frac{uv}{w}\right) + \frac{1}{2}\log(u)\log(v)\log(w) + \frac{1}{12}\log^3(w) + (u \longleftrightarrow v) \Big] \\
&+ \mathrm{Li}_3(1-v) - \mathrm{Li}_3(u) + \frac{1}{2}\log^2(v)\log\left(\frac{1-v}{u}\right) - \zeta_2 \log\left(\frac{uv}{w}\right),
\end{aligned}
\tag{25}
$$

plus simple $\pi^2 \times \log$ and ζ_3 terms. Building blocks of degree-2 part are $T_2(u,v)$:

$$
T_2(u,v) := \mathrm{Li}_2(1-u) + \mathrm{Li}_2(1-v) + \log(u)\log(v) - \zeta_2 ,
\tag{26}
$$

together with $\log \times \log$ and π^2 terms. When expanding the remainders in these deg-3 and deg-2 building blocks, the coefficients in front of them are just rational functions of u, v, w. Similar transcendental functions also appeared in the $\mathcal{N}=4$ form factors [39, 45, 46].

5 Conclusion

We construct the operator basis for high-dimensional YM operators. The efficient on-shell unitarity-IBP method is applied to compute the loop form factors associated with these operators, which are also related to Higgs EFT amplitudes. The two-loop renormalization is performed for operators up to dimension 16, and the finite remainder functions are also obtained. We can summarize the main steps and results in the following graph:

$$
\boxed{\begin{array}{c}\text{Operators}\\ \text{in YM}\end{array}} \xrightarrow[\text{BI}]{\text{EoM}} \boxed{\begin{array}{c}\text{Operator basis}\\ \text{in } D\text{-dim}\end{array}} \xRightarrow{\text{Unitarity-IBP}} \boxed{\begin{array}{c}\text{Loop}\\ \text{form factors}\end{array}} \to \left\{ \begin{array}{l} \boxed{\begin{array}{c}\text{UV}\\ \text{renormalization}\end{array}} \\ \boxed{\text{Remainders}} \end{array} \right.
$$

Although our discussion has mostly focused on the length-3 operators, we would like to stress that similar methods can be generalized to deal with more general higher length operators. Since our method works in full D dimensions in dimensional regularization, the strategy can be also used to study the so-called evanescent operators which will be discussed elsewhere [47].

Acknowledgements

This work is supported in part by the National Natural Science Foundation of China (Grants No. 12175291, 11822508, 11935013, 12047502, 12047503, 11947301). We also thank the support of the HPC Cluster of ITP-CAS.

References

[1] J. Ellis, M. K. Gaillard and D. V. Nanopoulos, *A phenomenological profile of the Higgs boson*, Nucl. Phys. B **106**, 292 (1976), doi:10.1016/0550-3213(76)90382-5.

[2] H. M. Georgi, S. L. Glashow, M. E. Machacek and D. V. Nanopoulos, *Higgs Bosons from Two-Gluon Annihilation in Proton-Proton Collisions*, Phys. Rev. Lett. **40**, 692 (1978), doi:10.1103/PhysRevLett.40.692.

[3] F. Wilczek, *Decays of Heavy Vector Mesons into Higgs Particles*, Phys. Rev. Lett. **39**, 1304 (1977), doi:10.1103/PhysRevLett.39.1304.

[4] M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, *Low-Energy Theorems for Higgs Boson Couplings to Photons*, Sov. J. Nucl. Phys. **30**, 711 (1979), [Yad. Fiz. **30**, 1368 (1979)].

[5] G. Yang, *On-shell methods for form factors in $\mathcal{N} = 4$ SYM and their applications*, Sci. China Phys. Mech. Astron. **63**, 270001 (2020), doi:10.1007/s11433-019-1507-0.

[6] Q. jun Jin, K. Ren and G. Yang, *Two-loop anomalous dimensions of QCD operators up to dimension-sixteen and Higgs EFT amplitudes*, J. High Energy Phys. **04**, 180 (2021), doi:10.1007/JHEP04(2021)180.

[7] Q. Jin and G. Yang, *Two-Loop QCD Corrections to the Higgs Plus Three-parton Amplitudes with Top Mass Correction*, J. High Energy Phys. **02**, 169 (2020), doi:10.1007/JHEP02(2020)169.

[8] N. Beisert, *On Yangian Symmetry in Planar $\mathcal{N} = 4$ SYM*, In Quantum chromodynamics and beyond: Gribov-80 memorial volume. Proceedings, Memorial Workshop devoted to the 80th birthday of V.N. Gribov, Trieste, Italy, May 26-28, 2010, 175 (2011), doi:10.1142/9789814350198_0039.

[9] B. I. Zwiebel, *From scattering amplitudes to the dilatation generator in $\mathcal{N} = 4$ SYM*, J. Phys. A: Math. Theor. **45**, 115401 (2012), doi:10.1088/1751-8113/45/11/115401.

[10] M. Wilhelm, *Amplitudes, Form Factors and the Dilatation Operator in $\mathcal{N} = 4$ SYM Theory*, J. High Energy Phys. **02**, 149 (2015), doi:10.1007/JHEP02(2015)149.

[11] Z. Bern, L. Dixon, D. C. Dunbar and D. A. Kosower, *One-loop n-point gauge theory amplitudes, unitarity and collinear limits*, Nucl. Phys. B **425**, 217 (1994), doi:10.1016/0550-3213(94)90179-1.

[12] Z. Bern, L. Dixon, D. C. Dunbar and D. A. Kosower, *Fusing gauge theory tree amplitudes into loop amplitudes*, Nucl. Phys. B **435**, 59 (1995), doi:10.1016/0550-3213(94)00488-Z.

[13] R. Britto, F. Cachazo and B. Feng, *Generalized unitarity and one-loop amplitudes in super-Yang–Mills*, Nucl. Phys. B **725**, 275 (2005), doi:10.1016/j.nuclphysb.2005.07.014.

[14] K. G. Chetyrkin and F. V. Tkachov, *Integration by parts: The algorithm to calculate β-functions in 4 loops*, Nucl. Phys. B **192**, 159 (1981), doi:10.1016/0550-3213(81)90199-1.

[15] F. V. Tkachov, *A theorem on analytical calculability of 4-loop renormalization group functions*, Phys. Lett. B **100**, 65 (1981), doi:10.1016/0370-2693(81)90288-4.

[16] A. V. Smirnov, *FIRE5: A C++ implementation of Feynman Integral REduction*, Comput. Phys. Commun. **189**, 182 (2015), doi:10.1016/j.cpc.2014.11.024.

[17] R. N. Lee, *LiteRed 1.4: a powerful tool for reduction of multiloop integrals*, J. Phys.: Conf. Ser. **523**, 012059 (2014), doi:10.1088/1742-6596/523/1/012059.

[18] P. Maierhöfer, J. Usovitsch and P. Uwer, *Kira—A Feynman integral reduction program*, Comput. Phys. Commun. **230**, 99 (2018), doi:10.1016/j.cpc.2018.04.012.

[19] A. von Manteuffel and C. Studerus, *Reduze 2 - Distributed Feynman Integral Reduction*, arXiv:1201.4330.

[20] Q. Jin and G. Yang, *Analytic Two-Loop Higgs Amplitudes in Effective Field Theory and the Maximal Transcendentality Principle*, Phys. Rev. Lett. **121**, 101603 (2018), doi:10.1103/PhysRevLett.121.101603.

[21] Q. Jin and G. Yang, *Hidden analytic relations for two-loop Higgs amplitudes in QCD*, Commun. Theor. Phys. **72**, 065201 (2020), doi:10.1088/1572-9494/ab7ed8.

[22] R. H. Boels, Q. Jin and H. Luo, *Efficient integrand reduction for particles with spin*, arXiv:1802.06761.

[23] Q. Jin and H. Luo, *Analytic Form of the Three-loop Four-gluon Scattering Amplitudes in Yang-Mills Theory*, arXiv:1910.05889.

[24] S. Abreu, F. Febres Cordero, H. Ita, M. Jaquier, B. Page and M. Zeng, *Two-Loop Four-Gluon Amplitudes from Numerical Unitarity*, Phys. Rev. Lett. **119**, 142001 (2017), doi:10.1103/PhysRevLett.119.142001.

[25] S. Abreu, F. Febres Cordero, H. Ita, B. Page and M. Zeng, *Planar two-loop five-gluon amplitudes from numerical unitarity*, Phys. Rev. D **97**, 116014 (2018), doi:10.1103/PhysRevD.97.116014.

[26] D. A. Kosower and K. J. Larsen, *Maximal unitarity at two loops*, Phys. Rev. D **85**, 045017 (2012), doi:10.1103/PhysRevD.85.045017.

[27] K. J. Larsen and Y. Zhang, *Integration-by-parts reductions from unitarity cuts and algebraic geometry*, Phys. Rev. D **93**, 041701 (2016), doi:10.1103/PhysRevD.93.041701.

[28] H. Ita, *Two-loop integrand decomposition into master integrals and surface terms*, Phys. Rev. D **94**, 116015 (2016), doi:10.1103/PhysRevD.94.116015.

[29] A. Georgoudis, K. J. Larsen and Y. Zhang, *Azurite: An algebraic geometry based package for finding bases of loop integrals*, Comput. Phys. Commun. **221**, 203 (2017), doi:10.1016/j.cpc.2017.08.013.

[30] T. Gehrmann and E. Remiddi, *Two-loop master integrals for jets: the planar topologies*, Nucl. Phys. B **601**, 248 (2001), doi:10.1016/S0550-3213(01)00057-8.

[31] T. Gehrmann and E. Remiddi, *Numerical evaluation of two-dimensional harmonic polylogarithms*, Comput. Phys. Commun. **144**, 200 (2002), doi:10.1016/S0010-4655(02)00139-X.

[32] S. Catani, *The singular behaviour of QCD amplitudes at two-loop order*, Phys. Lett. B **427**, 161 (1998), doi:10.1016/S0370-2693(98)00332-3.

[33] A. Y. Morozov, *Matrix of mixing of scalar and vector mesons of dimension $D \leq 8$ in QCD*, Sov. J. Nucl. Phys. **40**, 505 (1984). [Yad. Fiz. **40**, 788 (1984)].

[34] J. A. Gracey, *Classification and one loop renormalization of dimension-six and dimension-eight operators in quantum gluodynamics*, Nucl. Phys. B **634**, 192 (2002), doi:10.1016/S0550-3213(02)00334-6, [Erratum: Nucl. Phys. B **696**, 295 (2004), doi:10.1016/j.nuclphysb.2004.06.053].

[35] D. Neill, *Two-Loop Matching onto Dimension Eight Operators in the Higgs-Glue Sector*, arXiv:0908.1573.

[36] R. V. Harlander and T. Neumann, *Probing the nature of the Higgs-gluon coupling*, Phys. Rev. D **88**, 074015 (2013), doi:10.1103/PhysRevD.88.074015.

[37] S. Dawson, I. M. Lewis and M. Zeng, *Effective field theory for Higgs boson plus jet production*, Phys. Rev. D **90**, 093007 (2014), doi:10.1103/PhysRevD.90.093007.

[38] T. Gehrmann, M. Jaquier, E. W. N. Glover and A. Koukoutsakis, *Two-loop QCD corrections to the helicity amplitudes for $H \to 3$ partons*, J. High Energy Phys. **02**, 056 (2012), doi:10.1007/JHEP02(2012)056.

[39] A. Brandhuber, M. Kostacińska, B. Penante and G. Travaglini, *Higgs Amplitudes from $\mathcal{N} = 4$ Supersymmetric Yang-Mills Theory*, Phys. Rev. Lett. **119**, 161601 (2017), doi:10.1103/PhysRevLett.119.161601.

[40] A. Brandhuber, M. Kostacińska, B. Penante and G. Travaglini, *$Tr(F^3)$ supersymmetric form factors and maximal transcendentality Part I: $\mathcal{N} = 4$ super Yang-Mills*, J. High Energy Phys. **12**, 076 (2018), doi:10.1007/JHEP12(2018)076.

[41] A. Brandhuber, M. Kostacińska, B. Penante and G. Travaglini, *$Tr(F^3)$ supersymmetric form factors and maximal transcendentality Part II: $0 < \mathcal{N} < 4$ super Yang-Mills*, J. High Energy Phys. **12**, 077 (2018), doi:10.1007/JHEP12(2018)077.

[42] A. V. Kotikov and L. N. Lipatov, *DGLAP and BFKL equations in the $N = 4$ supersymmetric gauge theory*, Nucl. Phys. B **661**, 19 (2003), doi:10.1016/S0550-3213(03)00264-5, [Erratum: Nucl. Phys. B **685**, 405 (2004), doi:10.1016/j.nuclphysb.2004.02.032].

[43] A. Kotikov, L. Lipatov, A. Onishchenko and V. Velizhanin, *Three loop universal anomalous dimension of the Wilson operators in N=4 SUSY Yang-Mills model*, Phys. Lett. B **595**, 521 (2004), doi:10.1016/j.physletb.2004.05.078.

[44] A. Brandhuber, G. Travaglini and G. Yang, *Analytic two-loop form factors in $\mathcal{N} = 4$ SYM*, J. High Energy Phys. **05**, 082 (2012), doi:10.1007/JHEP05(2012)082.

[45] F. Loebbert, D. Nandan, C. Sieg, M. Wilhelm and G. Yang, *On-shell methods for the two-loop dilatation operator and finite remainders*, J. High Energy Phys. **10**, 012 (2015), doi:10.1007/JHEP10(2015)012.

[46] F. Loebbert, C. Sieg, M. Wilhelm and G. Yang, *Two-loop SL(2) form factors and maximal transcendentality*, J. High Energy Phys. **12**, 090 (2016), doi:10.1007/JHEP12(2016)090.

[47] Q. Jin, K. Ren, G. Yang and R. Yu, *Gluonic evanescent operators: classification and one-loop renormalization*, arXiv:2202.08285.