Forward trijet production and saturation

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Abstract

We present a multi-parton extension of the so-called small-x Improved Transverse Momentum Dependent (ITMD) factorization for hadroproduction of three or more jets in the forward rapidity region, allowing to study gluon saturation using such processes.

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1 Introduction

Small-x Improved Transverse Momentum Dependent (ITMD) factorization [1] accounts for gluon saturation effects, off-shell hard matrix elements, and involves several transverse momentum dependent (TMD) gluon distributions. It can be understood as a generalization of k_T*-factorization,* also called *high energy factorization*, which focuses on events for which small fractions x of the total scattering energy enter the hard process, while not neglecting powers of the initial-state transverse momenta k_T [2,3]. Thus, the incoming partons have non-vanishing transverse momentum, and the initial-state momenta entering both the PDFs and the matrix elements are space-like rather than light-like. The TMD PDFs are also called unintegrated PDFs in this context. The corresponding evolution equations typically resum logarithms of $1/x$.

For linear evolution equations of this type, gluon densities can grow power-like with energy, eventually violating unitarity. QCD predicts a cure via a nonlinear equation, called the BK equation [4, 5], which exhibits *gluon saturation*, *i.e.* a state in which almost all gluons have momenta of the size of the *saturation scale* Q_s. It is the mean field approximation to the more general B-JIMWLK system of equations [4,6–8], which describe the evolution of various gluon operators supplemented with Wilson lines, which have very different behavior for small k_T but coincide (or vanish very quickly) in the linear regime at large $k_T \gg Q_s$.

Color Glass Condensate (CGC) theory is a QCD-based model that incorporates this saturation (see e.g. [9]). In dijet phenomenology at LHC, with a hard scale of at least around 15 GeV and at sufficiently forward rapidity, one is sensitive to saturation. In that regime, many simplifications with respect to CGC theory occur leading to an effective TMD factorization with several small-x leading power TMD gluon distributions containing various Wilson line operators ensuring gauge invariance and resumming collinear gluons. By keeping the incoming gluon off-shell in the amplitude, the kinematic twist corrections are incorporated and one has a full description of the jet imbalance. This ITMD factorization is equivalent to CGC expressions for dilute-dense collisions [10] with all kinematic twist corrections isolated and resummed, while neglecting the genuine twist corrections [11].

Here, we present the extension of ITMD to three or more jets. The operator structures for three-and four-jet processes have been explicitly calculated in [12]. As presented here, the ITMD formalism does not account for the linearly-polarized gluons in unpolarized targets. Such a contribution is absent in CGC theory for massless two-particle production, but appears in heavy quark production [13, 14] and has been already observed in the correlation limit for the three-parton final state [15]. Therefore, in the following the extension of ITMD to multipartonic processes will be denoted ITMD*.

2 The formalism

The ITMD* formula of forward particle production, where a dilute proton p (probed at large x) collides with a dense target A (probed at small x) is given by:

$$d\sigma_{pA\to n} = \int \frac{dx_1}{x_1} \sum_a x_1 f_{a/p}(x_1,\mu)\, \frac{dx_2}{x_2} \int \frac{d^2k_T}{\pi}\, d\Phi_{n_J} \sum_{b_1,\dots,b_{n_J}} \frac{\left|\mathcal{M}_{ag\to b_1\dots b_{n_J}}\right|^2_{\mathrm{ITMD}^*}}{\mathrm{flux}_{ag\to b_1\dots b_{n_J}}}\,, \tag{1}$$

where $f_{a/p}$ is the collinear PDF for parton a, where $d\Phi_{n_J}$ is the n_J-particle differential phase space, and where $b_1,\dots,b_{n_J}$ are the various final state partons contributing to the partonic sub-process $ag \to b_1 \dots b_{n_J}$. The factor $\mathrm{flux}_{ag\to b_1\dots b_{n_J}}$ is assumed to contain the flux factor, the factors to turn the summations implied by the matrix element into averages regarding the initial-state partons, and the necessary factors in case there are identical final-state particles. The matrix element is given by

$$\begin{aligned}\left|\mathcal{M}_{ag\to b_1\dots b_{n_J}}\right|^2_{\mathrm{ITMD}^*} &= (N_c^2-1) \sum_{i_1,\dots,i_n} \sum_{j_1,\dots,j_n} \sum_{\bar{\imath}_1,\dots,\bar{\imath}_n} \sum_{\bar{j}_1,\dots,\bar{j}_n} \left(\tilde{\mathcal{M}}^{i_1 i_2\cdots i_n}_{j_1 j_2\cdots j_n}\right)^* \left(\tilde{\mathcal{M}}^{\bar{\imath}_1\bar{\imath}_2\cdots\bar{\imath}_n}_{\bar{j}_1\bar{j}_2\cdots\bar{j}_n}\right)\\ &\times \left\langle\!\!\left\langle 2\left(\hat{F}^+(\xi)\right)_{i_1}^{j_1}\left(\hat{F}^+(0)\right)_{\bar{\imath}_1}^{\bar{j}_1}\left(\mathcal{U}^{[\lambda_2]}\right)_{i_2\bar{\imath}_2}\left(\mathcal{U}^{[\lambda_2]\dagger}\right)^{j_2\bar{j}_2}\cdots\left(\mathcal{U}^{[\lambda_n]}\right)_{i_n\bar{\imath}_n}\left(\mathcal{U}^{[\lambda_n]\dagger}\right)^{j_n\bar{j}_n}\right\rangle\!\!\right\rangle. \end{aligned}\tag{2}$$

Here, $\tilde{\mathcal{M}}^{i_1 i_2\cdots i_n}_{j_1 j_2\cdots j_n}$ is the parton-level scattering amplitude in the incarnation of the colorflow representation as given in [16, 17]. Such a representation treats gluons on the same footing as quark-antiquark pairs in the color sum. The symbol n is the number of color-pairs in this representation, so the number of gluons plus the number of quarks, where the latter is equal to the number of antiquarks. The field strength operators $\hat{F}^+$ are separated in the light-cone 'minus' and transverse directions $\xi = \left(\xi^+ = 0, \xi^-, \vec{\xi}_T\right)$. The symbols $\mathcal{U}^{[\lambda]}$ denote two staple-like fundamental representation Wilson lines connecting the fields

$$\begin{aligned}\mathcal{U}^{[\pm]} = &\left[\left(0^+,0^-,\vec{0}_T\right),\left(0^+,\pm\infty^-,\vec{0}_T\right)\right]\left[\left(0^+,\pm\infty^-,\vec{0}_T\right),\left(0^+,\pm\infty^-,\vec{\xi}_T\right)\right]\\ &\times\left[\left(0^+,\pm\infty^-,\vec{\xi}_T\right),\left(0^+,\xi^-,\vec{\xi}_T\right)\right], \end{aligned}\tag{3}$$

where the square brackets are the straight segments of the Wilson link. The value $\lambda = \pm$ depends on whether the parton whose color the Wilson link connects is incoming or outgoing. The Wilson loop obtained by two staples glued together is denoted

$$\mathcal{U}^{[\square]} = \mathcal{U}^{[-]\dagger}\mathcal{U}^{[+]}\,. \tag{4}$$

The angular double brackets denote the Fourier transform of the hadronic matrix element:

$$\left\langle\!\!\left\langle \cdots \right\rangle\!\!\right\rangle = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3\, P^+} \exp\left(ix_2 P^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T\right) \langle P|\cdots|P\rangle\,, \tag{5}$$

where P^+ is the longtudinal 'plus' momentum component of the hadron. In this Fourier transform, the variables x_2 and $\vec{k}_T$ from Eq. (1) appear. One feature of the chosen color representation is that the amplitude decomposes as

$$\tilde{\mathcal{M}}^{i_1 i_2\cdots i_n}_{j_1 j_2\cdots j_n} = \sum_{\sigma\in S_n} \delta^{i_1}_{j_{\sigma(1)}}\delta^{i_2}_{j_{\sigma(2)}}\cdots\delta^{i_n}_{j_{\sigma(n)}}\mathcal{A}_\sigma\,, \tag{6}$$

where the *partial amplitudes* $\mathcal{A}_\sigma$ only depend on momenta, helicity, and the permutation σ, but not on color. They can be calculated using color-ordered Feynman rules. Inserting this decompostion, Eq. (2) collapses to

$$\left|\mathcal{M}_{ag\to b_1\ldots b_{n_J}}\right|^2_{\mathrm{ITMD}^*} = (N_c^2-1)\sum_{\sigma\in S_n}\sum_{\tau\in S_n}\mathcal{A}^*_\sigma\, \mathcal{C}_{\sigma\tau}(x_2,k_T)\,\mathcal{A}_\tau\,, \tag{7}$$

where the entries of the "TMD-valued color matrix" $\mathcal{C}_{\sigma\tau}(x_2,k_T)$ consist of exactly a single power of N_c times one of the following 10 TMDs

$$\mathcal{F}^{(1)}_{qg}(x,k_T) = \left\langle\!\!\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle, \tag{8}$$

$$\mathcal{F}^{(2)}_{qg}(x,k_T) = \left\langle\!\!\left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle, \tag{9}$$

$$\mathcal{F}^{(3)}_{qg}(x,k_T) = \left\langle\!\!\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle, \tag{10}$$

$$\mathcal{F}^{(1)}_{gg}(x,k_T) = \left\langle\!\!\left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle, \tag{11}$$

$$\mathcal{F}^{(2)}_{gg}(x,k_T) = \frac{1}{N_c}\left\langle\!\!\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[\square]\dagger}\right]\mathrm{Tr}\left[\hat{F}^{i+}(0)\mathcal{U}^{[\square]}\right]\right\rangle\!\!\right\rangle, \tag{12}$$

$$\mathcal{F}^{(3)}_{gg}(x,k_T) = \left\langle\!\!\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle, \tag{13}$$

$$\mathcal{F}^{(4)}_{gg}(x,k_T) = \left\langle\!\!\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[-]}\right]\right\rangle\!\!\right\rangle, \tag{14}$$

$$\mathcal{F}^{(5)}_{gg}(x,k_T) = \left\langle\!\!\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle, \tag{15}$$

$$\mathcal{F}^{(6)}_{gg}(x,k_T) = \left\langle\!\!\left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle, \tag{16}$$

$$\mathcal{F}^{(7)}_{gg}(x,k_T) = \left\langle\!\!\left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle. \tag{17}$$

Below is the explicit example of the matrix $\mathcal{C}_{\sigma\tau}(x_2,k_T)$ and the column vector of partial amplitudes for both the processes $g_1^* q_2 \to q_4\bar{q}_3' q_5'$ and $g_1^* q_2 \to q_4\bar{q}_3 q_5$:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_c\mathcal{F}^{(1)}_{qg} & \mathcal{F}^{(1)}_{qg} & 0 & \mathcal{F}^{(1)}_{qg} & 0 \\ 0 & \mathcal{F}^{(1)}_{qg} & N_c\mathcal{F}^{(2)}_{qg} & \mathcal{F}^{(3)}_{qg} & 0 & 0 \\ 0 & 0 & \mathcal{F}^{(3)}_{qg} & N_c\mathcal{F}^{(2)}_{qg} & \mathcal{F}^{(1)}_{qg} & 0 \\ 0 & \mathcal{F}^{(1)}_{qg} & 0 & \mathcal{F}^{(1)}_{qg} & N_c\mathcal{F}^{(1)}_{qg} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \mathcal{A}_{12345} \\ \mathcal{A}_{21345} \\ \mathcal{A}_{23145} \\ \mathcal{A}_{32145} \\ \mathcal{A}_{31245} \\ \mathcal{A}_{13245} \end{pmatrix}. \tag{18}$$

The processes have 3 color pairs, so there are (at most) 3! partial amplitudes. They are explicitly labeled with their associated permutation, and the logic in the enumation of the partons is that gluons come first, then anti-quarks, and then quarks. Initial-state quarks count as negative-energy antiquarks. As can be seen, the first and last partial amplitudes do not contribute at all, but are included here for clearity. For processes with only gluons, the number of contributing partial amplitudes is only $(n-1)!$ rather than $n!$.

In order to evaluate the TMDs necessary for trijet production, the same path as in [18] can be employed. The starting point is the so-called dipole distribution (8), which appears in inclusive DIS processes. In particular, one can use the TMD coming from the BK equation augmented with subleading corrections following the framework of [19] and fitted to F_2 data [20]. Having the dipole gluon distribution, all other distributions appearing in trijet production can be calculated in the mean field approximation often used in CGC theory and to leading number of colors.

3 Conclusion

We gave a concise description of the ITMD* formalism for multi-jet production at hadron colliders, which allows to study saturation in these process. Such a study was performed recently in [21], where various azimuthal angle distributions for three jets produced in the forward rapidity region were calculated for proton-proton and proton-lead collisions.

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