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Multi-particle production in proton-nucleus collisions in the Color Glass Condensate

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Abstract

We compute multi-gluon production in the Color Glass Condensate approach in dilute-dense collisions, p*A*. We include the contributions that are leading in the overlap area of the collision but keep all orders in the expansion in the number of colors. We use a form of the Lipatov vertices that leads to the Wigner function approach for the projectile previously employed, that we generalise to take into account quantum correlations in the projectile wave function. We compute four gluon correlations and we find that the second order four particle cumulant is negative, so a sensible second Fourier azimuthal coefficient can be defined.

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1 Introduction

Small size collision systems, pp and pA, probed at the Large Hadron Collider (LHC) show many properties that are typical of dense systems generated in heavy ion collisions (HICs) such as the existence of azimuthal correlations in the two-particle distribution that show a maxima when the particles move in the same or opposite direction. This phenomenon, known as the *ridge*, introduces the question of whether the source of such a collective behavior is the same in small systems and in HICs.

The standard explanation of collective behavior in HICs, where the partonic density is large, is the existence of strong final state interactions that lead to a situation where viscous relativistic hydrodynamics can be applied. On the other hand, the application of hydrodynamics in pp and pA collisions has lead to a successful description of azimuthal anisotropies. However, these systems are defined by low particle densities and small collision area where non-hydrodynamic modes play an important role. Thus, it makes sense to look for other explanations of collectivity effects in these systems. The Color Glass Condensate (CGC), a weak coupling but non-perturbative description of partonic systems, offers a framework where azimuthal asymmetries can be calculated from first principles [1]. In the CGC picture, correlations in the final state reflect those found in the wave function of the target and projectile, assuming that final state effects, including hadronization, do not wash them out.

The initial versus final state origin of azimuthal correlations in small systems has been subject to intense scrutiny in recent years. At present, no CGC-based model is able to fully describe experimental data. Still, the search for observables that may discriminate initial from final effect continues such as multi-particle cumulants.

2 Multi-gluon production in dilute-dense scatterings within the CGC framework

In our approach we consider the projectile as a highly boosted dilute system that is composed, mostly, by large-x partons that act each as a color source with color charge density $\rho^a(\mathbf{x}, x^+)$, with a denoting color, $\mathbf{x}$ the transverse position and x^+ the longitudinal position. The target is defined by a strong field $A^\mu(\mathbf{x}, x^+) = A^\mu_a(\mathbf{x}, x^+)T^a$, with T^a the generators of the $SU(N_c)$ group in the adjoint representation. Furthermore, the nucleus ensemble is supposed to be much larger than the projectile in the transverse plane. In this picture, working in the light-cone gauge $A^+ = 0$ and neglecting the transverse components of the field, the amplitude for producing a gluon with transverse momentum $\mathbf{k}$, color a, polarization λ and pseudorapidity η is given by

$$\mathcal{M}^a_\lambda(\eta, \mathbf{k}) = g \int \frac{d^2\mathbf{q}}{(2\pi)^2} \overline{\mathcal{M}}^{ab}_\lambda(\eta, \mathbf{k}, \mathbf{q}) \rho^b(\mathbf{k} - \mathbf{q}), \tag{1}$$

where $\rho^a(\mathbf{q})$ is the Fourier transform of the projectile's color charge density and $\overline{\mathcal{M}}^{ab}_\lambda(\eta, \mathbf{k}, \mathbf{q})$ is the reduced matrix amplitude [2] that, in the eikonal approximation, reads[1] [3]

$$\overline{\mathcal{M}}^{ab}_\lambda(\mathbf{k}, \mathbf{q}) = 2i\epsilon^{i*}_\lambda(\mathbf{k}) L^i(\mathbf{k},\mathbf{q}) \int d^2\mathbf{y} e^{-i\mathbf{q}\mathbf{y}} U^{ab}(\mathbf{y}). \tag{2}$$

In this equation $\epsilon^{i*}_\lambda(\mathbf{k})$ is the gluon polarization vector, $\mathbf{q}$ is the transverse momentum transferred from the target during the interaction, $U^{ab}(\mathbf{y}) = \mathcal{P}^+ \exp\left\{ig \int dz^+ A^-(\mathbf{y}, z^+)\right\}^{ab}$ is the eikonal Wilson line and

$$L^i(\mathbf{k},\mathbf{q}) = \frac{\mathbf{k}^i}{\mathbf{k}^2} - \frac{(\mathbf{k} - \mathbf{q})^i}{(\mathbf{k} - \mathbf{q})^2}, \tag{3}$$

is the Lipatov vertex.

In this setup the spectrum for the production of n gluons, each of them with momentum $\mathbf{k}_i$ $(i = 1, \dots, n)$, can be written as

$$\begin{aligned} 2^n(2\pi)^{3n} \frac{d^n N}{\prod_{i=1}^n d^2\mathbf{k}_i} =& g^{2n} \int \left(\prod_{i=1}^{2n} \frac{d^2\mathbf{q}_i}{(2\pi)^2} \right) \\ &\times \left\langle \rho^{b_1}(\mathbf{k}_1 - \mathbf{q}_1) \rho^{b_2 \dagger}(\mathbf{k}_1 - \mathbf{q}_2) \cdots \rho^{b_{2n-1}}(\mathbf{k}_n - \mathbf{q}_{2n-1}) \rho^{b_{2n}\dagger}(\mathbf{k}_n - \mathbf{q}_{2n}) \right\rangle_p \\ &\times \left\langle \overline{\mathcal{M}}^{a_1 b_1}_{\lambda_1}(\mathbf{k}_1, \mathbf{q}_1) \overline{\mathcal{M}}^{b_2 a_1 \dagger}_{\lambda_1}(\mathbf{k}_1, \mathbf{q}_2) \cdots \overline{\mathcal{M}}^{a_n b_{2n-1}}_{\lambda_n}(\mathbf{k}_n, \mathbf{q}_{2n-1}) \overline{\mathcal{M}}^{b_{2n} a_n \dagger}_{\lambda_n}(\mathbf{k}_n, \mathbf{q}_{2n}) \right\rangle_T. \end{aligned} \tag{4}$$

[1]We have dropped the η dependence of the gluon amplitude due to the rapidity invariance of the multi-gluon spectrum in the eikonal approximation.

3 The area enhancement argument

In order to evaluate Eq. (4) we should compute the average over the target sources of $2n$ matrix amplitudes (or equivalently, Wilson lines) and the average over the projectile color charge density of $2n$ sources. For the second object we assume a Gaussian distribution for the projectile sources, i.e. the generalized MV model, in such a way that it can only be written in terms of the 2-point function through the Wick's theorem

$$\left\langle \rho^{b_1}(\mathbf{k}_1-\mathbf{q}_1)\rho^{b_2\dagger}(\mathbf{k}_1-\mathbf{q}_2)\cdots\rho^{b_{2n-1}}(\mathbf{k}_n-\mathbf{q}_{2n-1})\rho^{b_{2n}\dagger}(\mathbf{k}_n-\mathbf{q}_{2n})\right\rangle_p = \sum_{\omega\in\Pi(\chi)}\prod_{\{i,j\}\in\omega}\left\langle \rho^{b_i}(\mathbf{k}_i-\mathbf{q}_i)\rho^{b_j}(\mathbf{k}_j-\mathbf{q}_j)\right\rangle_p, \tag{5}$$

where $\chi=\{1,2,\ldots,2n\}$ and $\Pi(\chi)$ the set of partitions of χ with disjoint pairs. In the generalized MV model this projectile 2-point function is proportional to a function, $\mu^2(\mathbf{k},\mathbf{q})$, that is peaked around $\mathbf{k}+\mathbf{q}=0$ [3].

The target average can be evaluated by using the so-called *area enhancement argument* [4]. In this model one uses the fact that the configuration of the coordinates $\mathbf{y}_i$ that maximizes the transverse integrals appearing in Eq. (4) through Eq. (2) is such that the coordinates are as far away as possible between them since, in this case, they cover a higher region of the phase space. On the other hand, due to the domain model [3], two objects that only depend on the target field, sitting at two different points $\mathbf{y}_i$ and $\mathbf{y}_j$, will have a negligible correlation when $|\mathbf{y}_i-\mathbf{y}_j|\gg Q_s^{-1}$, where Q_s is the saturation scale that defines the target. This implies that the only way of obtaining a non vanishing target correlator is by grouping the legs in, at least, pairs where the distance between the coordinates is smaller than the correlation length (Q_s^{-1}). Moreover, the terms where more than two legs are sitting in the same domain will cover a lower region of the phase space and will be suppressed by the overlap area of the interaction. Thus, at leading order in the inverse of the area of the interaction, only these configurations where the coordinates are grouped in pairs that are far away from each other will contribute to the target multipole.

This assumption is equivalent to assuming Gaussian statistics for the Wilson lines. Therefore we can use the Wick's theorem and write (see [3] for a detailed derivation)

$$\left\langle \overline{\mathcal{M}}^{a_1b_1}_{\lambda_1}(\mathbf{k}_1,\mathbf{q}_1)\overline{\mathcal{M}}^{b_2a_1\dagger}_{\lambda_1}(\mathbf{k}_1,\mathbf{q}_2)\cdots\overline{\mathcal{M}}^{a_nb_{2n-1}}_{\lambda_n}(\mathbf{k}_n,\mathbf{q}_{2n-1})\overline{\mathcal{M}}^{b_{2n}a_n\dagger}_{\lambda_n}(\mathbf{k}_n,\mathbf{q}_{2n})\right\rangle_T = \sum_{\sigma\in\Pi(\chi)}\prod_{\{\alpha,\beta\}\in\sigma}\left\langle \overline{\mathcal{M}}^{a_\alpha b_\alpha}_{\lambda_\alpha}(\mathbf{k}_\alpha,\mathbf{q}_\alpha)\overline{\mathcal{M}}^{a_\beta b_\beta}_{\lambda_\beta}(\mathbf{k}_\beta,\mathbf{q}_\beta)\right\rangle_T, \tag{6}$$

where the target 2-point function can be written as

$$\left\langle \overline{\mathcal{M}}^{a_\alpha b_\alpha}_{\lambda_\alpha}(\mathbf{k}_\alpha,\mathbf{q}_\alpha)\overline{\mathcal{M}}^{a_\beta b_\beta}_{\lambda_\beta}(\mathbf{k}_\beta,\mathbf{q}_\beta)\right\rangle_T = 4\frac{\delta^{a_\alpha a_\beta}\delta^{b_\alpha b_\beta}}{N_c^2-1}(2\pi)^2\delta^{(2)}[\mathbf{q}_\alpha+(-1)^{\alpha+\beta}\mathbf{q}_\beta]L^{\lambda_\alpha}(\mathbf{k}_\alpha,\mathbf{q}_\alpha)L^{\lambda_\beta}(\mathbf{k}_\beta,\mathbf{q}_\beta)d(\mathbf{q}_\alpha), \tag{7}$$

being $d(\mathbf{q})$ is the Fourier transform of the dipole operator $\frac{1}{N_c^2-1}\langle Tr\,[U(\mathbf{x})U(\mathbf{y})]\rangle_T$.

4 Numerical results

In order to compute Eq. (4) we need to evaluate Eqs. (7) and (5) which contain two functions that need to be modeled, $\mu^2(\mathbf{k},\mathbf{q})$ and $d(\mathbf{q})$. Since $\mu^2(\mathbf{k},\mathbf{q})$ is a function that is peaked around

$\mathbf{k}+\mathbf{q}=0$ it is reasonable to assume a Gaussian shape for this object

$$\mu^2(\mathbf{k},\mathbf{q})=e^{-\frac{(\mathbf{k}+\mathbf{q})^2}{4B_p^{-1}}}, \tag{8}$$

where B_p is the gluonic transverse area of the projectile. For the dipole function we use the Fourier transform of the GBW model

$$d(\mathbf{q})=\frac{4\pi}{Q_s^2}e^{-\frac{\mathbf{q}^2}{Q_s^2}}. \tag{9}$$

Moreover, we have to regulate the infrared divergences that appear in the Lipatov vertex. In order to do so we use the following expression for the product of two Lipatov vertices

$$L^i(\mathbf{k},\mathbf{q}_1)L^i(\mathbf{k},\mathbf{q}_2)=\frac{(2\pi)^2}{\xi^2}\exp\left\{-\frac{[\mathbf{k}-(\mathbf{q}_1+\mathbf{q}_2)/2]^2}{\xi^2}\right\}, \tag{10}$$

where ξ^2 is a parameter with dimensions of momentum squared. This choice breaks some of the properties of the Lipatov vertices[2] but, apart from making the numerical implementation much simpler, it introduces a connection with the Wigner function approach presented in [5] with the difference that it includes quantum correlations in the projectile wave function.

With the models and approximation presented above we have obtained an analytical expression for the flow coefficients for double gluon production as a function of $B_pQ_s^2$, ξ^2/Q_s^2 and $p_\perp$. These results can be found in [3]. More interestingly, we have also evaluated the flow harmonics for quadruple gluon production as a function of Q_s^2 and $p_\perp$. These results are summarized in Figs. (1) and (2) and were computed at all orders in $1/(N_c^2-1)$. The values obtained for the 4-particle cumulants are negative (implying a real $v_n\{4\}$) and in the ballpark of experimental data. Moreover, it has been seen that at low multiplicity the 4-particle cumulant is positive [6]. The (naive) assumption that the multiplicity is proportional to the saturation momentum suggests that, at low Q_s, the cumulant should be positive. In fact, in the glasma graph approximation, suitable for pp (dilute-dilute) collisions and therefore for lower multiplicities, arguments suggested that $c_2\{4\}>0$. This property is not seen in Fig. (1) implying that a more detailed calculation should be done in the regime where the transition from dense to dilute is expected to occur, that is, at low Q_s^2.

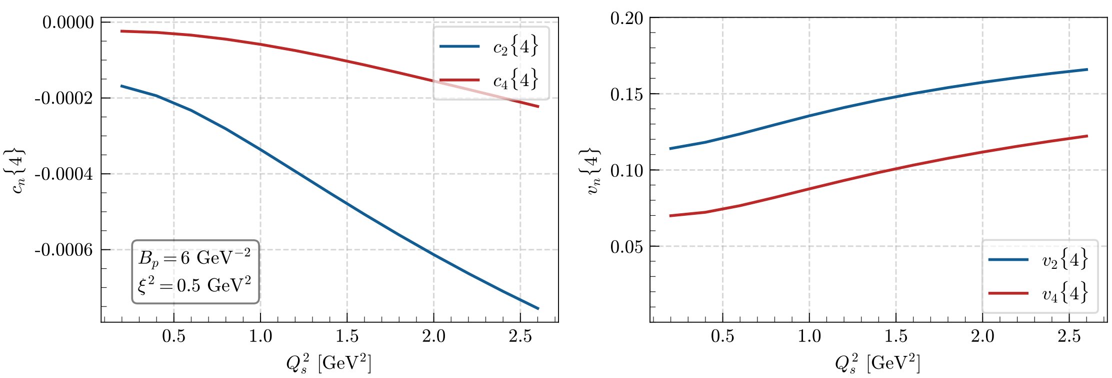

Figure 1: Dependence of the 4-particle cumulant (left) and azimuthal harmonic (right) of second and fourth order with Q_s^2.

[2]See [3] for a detailed discussion on the validity of this approach.

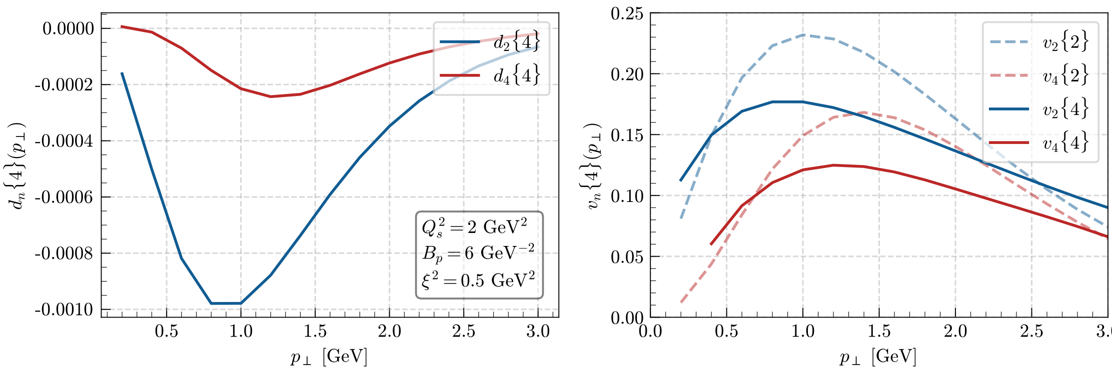

Figure 2: Dependence of the differential 4-particle cumulant (left) and azimuthal harmonic (right) of second and fourth order with $p_\perp$. For the latter we also show the results obtained from 2-particle correlations.

5 Conclusion

We have shown the results computed in [3] for multi-gluon production in the CGC in pA collisions. This calculation includes the contributions that are leading in the overlap area of the collision, while keeping all orders in the expansion in the number of colors. We have used the generalised MV model for computing projectile averages and the GBW model for the dipole functions. In order to proceed analytically as far as possible, we use the Wigner function approach [5] that we extend to include quantum correlations in the projectile wave function.

Our results can be summarized in Figs. 1 and 2. For four gluon correlations we find that the second order four particle cumulant $c_2\{4\} < 0$ – thus providing a real second order Fourier coefficient $v_2\{4\}$. The numerical results presented here, due to the Gaussian forms that we employ for the dipole and Wigner functions, should not be considered reliable for $p_\perp$ much larger than Q_s. They lie in the ballpark of experimental data, for values of parameters that look reasonable. However, we should note that further analytic understanding is still required, and several pieces are still missing in our formalism: the contribution from quarks, more involved projectile and target averages, fragmentation functions, ... All these aspects should be explored before we can establish a model ready for phenomenology.

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