Next-to-leading power SCET in Higgs amplitudes induced by light quarks

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Abstract

In this article, we report how to use the "plus-type" subtraction scheme to deal with end-point divergences in $h \to \gamma\gamma$ or $gg \to h$ amplitudes induced by light quark loop, which can be formulated by next-to-leading power (NLP) SCET. This subtraction is ensured by two re-factorization conditions, which have been proven to all orders in α_s. Based on these conditions, cutoffs emerge naturally after some re-arrangements to handle endpoint divergences and renormalization is compatible with that. Our formalism can analytically reproduce three-loop amplitudes up to $\ln^3(-M_h^2/m_b^2 - i0)$ and resum to next-to-leading logarithm and beyond.

1 Introduction

Soft-collinear effective theory (SCET) [1–3] has been proven a powerful tool at leading power to deal with multi-scale problems at colliders. Recently, there have been a lot of works for different processes on understanding the structure beyond the leading order in power counting, including event shape [4, 5], Drell-Yan [6–8], Higgs decay via light quark loops [9, 10] and off-diagonal splitting function [11]. It turns out that endpoint divergence occurring for convolutions of factorized quantities is a common feature at next-to-leading power (NLP). It makes renormalization and therefore resummation rather non-trivial and is not yet fully understood how to handle it. Recently, Liu and Neubert [9] found that rapidity divergences in Higgs amplitudes induced by a light quark loop can help to find two re-factorization conditions. We have proven these conditions to all orders in the language of SCET in [12]. We have also shown that based on these conditions, endpoint divergences can be cured from the start by a "plus-type"

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[1]The speaker, based on works with Ze Long Liu, Bianka Mecaj, Matthias Neubert and Marvin Schnubel.

subtraction scheme and this scheme is compatible with renormalization. This formalism has been validated to three loops, which is a key step to understand NLP SCET further. We can resum large logarithms to next-to-next-to-leading logarithm (NLL) and even beyond, with the logarithm being the ratio of the Higgs mass and the light quark mass: $L = \ln(-M_h^2/m_b^2 - i0)$.

In this article, we first briefly review the factorization formula for the Higgs amplitude induced by a b quark loop in Sec. 1. Then we present the logic of the subtraction method in Sec. 3. Sec. 4 is devoted to illustrate how to renormalize and discuss the RGEs and resummation. We conclude in Sec. 5.

2 Factorization

In this section, we use $h \to \gamma\gamma$ by a b quark loop as an example to summarize the operator basis and review the factorization formula at the bare level [9]. The operator basis and the factorization formula for $gg \to h$ are rather similar [13].

The bare decay amplitude $\mathcal{M}$ contains three terms consisting of unrenormalized SCET operators multiplied (convoluted) by bare Wilson coefficients. The third operator O_3 can be further factorized into the convolution of two radiative jet functions and the soft quark soft function. To this end, the bare factorization reads:

$$\mathcal{M} = H_1 \langle O_1 \rangle + 2 \int_0^1 dz \, H_2(z) \langle O_2(z) \rangle + H_3 \int_0^\infty \frac{d\ell_-}{\ell_-} \int_0^\infty \frac{d\ell_+}{\ell_+} J(M_h \ell_-) J(-M_h \ell_+) S(\ell_- \ell_+). \tag{1}$$

We will call the three terms T_1^{unsub}, T_2^{unsub} and T_3^{unsub} respectively. The convolutions in the last two terms are a common feature at NLP. On the one hand, unlike the leading power cases where there is only one collinear field at one collinear sector, there can be several collinear fields at one collinear sector. As a result, one has to integrate over the momentum fraction z distributed among these collinear particles[2]. On the other hand, there can be some power suppressed interactions now. Hence one has to integrate over the inserting position of such interactions, which is (ℓ_-, ℓ_+) in this example after multiple expansions in momentum space.

It turns out that such convolutions are usually divergent when approaching to endpoints: $z \to 0, 1$ and $\ell_-, \ell_+ \to \infty$ in (1). This can be easily shown by noticing that the leading order $\langle O_2(z) \rangle$ is a constant, while the leading order $H_2(z)$ goes like $1/z(1-z)$. Only part of the divergences can be regularized by dimensional regulator, while the remaining ones are rapidity divergences. Consequently, one can't renormalize the operators as usual and naively replace the bare ones in (1) with the their renormalized counterparts because rapidity divergences still exist when $z \to 0, 1$ and $\ell_-, \ell_+ \to \infty$. These superficially troublesome divergences turn to be useful to construct our prescription to cure endpoint divergences.

3 Plus-type subtraction

Subtraction is a widely used method to deal with divergences. In this section, we show how to use a "plus-type" subtraction scheme to cure endpoint divergences and how it works with renormalization.

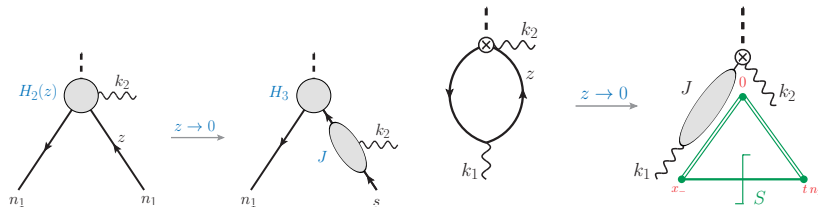[2]It is symmetric under $z \leftrightarrow \bar{z} = 1 - z$.

Figure 1: Graphical illustration of the two refactorization condition.

3.1 Re-factorization conditions

As explained in the previous section, both T_2^{unsub} and T_3^{unsub} have rapidity divergences. However the amplitude is free of rapidity divergences, which means that there is a cancellation between T_2^{unsub} and T_3^{unsub}. This indicates that there are some close relations between them.

By using a suitable rapidity regulator [9], it has been shown how the rapidity divergences (regularized as $1/\eta$, where η is the regulator) cancel among the sum of T_2^{unsub} and T_3^{unsub}. This is highly non-trivial because H_2 and O_2 depend on the hard scale M_h^2, and the collinear scale m_b^2 respectively, while T_3^{unsub} also involves the object depending on the hard-collinear scale $M_h m_b$, which is the jet function. Objects depending on different scales are related indicates that there are some re-factorization conditions to make the scale separation consistently, just resembles the structure of O_3. These conditions were observed and calculated at NLO in [9]. They were proved to all orders in α_s in [14] using the language of SCET. Here we just want to illustrate them by Fig. 1 and they read:

$$[\![H_2(z)]\!] = -\frac{H_3}{z} J(z M_h^2), \quad [\![\langle O_2(z) \rangle]\!] = -\frac{\varepsilon_\perp^*(k_1) \cdot \varepsilon_\perp^*(k_2)}{2} \int_0^\infty \frac{\mathrm{d}\ell_+}{\ell_+} J(-M_h \ell_+) S(z M_h \ell_+), \quad (2)$$

where $[\![f(z)]\!]$ means only keeping the leading terms for $z \to 0$. Note that the second one involves a convolution, such that it only makes sense at the bare level, otherwise it will develop some divergence for $\ell_+ \to \infty$. A similar formula as the first one also appears in [11]. For $gg \to h$, two very similar re-fractorization conditions hold as well.

3.2 Subtraction and renormalization

Note that $[\![H_2(z)]\!]$ and $[\![\langle O_2(z) \rangle]\!]$ are the endpoint versions of $H_2(z)$ and $\langle O_2(z) \rangle$ respectively. This means we can subtract $[\![H_2(z)]\!][\![\langle O_2(z) \rangle]\!]$ from the bare integrand of T_2^{unsub}, such that the subtracted T_2^{unsub} does not have endpoint divergences any more. Then we have to add the subtracted terms back to T_3^{unsub}. Thanks to the two re-fractorization conditions, the subtraction has the same integrand as in T_3^{unsub}, but with a different integral limit, such that the neat effect is the change of integration upper boundaries from infinity to the hard scale! To this end, the amplitude $\mathcal{M}$ in (1) becomes [9]:

$$(H_1 + \Delta H_1)\langle O_1 \rangle + 2 \int_0^1 \mathrm{d}z \Big[H_2(z) \langle O_2(z) \rangle - [\![H_2(z)]\!][\![\langle O_2(z) \rangle]\!] - [\![H_2(\bar{z})]\!][\![\langle O_2(\bar{z}) \rangle]\!] \Big]$$

$$+ \lim_{\sigma \to -1} H_3 \int_0^{M_h} \frac{\mathrm{d}\ell_-}{\ell_-} \int_0^{\sigma M_h} \frac{\mathrm{d}\ell_+}{\ell_+} J(M_h \ell_-) J(-M_h \ell_+) S(\ell_- \ell_+) \Big|_{\text{LP}}, \quad (3)$$

where the analytic continuation and leading power expansion have to be taken in the end. ΔH_1 is the so-called infinity bin contribution, which has been subtracted twice, so we need to add it back to H_1. We now denote the three contributions above as T_1, T_2 and T_3. A key point

is that T_i's are now free of endpoint divergences and we no longer need rapidity regulator. Note that (3) is still at the bare level.

Now turn to renormalization of (3). Notice that O_3 is a time product between the scalar current and the NLP Lagrangian, which does not need renormalization. So O_3 renormalizes like the scalar current via Z_{33}, which has been known to three loops [15]. Then it is straightforward to get $H_3(\mu) = Z_{33}^{-1}H_3$. We know that O_3 factorizes as a convolution of the jet functions and the soft function and the jet function also appears in the factorization formula of $B^- \to \gamma \ell^- \bar{\nu}$. Consequently Z_J can be deduced from the RG consistency of this decay amplitude. Recently its renormalization has also been studied directly in [16]. Then we deduced Z_S for the soft function from the information of O_3 and the jet function and it was confirmed directly in [17]. Again thanks to the first re-fractorization condition in (2), we also know the $Z-$factor for $[\![H_2(z)]\!]$.

$$J(p^2,\mu) = \int_0^\infty \mathrm{d}x\, Z_J(p^2,xp^2)J(xp^2), \quad S(w,\mu) = \int_0^\infty \mathrm{d}w'\, Z_S(w,w')S(w'). \tag{4}$$

The $Z-$factor of $O_2(z)$ can be calculated directly, and the corresponding diagrams are given in [14]. It turns out that it is not only renormalized multiplicatively, but it mixes with O_1 under renormalization as well. Since the colour fields in O_2 are the same as those in the leading-twist light-cone distribution amplitude of a transversely polarized vector meson, it is not surprising that the well-known Brodsky-Lepage kernel enters here. By taking the endpoint limit, we can directly obtain the factors for $[\![O_2(z)]\!]$, whose non-mixing term is nothing but the inverse of that of $[\![H_2(z)]\!]$, serving as a crosscheck.

$$\langle O_2(z,\mu)\rangle = \int_0^1 \mathrm{d}z'\, Z_{22}(z,z')\langle O_2(z')\rangle + Z_{21}(z)\langle O_1\rangle,$$

$$[\![\langle O_2(z,\mu)\rangle]\!] = \int_0^\infty \mathrm{d}z'\, [\![Z_{22}(z,z')]\!][\![\langle O_2(z')\rangle]\!] + [\![Z_{21}(z)]\!]\langle O_1\rangle,$$

$$H_2(z,\mu) = \int_0^1 \mathrm{d}z'\, H_2(z')Z_{22}^{-1}(z',z), \tag{5}$$

$$[\![H_2(z,\mu)]\!] = \int_0^\infty \mathrm{d}z'\, [\![H_2(z')]\!][\![Z_{22}^{-1}(z',z)]\!].$$

Renormalizing O_1 is trivial: $\langle O_1(\mu)\rangle = Z_{11}\langle O_1\rangle$, but renormalizing H_1 is the most complicated throughout this project. This complexity originates from the fact that there are no cutoffs when renormalizing, see (4), but there exist cutoffs in the convolutions of the factorization formula (3) both for T_3 and the subtracted terms in T_2. As a result, when we re-express the bare quantities in T_2 and T_3 in terms of their renormalized counterparts, there will be some extra contribution, because exchanging the integration boundaries does not commute here (or there is mismatch for the integral boundaries) We call them "left-over" terms. By a careful proof to all orders in α_s, we find that the sum of "left-over" terms in T_2 and T_3, denoted as $\delta H_1^{\mathrm{tot}}$, is purely hard, which can be naturally absorbed into H_1. This happens because of the two re-fractorization conditions. On top of that, the renormalized H_1 also obtains contribution from operator mixing in (5). To this end, we arrive at the formula:

$$H_1(\mu) = \left[H_1 + \Delta H_1 + \delta H_1^{\mathrm{tot}}\right]Z_{11}^{-1} + 2\int_0^1 \mathrm{d}z\Big(H_2(z)Z_{21}^{-1}(z) - [\![H_2(z)]\!][\![Z_{21}^{-1}(z)]\!]$$

$$-[\![H_2(\bar{z})]\!][\![Z_{21}^{-1}(\bar{z})]\!]\Big), \tag{6}$$

where Z_{21}^{-1} is calculated from Z_{22}^{-1}, Z_{21} and Z_{11}^{-1}, likewise for $[\![Z_{21}^{-1}]\!]$.

At the end of the day, we get the factorization formula with renormalized quantities:

$$
\begin{aligned}
T_1^R =& H_1(\mu)\langle O_1(\mu)\rangle, \\
T_2^R =& 2\int_0^1 \mathrm{d}z\Big[H_2(z,\mu)\langle O_2(z,\mu)\rangle - [\![H_2(z,\mu)]\!][\![\langle O_2(z,\mu)\rangle]\!] - [\![H_2(\bar{z},\mu)]\!][\![\langle O_2(\bar{z},\mu)\rangle]\!]\Big], \\
T_3^R =& \lim_{\sigma\to-1} H_3(\mu)\int_0^{M_h}\frac{\mathrm{d}\ell_-}{\ell_-}\int_0^{\sigma M_h}\frac{\mathrm{d}\ell_+}{\ell_+}J(M_h\ell_-,\mu)J(-M_h\ell_+,\mu)S(\ell_-\ell_+,\mu)\Big|_{\text{LP}}.
\end{aligned}
\tag{7}
$$

Now the amplitude is just $T_1^R + T_2^R + T_3^R$, and it is free of any divergences. At first sight, the cutoffs may seem artificial. However they are not. They emerge as a result of the subtracted terms. Subtracted terms in T_2 and T_3 are the same up to a minus sigh, which is the result of re-factorization conditions, which are of physical nature of this problem. Emergence of cutoffs come at a price when writing down the renormalized factorization formula, but again re-factorization conditions play a key role.

4 Renormalization and resummation

It is straightforward to derive the anomalous dimensions from the renormalization factors [18]. And based on the knowledge of two-loop anomalous dimension of the jet function, we can go even beyond one-loop except $O_2(z,\mu)$. We present results for $S(w,\mu)$ and $H_1(\mu)$ as two examples. For the soft function, the RGE reads

$$
\frac{\mathrm{d}S(w,\mu)}{\mathrm{d}\ln\mu} = -\int_0^\infty \mathrm{d}x\,\gamma_S(w,w/x)S(w/x,\mu),
\tag{8}
$$

with[3]

$$
\gamma_S(w,w/x) = -\Big[\Gamma_{\text{cusp}}\,(\alpha_s)\ln\frac{w}{\mu^2} - \gamma_s(\alpha_s)\Big]\delta(1-x) - 2\Gamma_{\text{cusp}}\,(\alpha_s)\Gamma(1,x) - 2\Big(\frac{\alpha_s}{4\pi}\Big)^2 h(x),
\tag{9}
$$

up to $\mathcal{O}(\alpha_s^3)$, where $\Gamma(1,x)$ is the Lange-Neubert kernel [19] and $h(x)$ has recently been derived in [20]. At one-loop[4] $\gamma_{s,0} = -6C_F$. One can find the explicit expression for γ_s at two-loop in [14, 21]. The solution of this equation has been studied in [21] at RG improved NLO accuracy. An interesting mathematical structure appears due to the two non-local kernels. After resummation, the soft function becomes continuous at the threshold point $w = m_b^2$. It is worthwhile to compare with the anomalous dimension of the soft quark soft function for $gg\to h$. The soft function for $gg\to h$ differs with that for $h\to\gamma\gamma$ by insertion of two colour matrices, which changes the UV behaviour. The RGE takes the same structure as (8) but with a different anomalous dimension. At one-loop order, it reads

$$
\gamma_S^{gg}(w,w/x) = -\frac{\alpha_s}{\pi}\left\{\Big[(C_F-C_A)\ln\frac{w}{\mu^2} + \frac{3C_F-\beta_0}{2}\Big]\delta(1-x) + 2\Big(C_F - \frac{C_A}{2}\Big)\Gamma(1,x)\right\}.
\tag{10}
$$

We define r_Γ as the ratio between the coefficient of the logarithmic term and that of the Lange-Neubert kernel. For $h\to\gamma\gamma$, $r_\Gamma = 2$, it allows the RG solution to be expressed in terms of Meijer-G function. But for $gg\to h$, $r_\Gamma = 1/5$, the solution is not even single-valued in the complex plane. The same story also goes for the jet function. But for NLL resummation, we

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[3]Here the cusp anomalous dimension is in the fundamental representation.

[4]In this report, we always adopt the convention: $\gamma_V(\alpha_s) = \sum_{i\geq 0}\big(\alpha_s/(4\pi)\big)^{i+1}\gamma_{V,i}$.

can simplify the result by Taylor expansion. To give a taste of this comparison for resummed soft functions at RG improved LO, we go to the regime $w \gg m_b^2$, where the solutions simplify significantly:

$$
S_{\text{RGi}}^{\text{LO}}(w,\mu) = -N_c \frac{\alpha_b}{\pi} m_b(\mu_s) \left(\frac{w}{\mu_s^2}\right)^{-a_\Gamma} e^{2S_\Gamma + a_{\gamma_s}} \left(\frac{\Gamma(1+a_\Gamma)}{\Gamma(1-a_\Gamma)} e^{2\gamma_E a_\Gamma}\right)^2,
$$

$$
S_{\text{RGi}}^{gg,\text{LO}}(w,\mu) = -T_F \delta_{ab} \frac{\alpha_s(\mu_s)}{\pi} m_b(\mu_s) \left(\frac{w}{\mu_s^2}\right)^{-a_{\Delta\Gamma}} e^{2S_{\Delta\Gamma} + a_{\gamma_s'}} \left(\frac{\Gamma(1+a_{\Delta\Gamma})}{\Gamma(1-a_{\Delta\Gamma})} e^{2\gamma_E a_{\Delta\Gamma}}\right)^{2r_\Gamma},
$$

(11)

where $\gamma_{s,0} = -6C_F$, $\gamma_{s,0}' = -(6C_F - 2\beta_0)$ are used to compute the RG function a_V. The RG functions above are defined as:

$$
S_V(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\gamma_V(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')}, \quad a_V(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\gamma_V(\alpha)}{\beta(\alpha)},
$$

(12)

where V can be any anomalous dimensions above, for example, $a_{\Delta\Gamma}$ takes $\Delta\Gamma = \alpha_s(C_F - C_A)/\pi + \cdots$ and S_Γ takes $\Gamma = \alpha_s C_F/\pi + \cdots$. And to save space, the arguments in the RG functions are suppressed. The full solution can be found in [13, 21, 22]

The RGE of $H_1(\mu)$ is again complicated due to its involved definition (6). With the knowledge of all the renormalization factors, at last, we find that

$$
\frac{dH_1(\mu)}{d\ln\mu} = \gamma_{11} H_1(\mu) + D_{\text{cut}}(\mu) + 2 \int_0^1 dz \Big(H_2(z,\mu)\gamma_{21}(z) - [\![H_2(z,\mu)]\!][\![\gamma_{21}(z)]\!]
$$
$$
- [\![H_2(\bar{z},\mu)]\!][\![\gamma_{21}(\bar{z})]\!] \Big),
$$

(13)

where the last term is obviously the mixing contribution, while the second term comes from the "left-over" contribution. We don't give the definition for $D_{\text{cut}}(\mu)$ here. But we want to emphasize that $D_{\text{cut}}(\mu) \ni \alpha_s^2 L_h^2$, so the RGE of $H_1(\mu)$ is not Sudakov-like. To solve this kind of RGE is still an open question, but we don't have to solve it to resum large logarithms, because we can choose to evolve the operators up to the hard scale μ_h. A slightly different RGE holds for the case of $gg \to h$ [13].

The solutions to the RGEs of $\langle O_2(z,\mu)\rangle$ and $[\![\langle O_2(z,\mu)\rangle]\!]$ are more involved due to the subtraction nature. We have found a way to solve them systematically by a semi-numerical method. They are not needed if we care about next-to-leading logarithmic (NLL) resummation of the amplitude. At this accuracy, all contributions come from T_3, as a subset of RG improved leading order T_3. To be more concrete, at NLL, the asymptotic solution (11) is enough for the soft function. Combined with the resummed jet and hard function, and Taylor expanding the integrand in T_3, we arrive at:

$$
T_3^{\text{NLL}} = N_c \frac{\alpha_b}{\pi} \frac{y_b(\mu_h)}{\sqrt{2}} m_b(\mu_h) \int_0^\infty dL_- \int_0^\infty dL_+ \theta(L - L_- - L_+) \exp\Bigg\{ -\frac{\alpha_s(\mu_h)}{4\pi} \times
$$
$$
\left[\frac{\Gamma_0}{4}\left(L_s^2 - L_-^2 - L_+^2 + \frac{\alpha_s(\mu_h)}{4\pi}\frac{\beta_0}{3}(L_s^3 - L_-^3 - L_+^3)\right) + \frac{\gamma_{s,0} + \gamma_{m,0}}{2} L_s\right]\Bigg\},
$$

(14)

where $\gamma_{m,0} = -6C_F$, $\Gamma_0 = 4C_F$, $\beta_0 = 11/3 C_A - 4/3 T_F n_f$ and $L_s = L_- + L_+$. After the integration, we obtain (9) in [23]. For $gg \to h$, a similar formula can be found in [13].

We can also use the RGEs to predict large logarithms in the three-loop amplitudes, from L^6 to L^3. After switching to on-shell scheme, and making the choice $\mu^2 = \mu_h^2 = -M_h^2 - i0$, we

find that the amplitude takes the form:

$$
\begin{aligned}
\mathcal{M} = {} & \frac{N_c \alpha_b}{\pi} \frac{m_b y_b}{\sqrt{2}} \varepsilon_\perp^*(k_1) \cdot \varepsilon_\perp^*(k_2) \left\{ \frac{L^2}{2} - 2 \right. \\
& + \frac{C_F \alpha_s(\mu_h)}{4\pi} \left[-\frac{L^4}{12} - L^3 - \frac{2\pi^2}{3} L^2 + \left(12 + \frac{2\pi^2}{3} + 16\zeta_3 \right) L - 20 + 4\zeta_3 - \frac{\pi^4}{5} \right] \\
& \left. + C_F \left(\frac{\alpha_s(\mu_h)}{4\pi} \right)^2 \left[\frac{C_F}{90} L^6 + \left(\frac{C_F}{10} - \frac{\beta_0}{30} \right) L^5 + d_4^{\mathrm{OS}} L^4 + d_3^{\mathrm{OS}} L^3 + \dots \right] \right\},
\end{aligned}
\tag{15}
$$

where

$$
\begin{aligned}
d_4^{\mathrm{OS}} &= \left(\frac{3}{2} + \frac{\pi^2}{18} \right) C_F + \left(-\frac{91}{27} + \frac{\pi^2}{36} \right) C_A + \frac{32}{27} T_F n_f, \\
d_3^{\mathrm{OS}} &= \left(-\frac{1}{2} + \frac{7\pi^2}{9} + \frac{20}{3} \zeta_3 \right) C_F + \left(-\frac{199}{18} - \frac{22\pi^2}{27} - 4\zeta_3 \right) C_A + \left(\frac{22}{9} + \frac{8\pi^2}{27} \right) T_F n_f.
\end{aligned}
\tag{16}
$$

This is in perfect agreement with [24] after decoupling the coupling constant. We have done the same thing for $gg \to h$ case, even though we don't know two loop anomalous dimension for the jet and the soft function exactly [13].

5 Conclusion

In this proceeding, we report our recent progress on NLP SCET, in the context of Higgs amplitudes induced by a light quark loop (both for colourless and colourful external states). SCET can be used to derive the factorization formulae systematically. We find that the "plus-type" subtraction scheme to cure endpoint divergences naturally is compatible with renormalization. We derive the RGEs of factorized quantities and obtain the solutions. New functional forms appear after resummation and new colour structures appear surprisingly. Based on the RGEs, we can resum large logarithms to NLL, and beyond to RG improved LO. We also reproduce three-loop amplitudes analytically up to $\mathcal{O}(L^3)$, which is a good cross-check of our formalism.

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