

# A spectator-model way to transverse-momentum-dependent gluon distribution functions

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## Abstract

We present exploratory analyses of the 3D gluon content of the proton via a study of unpolarized and polarized gluon TMDs at twist-2, calculated in a spectator model for the parent nucleon. Our approach embodies a flexible parametrization for the spectator-mass function, suited to describe both moderate and small- $x$  effects. All these studies can serve as a useful guidance in the investigation of the gluon dynamics inside nucleons and nuclei, which constitutes one of the major goals of new-generation colliding machines, as the EIC, the HL-LHC, NICA, and the FPE.



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## 1 Introduction

The study of the proton content via transverse-momentum-dependent (TMD) parton distribution functions represents a challenging line of research plans at current and new-generation colliding machines. While in the last years the investigation of the quark-TMD field has reached important milestones, from the deep knowledge of formal properties to the more and more accurate extraction of quark densities from global fits, the gluon-TMD sector still represents a largely unexplored territory. A first classification of unpolarized and polarized gluon TMD

distributions was first made in Ref. [1] and subsequently extended in Refs. [2–4]. Recent phenomenological analyses on gluon TMDs can be found in Refs. [5–9]. A major difficulty that emerges in formal studies of gluon TMDs is their process dependence. Different kinds of reactions are sensitive to distinct *gauge-link* structures, and this leads to a more intricate *modified universality* with respect to what we observe for quark TMDs. Two main gauge links can be identified. They have been classified in the context of small- $x$  analyses as Weiszäcker–Williams and dipole TMDs [10]. They are strictly related to gluon correlators where for  $T$ -odd TMDs the  $f_{abc}$  and  $d_{abc}$  QCD color structures respectively emerge. Therefore, they are also known among the TMD community as  $f$ -type and  $d$ -type gluon TMDs. At low- $x$  values and large transverse momenta, the gluon content of the proton is described by the so-called *unintegrated gluon distribution* (UGD), whose evolution is governed by the Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation [11, 12] (for recent applications see Refs. [13–23]). Its relation to the low- $x$  limit of gluon TMDs and, more in general, to the Collins–Soper–Sterman (CSS) evolution [24, 25] has been investigated in Refs. [10] and [26, 27], respectively. In this work we present a study on leading-twist  $T$ -even gluon TMDs calculated in a *spectator model* for the parent proton. Our framework is suited to analyses both in moderate and small- $x$  ranges.

## 2 TMD gluon distribution functions

According to the spectator-model approximation, the proton can emit a gluon with longitudinal-momentum fraction  $x$  and transverse momentum  $\mathbf{p}_T$ , and the remainders are treated as an effective colored particle with mass  $M_X$  and possessing the quantum numbers of a fermion, that we call spectator. The nucleon-gluon-spectator coupling is encoded in a effective vertex that contains two form factors, chosen as dipolar functions of  $\mathbf{p}_T^2$ . The main advantage of using dipolar form factors consists in the possibility of cancelling gluon-propagator singularities, quenching the effects of large transverse momenta where a pure TMD description is not anymore adequate, and removing logarithmic divergences emerging in  $\mathbf{p}_T$ -integrated densities.

In Ref. [28] a pioneering study on quark TMDs was proposed, by considering different di-quark spectator polarization states and nucleon-parton-spectator form factors. In Ref. [29] the weight of azimuthal asymmetries was assessed.

In the present study we present our calculation in the spectator model of  $T$ -even gluon TMDs at twist-2. We improved the genuine spectator-model approach by allowing the spectator mass,  $M_X$ , to be in a range of values weighed by the following 7-parameter spectral function

$$\rho_{[\text{spect.}]}(M_X) = \mu^{2a} \left( \frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right). \quad (1)$$

The expression for a given TMD reads

$$\mathcal{F}^g(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_{[\text{spect.}]}(M_X) \hat{\mathcal{F}}^g(x, \mathbf{p}_T^2; M_X), \quad (2)$$

with  $\hat{\mathcal{F}}^g$  the corresponding TMD obtained in a pure spectator-model calculation. Model parameters were fitted to simultaneously reproduce the gluon unpolarized ( $f_1^g(x)$ ) and helicity ( $g_1^g(x)$ ) collinear parton distribution functions (PDFs), obtained in global fits at the initial scale  $Q_0 = 1.64$  GeV (see Fig. 1). We performed our fit by making use of the so-called bootstrap method. We created  $N$  replicas of the central value of the NNPDF parametrization by randomly varying it with a Gaussian noise that keeps the same variance of the original parametrization uncertainty. We fitted each replica separately and we obtained  $N$ -dimensional vector for each parameter of the model. A complete description of our model together all technical details of our fit procedure can be found in Ref. [30] (see also Refs. [31–36]). We show in Fig. 2 the

$p_T^2$ -dependence of two  $T$ -even gluon TMDs calculated at  $x = 0.001$  and at the same initial scale,  $Q_0 = 1.64$  GeV. Each one of our TMDs exhibit a distinct shape. The unpolarized gluon density  $xf_1^g(x, p_T^2)$  (left panel) shows a non-Gaussian pattern in  $p_T^2$ , a large flattening tail in the  $p_T^2 \rightarrow 1$  GeV limit, and it goes to a quite small value when  $p_T^2 \rightarrow 0$ . Conversely, the Boer–Mulders gluon distribution  $xh_1^{\perp g}(x, p_T^2)$  (right panel), that is connected to the density of transversely polarized gluons inside an unpolarized proton, starts from a finite value at  $p_T^2 = 0$  and decreases very fast when  $p_T^2$  grows.

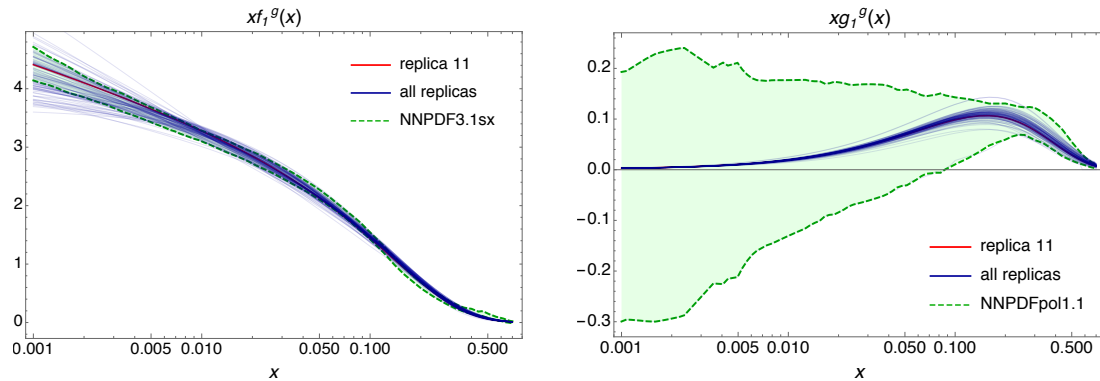


Figure 1:  $x$ -dependence of the unpolarized (left) and helicity (right) gluon PDFs densities calculated in the spectator model at the initial scale  $Q_0 = 1.64$  GeV. Green bands with dashed borders stand for the NNPFD3.1x [37] and the NNPDFpol1.1 [38] parametrizations. Blue curves depict the 100 replicas for our integrated TMDs. Red curve for the most representative replica #11.

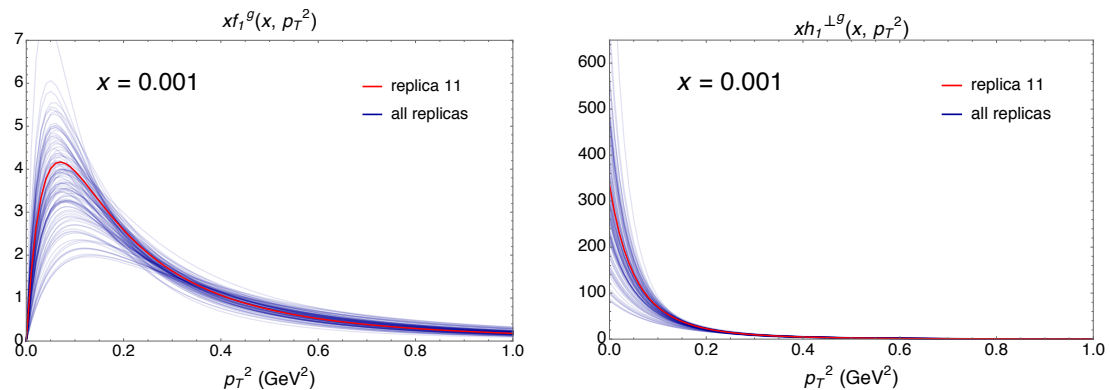


Figure 2: Transverse-momentum dependence of the unpolarized (left) and Boer–Mulders (right) gluon TMD densities calculated in the spectator model, for  $x = 10^{-3}$  and at the initial scale  $Q_0 = 1.64$  GeV. Red curve for the most representative replica #11.

### 3 Conclusion

We presented a model dependent calculation of all twist-2  $T$ -even gluon TMDs based on the assumption that what remains of the proton after gluon emission can be described as an effective spin-1/2 spectator particle. We improved the genuine spectator-model description by weighing its mass via a versatile spectral function. We fitted model parameters to reproduce the  $x$ -shape of collinear unpolarized and helicity gluon PDFs that were extracted from global

fits. At the current level, our model does not incorporate any gauge-link dependence, and the extension to twist-2  $T$ -odd gluon TMD distributions is underway. Another intriguing perspective is represented by encoding in the description of the unpolarized gluon TMD genuine small- $x$  effect from the BFKL resummation [11, 12]. Exploratory studies on gluon-TMD phenomenology via our model can represent a useful guidance in accessing the proton content at new-generation colliding machines, as the *Electron-Ion Collider* (EIC) [39], the *High-Luminosity Large Hadron Collider* (HL-LHC) [40], NICA [41], and the *Forward Physics Facility* [42, 43].

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