SciPost Phys. Proc. 7, 045 (2022)

Background field method and generalized field redefinitions in effective field theories

Andrea Quadri

INFN, Sezione di Milano, via Celoria 16, I-20133 Milan, Italy

andrea.quadri@mi.infn.it

15th International Symposium on Radiative Corrections:
Applications of Quantum Field Theory to Phenomenology,
FSU, Tallahasse, FL, USA, 17-21 May 2021
doi:10.21468/SciPostPhysProc.7

Abstract

We study the non-linear background field redefinitions arising at the quantum level in a spontaneously broken effective gauge field theory. The non-linear field redefinitions are crucial for the symmetric (i.e. fulfilling all the relevant functional identities of the theory) renormalization of gauge-invariant operators. In a general R_ξ-gauge the classical background-quantum splitting is also non-linearly deformed by radiative corrections. In the Landau gauge these deformations vanish to all orders in the loop expansion.

Received 29-09-2021
Accepted 17-12-2021
Published 22-06-2022
doi:10.21468/ciPostPhysProc.7.045

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1 Introduction

Spontaneously broken effective gauge field theories [1] have become in recent years an increasingly important phenomenological tool [2], since they allow to parameterize in a model-independent way possible signals of new physics beyond the Standard Model (BSM) at the LHC [3, 4].

Spontaneously broken effective gauge field theories involve higher dimensional operators suppressed by some large mass scale Λ. These models are not power-counting renormalizable and therefore they give rise to new ultra-violet (UV) divergences order by order in the loop expansion.

Nevertheless, in these models the recursive subtraction of UV divergences can still be achieved by local (in the sense of formal power series in the fields, the external sources and their derivatives) counter-terms in such a way to fulfill the so-called Batalin-Vilkovisky (BV) master equation [5] (equivalently, the Slavnov-Taylor (ST) identity [6, 7], encoding the BRST invariance of the gauge-fixed classical action), provided that the underlying gauge group is non-anomalous.

This result has been established long ago in Ref. [8] by proving that the UV divergences can be subtracted by a combination of generalized (i.e. non-linear) field renormalizations and a redefinition of the coupling constants of gauge-invariant operators.

The fulfillment of the BV master equation (equivalently, of the ST identity) ensures that physical unitarity holds, i.e. that the cancellations of intermediate unphysical ghost states happens [9–11].

Due to the complexity of these computations, attempts have been made to implement the background field method (BFM) [12–19] technique in order to simplify the evaluation of the required counter-terms in the context of the (geometric) Standard Model Effective Field Theory (SMEFT) [20–22].

The main virtue of the BFM is that it allows to retain (background) gauge invariance to all orders in perturbation theory. The resulting background Ward identity is linear in the quantum fields, unlike the ST identity, and hence the additional relations imposed by the BFM on the one-particle irreducible (1-PI) Green's functions are easier to study.

For instance, in the power-counting renormalizable case, with linear field and background redefinitions, the BFM ties together the background gauge field $\widehat{\Phi}$ and the coupling constant renormalizations so that the charge renormalization factor can be obtained by evaluating just the gauge two-point 1-PI amplitude.

An important question is to clarify whether the same simplifications arise in the context of effective field theories. In fact, while the background Ward identity is still linear, the field and background field redefinitions are in general not [23, 24]. More precisely one finds that that [24]: *i)* the tree-level background-quantum splitting

$$\Phi = \widehat{\Phi} + Q_\Phi$$

is in general deformed in a non-linear (and gauge-dependent) way, unlike in the power-counting renormalizable case where only multiplicative Z-factors arise both for background and quantum fields; *ii)* in the Landau gauge no deformation of the tree-level background-quantum splitting happens, to all orders in the loop expansion; *iii)* also the background fields renormalize non-linearly, similarly to what happens to quantum fields when no backgrounds are switched on [25–27]; *iv)* the redefinition of the background fields is background gauge invariant. This follows from non-trivial cancellations between the non gauge-invariant contributions to the background-quantum splitting and the non gauge-invariant terms in the generalized field redefinitions of the quantum fields; *v)* both the background and the quantum field redefinitions need to be taken into account in order to ensure gauge-invariance of the coupling constants renormalization.

Full control of the background and quantum fields renormalization is required in order to consistently subtract the SMEFT at higher orders in the loop expansion.

For the sake of simplicity we study here the Abelian Higgs-Kibble model supplemented by dim.6 operators. This is a toy model illustrating the new features of the BFM arising in the effective field theory context. The techniques and the results presented here can be easily generalized to non-Abelian gauge groups.

2 The model and its symmetries

We consider the Abelian Higgs-Kibble model supplemented by the following set of dimension 6 operators:

$$\int \Big[\frac{z}{2v^2}\partial^\mu(\phi^\dagger\phi)\partial_\mu(\phi^\dagger\phi) + \frac{g_1}{\Lambda^2}\Big(\phi^\dagger\phi - \frac{v^2}{2}\Big)(D^\mu\phi)^\dagger(D_\mu\phi) + \frac{g_2}{\Lambda^2}\Big(\phi^\dagger\phi - \frac{v^2}{2}\Big)F^2_{\mu\nu} + \frac{g_3}{6\Lambda^2}\Big(\phi^\dagger\phi - \frac{v^2}{2}\Big)^3 \Big]. \quad (1)$$

We work in the so-called X-formalism [28–30]. In the latter an additional set of auxiliary fields $X_{1,2}$ is introduced, X_1 being a Nakanishi-Lautrup field. The gauge-invariant field coordinate X_2, that describes the physical scalar mode, obeys the following on-shell condition (imposed by the X_1-equation of motion):

$$X_2 \sim \frac{1}{v}\Big(\phi^\dagger\phi - \frac{v^2}{2}\Big). \tag{2}$$

The convenience of the X-formalism is related to the existence of a set of functional identities fixing the 1-PI amplitudes of the auxiliary fields in terms of ancestor amplitudes with a better UV degree of divergence than in the ordinary formalism. One recovers the results for the model in the standard approach by going on-shell with $X_{1,2}$. The procedure has been studied to all orders in the loop expansion in Refs. [28–30].

We adopt the following background gauge-fixing condition:

$$\widehat{\mathcal{F}}_\xi = \partial^\mu(A_\mu - \widehat{A}_\mu) + \xi e(\widehat{\phi}_0\chi - \widehat{\chi}\phi_0). \tag{3}$$

Hats denote the background fields, A_μ is the Abelian gauge field and $\phi \equiv \frac{1}{\sqrt{2}}(\phi_0 + i\chi)$ is the scalar doublet, with $\phi_0 = \sigma + v$. Notice that the replacement in Eq.(2) is carried out in the classical action only whenever the combination $\phi^\dagger\phi - \frac{v^2}{2}$ appears. The complete classical vertex functional of the theory is reported in Ref. [24].

The gauge-fixed model exhibits a U(1) gauge BRST invariance, obtained by promoting the infinitesimal parameter of the gauge transformation of the fields to a ghost ω and then supplementing the transformation of the antighost $\bar{\omega}$ into the Nakanishi-Lautrup field b:

$$sA_\mu = \partial_\mu\omega\,, \quad s\phi = ie\omega\phi\,, \quad s\omega = 0\,, \quad s\bar{\omega} = b\,, \quad sb = 0\,. \tag{4}$$

A further BRST symmetry encodes the decoupling of the X_1-modes from physical states and is given by ($c, \bar{c}$ are the ghost and antighost associated with this additional *constraint* BRST invariance):

$$sX_1 = vc\,, \quad sc = 0\,, \quad s\bar{c} = \phi^\dagger\phi - \frac{v^2}{2} - vX_2\,. \tag{5}$$

The quantum dependence of the effective action on the background fields can be constrained by algebraic tools based on the extension of the BRST differential in such a way that for each background field a classical ghost partner is introduced, i.e.

$$s\widehat{\Phi} = \Omega_{\widehat{\Phi}}\,, \qquad s\Omega_{\widehat{\Phi}} = 0\,.$$

The vertex functional Γ obeys then the following Slavnov-Taylor (ST) identity:

$$\mathcal{S}(\Gamma) = \int d^4x \Big[\partial_\mu\omega\frac{\delta\Gamma}{\delta A_\mu} + \frac{\delta\Gamma}{\delta\sigma^*}\frac{\delta\Gamma}{\delta\sigma} + \frac{\delta\Gamma}{\delta\chi^*}\frac{\delta\Gamma}{\delta\chi} + b\frac{\delta\Gamma}{\delta\bar{\omega}} + \Omega_\mu\frac{\delta\Gamma}{\delta\widehat{A}_\mu} + \Omega_{\widehat{\sigma}}\frac{\delta\Gamma}{\delta\widehat{\sigma}} + \Omega_{\widehat{\chi}}\frac{\delta\Gamma}{\delta\widehat{\chi}}\Big] = 0\,, \tag{6}$$

and in addition the following linear background Ward identity:

$$\begin{aligned}\mathcal{W}(\Gamma) = &-\partial^\mu\frac{\delta\Gamma}{\delta A^\mu} - e\chi\frac{\delta\Gamma}{\delta\sigma} + e(\sigma+v)\frac{\delta\Gamma}{\delta\chi} - \partial^\mu\frac{\delta\Gamma}{\delta\widehat{A}^\mu} - e\widehat{\chi}\frac{\delta\Gamma}{\delta\widehat{\sigma}} + e(\widehat{\sigma}+v)\frac{\delta\Gamma}{\delta\widehat{\chi}}\\ &-e\chi^*\frac{\delta\Gamma}{\delta\widehat{\sigma^*}} + e\sigma^*\frac{\delta\Gamma}{\delta\chi^*} = 0\,.\end{aligned} \tag{7}$$

In the above equations σ^*, χ^* denote the external sources coupled to the non-linear BRST transformations of the fields (the so-called antifields [5]).

3 Quantum background-splitting deformation

By taking a derivative of the ST identity in Eq.(6) w.r.t $\Omega_\mu, \Omega_{\widehat{\sigma}}, \Omega_{\widehat{\chi}}$ and then setting all the fields and external sources with positive ghost number to zero as well as $b = 0$ we get [1]

$$\Gamma'_{\widehat{\sigma}} = -\int \Big[\Gamma'_{\Omega_{\widehat{\sigma}}\sigma^*} \Gamma'_\sigma + \Gamma'_{\Omega_{\widehat{\sigma}}\chi^*} \Gamma'_\chi \Big], \qquad \Gamma'_{\widehat{\chi}} = -\int \Big[\Gamma'_{\Omega_{\widehat{\chi}}\sigma^*} \Gamma'_\sigma \Gamma'_{\Omega_{\widehat{\chi}}\chi^*} \Gamma'_\chi \Big]. \tag{8}$$

In the above equation we have denoted by a prime the functionals evaluated at zero ghost number and at $b = 0$. Let us project Eq.(8) at first order in the loop expansion.

There is no tree-level 1-PI amplitude involving the antifields σ^*, χ^* together with the background ghosts $\Omega_{\widehat{\sigma}}, \Omega_{\widehat{\chi}}$, since the background field only enters in the gauge-fixing.[2] Thus

$$\Gamma^{(1)'}_{\widehat{\sigma}} = -\int \Big[\Gamma^{(1)'}_{\Omega_{\widehat{\sigma}}\sigma^*} \Gamma^{(0)'}_\sigma + \Gamma^{(1)'}_{\Omega_{\widehat{\sigma}}\chi^*} \Gamma^{(0)'}_\chi \Big], \qquad \Gamma^{(1)'}_{\widehat{\chi}} = -\int \Big[\Gamma^{(1)'}_{\Omega_{\widehat{\chi}}\sigma^*} \Gamma^{(0)'}_\sigma + \Gamma^{(1)'}_{\Omega_{\widehat{\chi}}\chi^*} \Gamma^{(0)'}_\chi \Big]. \tag{9}$$

Since we look at the UV divergent part of the amplitudes, we can limit ourselves to solve the above equation for local functionals. Again at $b = 0$ there is no background dependence of the classical action. Eq.(9) fixes the full dependence on the background fields in terms of the amplitudes at zero background, once the kernels $\Gamma^{(1)'}_{\Omega_{\widehat{\sigma}}\sigma^*}, \Gamma^{(1)'}_{\Omega_{\widehat{\sigma}}\chi^*}, \Gamma^{(1)'}_{\Omega_{\widehat{\chi}}\sigma^*}, \Gamma^{(1)'}_{\Omega_{\widehat{\chi}}\chi^*}$ are known. The kernels are gauge-dependent. Therefore we consider the Feynman gauge and the Landau gauge separately.

3.1 Feynman gauge

In the Feynman gauge $\xi = 1$ the kernels are non-vanishing. By power-counting they contain at most logarithmic divergences. The kernels have been computed to the accuracy required to renormalize dim.6 operators in Ref. [25]. Let us denote by a bar the UV divergent part of a given amplitude or of the full vertex functional.

We notice that the kernels in the (A_μ, σ, χ)-basis do not depend on the background fields, so by integrating Eq.(9) and projecting on the UV divergent sector we find a linear dependence on the $\widehat{\Phi}$'s:

$$\overline{\Gamma}^{(1)'} = -\int \Big[\Big(\widehat{\sigma}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\sigma}}\sigma^*} + \widehat{\chi}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\chi}}\sigma^*}\Big)\Gamma^{(0)'}_\sigma + \Big(\widehat{\sigma}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\sigma}}\chi^*} + \widehat{\chi}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\chi}}\chi^*}\Big)\Gamma^{(0)'}_\chi \Big] + \overline{\Gamma}^{(1)'}\Big|_{\widehat{\sigma}=\widehat{\chi}=0}. \tag{10}$$

The last term in Eq.(10) denotes the UV divergent part of the vertex functional at zero background. It has been evaluated in [25] for the relevant sector of operators up to dimension 6. Eq.(10) can be interpreted as follows. It entails that the classical background-quantum splitting $\Phi = Q_\Phi + \widehat{\Phi}$ does not survive at the quantum level, being deformed according to

$$\sigma \to \widehat{\sigma} + q_\sigma - \widehat{\sigma}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\sigma}}\sigma^*} - \widehat{\chi}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\chi}}\sigma^*}, \qquad \chi \to \widehat{\chi} + q_\chi - \widehat{\sigma}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\sigma}}\chi^*} - \widehat{\chi}\, \overline{\Gamma}^{(1)'}_{\Omega_{\widehat{\chi}}\chi^*}. \tag{11}$$

By inserting the redefinitions in Eq.(11) into the tree-level vertex functional $\Gamma^{(0)}$, one gets back the terms in square brackets in Eq.(10). The kernels $\overline{\Gamma}^{(1)'}_{\Omega_\Phi \Phi}$ depend on the fields and external sources in a complicated way. Eq.(11) is therefore a highly non-linear redefinition w.r.t. the quantum fields.

[1] We denote by a subscript the functional differentiation w.r.t. the argument, i.e. $\Gamma_X = \frac{\delta \Gamma}{\delta X}$.

[2] A different basis is often equivalently used in BFM calculations, namely the quantum fields are defined as $Q_\mu = A_\mu - \widehat{A}_\mu, q_\sigma = \sigma - \widehat{\sigma}, q_\chi = \chi - \widehat{\chi}$. We prefer the basis (A_μ, σ, χ) since it simplifies the solution of the extended ST identity in Eq.(8), due to the fact that the dependence on the background fields in limited to the gauge-fixing sector

3.2 Landau gauge

In the Landau gauge $\xi = 0$ all the four kernels are identically zero since there are no interaction vertices that could generate them. Therefore the classical background-quantum splitting $\Phi = \widehat{\Phi} + Q_\Phi$ does not receive any radiative corrections. This is an all-order result.

4 Generalized background field redefinitions

The final form of the generalized background field redefinitions in the target theory can be read off by going on-shell with $X_{1,2}$ through the substitution on the external sources described in Refs. [24]. The resulting expressions are too long to be reported here and can be found in [24]. In particular at zero quantum fields $Q_\Phi = 0$ they reduce (to the accuracy required for the renormalization of dim.6 operators) to the following expressions:

$$\begin{pmatrix} \widehat{\sigma}_R \\ \widehat{\chi}_R \end{pmatrix} = \Big[a_0 + \Big(\frac{a_1 g_1}{\Lambda^2} + a_2 \Big) \Big(\widehat{\phi}^\dagger \widehat{\phi} - \frac{v^2}{2} \Big) + a_3 \Big(\widehat{\phi}^\dagger \widehat{\phi} - \frac{v^2}{2} \Big)^2 + \dots \Big] \begin{pmatrix} \widehat{\sigma} + v \\ \widehat{\chi} \end{pmatrix} , \tag{12}$$

with (gauge-dependent) coefficients a_j's. Their explicit expression has been given in Ref. [24]. As can be seen from Eq.(12) this is non-linear gauge-invariant (albeit gauge-dependent) field redefinition.

5 Conclusion

Application of the BFM to spontaneously broken gauge effective field theories exhibits novel features with respect to the usual power-counting renormalizable case. In particular the background-quantum splitting is non linearly deformed by quantum corrections and also the background fields are non-trivially redefined, at variance with the power-counting renormalizable case, where both the quantum and the background fields renormalize linearly. These redefinitions are crucial in order to preserve the locality of the recursive subtraction of the UV divergences in a way compatible with the symmetries of the model. The tools described here provide a constructive approach to the renormalization of the sponteaneously broken effective gauge theories in the modern sense of Ref. [8].

Acknowledgements

Useful discussions with D. Anselmi and D. Binosi are gratefully acknowledged.

Funding information Partial financial support by INFN is acknowledged.

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