Zero modes and matching for the twist-3 PDFs

Shohini Bhattacharya[1⋆], Krzysztof Cichy[2], Martha Constantinou[1], Andreas Metz[1], Aurora Scapellato[1] and Fernanda Steffens[3]

1 Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA
2 Faculty of Physics, Adam Mickiewicz University, ul. Uniwersytetu Poznańskiego 2, 61-614 Poznań, Poland
3 Institut für Strahlen- und Kernphysik, Rheinische Friedrich-Wilhelms-Universität Bonn, Nussallee 14-16, 53115 Bonn

⋆ tug23108@temple.edu

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Abstract

The quasi-PDF approach, proposed by X. Ji in 2013, has made it possible to directly extract light-cone PDFs from lattice QCD. This approach relies on the extraction of matrix elements of space-like operators for fast-moving hadrons. Quasi-PDFs can be related to the light-cone PDFs through a perturbatively calculable matching coefficient. We explore the formalism of matching, for the very first time, for the twist-3 PDFs $g_T(x)$, $e(x)$ and $h_L(x)$. In this work, we address the non-trivialities involved in the extraction of the matching coefficient due to the presence of (singular) zero-mode contributions.

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1 Introduction

Parton Distribution Functions (PDFs) are the fundamental quantities that describe the (one-dimensional) structure of the hadrons. They can be characterized according to their "twist", which dictates the order at which the PDFs contribute to physical observables. While leading-twist (twist-2) PDFs are dominant, the three twist-3 PDFs, $g_T(x)$, $e(x)$ and $h_L(x)$, suffer from kinematical suppressions which prevents an easy extraction of these functions from experiments. Pinning down these functions therefore from first principles in lattice Quantum Chromodynamics (QCD) is of utmost importance. Unfortunately, because the correlation functions underlying the PDFs have time-dependence, they cannot be directly calculated on (Euclidean) lattices. In 2013, X. Ji proposed the concept of parton quasi-distributions (quasi-PDFs) [1]. Quasi-PDFs are auxiliary quantities that differ from the (physical) light-cone PDFs in the ultraviolet regime, and hence can be related to the light-cone PDFs through a "matching coefficient"

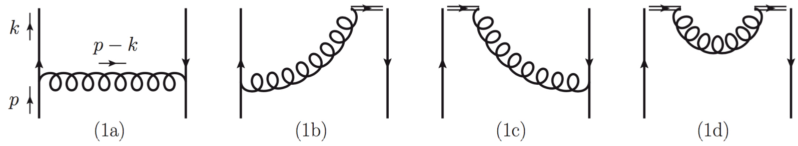

Figure 1: Real diagrams in the QTM at one-loop order.

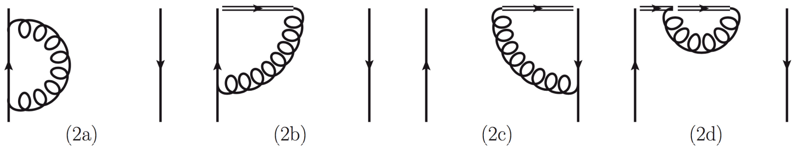

Figure 2: Virtual diagrams in the QTM at one-loop order. The Hermitian conjugate diagrams for (2a) and (2d) have not been shown.

that can be calculated order by order in perturbation theory. While quite some work on the matching for the twist-2 PDFs already existed, only recently the matching for the twist-3 PDFs was addressed in Refs. [2,3]. It is well known that the twist-3 PDFs can be contaminated with $\delta(x)$ singularities, popularly known as the zero-mode contributions. In this work, we show that these zero-mode contributions make the extraction of the matching coefficient difficult. In Sec. 2 we discuss the non-trivialities involved in the analysis. In Sec. 3 we present the final results of the matching coefficient. In Sec. 4 we provide a summary of this work.

2 Non-trivialities related to the singular terms

The light-cone PDFs and the quasi-PDFs are defined through the light-cone and spatial correlation functions,

$$\Phi^{[\Gamma]}(x,S) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle P,S|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|P,S\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}, \tag{1}$$

$$\Phi^{[\Gamma]}_{\mathrm{Q}}(x,S;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle P,S|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\,\psi(\tfrac{z}{2})|P,S\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}, \tag{2}$$

where Γ is a generic gamma matrix and and $\mathcal{W}/\mathcal{W}_{\mathrm{Q}}$ is the Wilson line. The twist-3 light-cone and quasi-PDFs are defined as,

$$\Phi^{[\gamma_\perp^i\gamma_5]} = \frac{M}{P^+}S_\perp^i g_T(x),\ \Phi_{\mathrm{Q}}^{[\gamma_\perp^i\gamma_5]} = \frac{M}{P^3}S_\perp^i g_{T,\mathrm{Q}}(x;P^3);\ \Phi^{[1]} = \frac{M}{P^+}e(x),\ \Phi_{\mathrm{Q}}^{[1]} = \frac{M}{P^3}e_{\mathrm{Q}}(x;P^3); \tag{3}$$

$$\Phi^{[i\sigma^{+-}\gamma_5]} = \frac{M}{P^+}\lambda h_L(x),\ \Phi_{\mathrm{Q}}^{[i\sigma^{30}\gamma_5]} = \frac{M}{P^3}\lambda h_{L,\mathrm{Q}}(x;P^3), \tag{4}$$

where M, $S_\perp^i$ and λ denotes the mass, transverse spin-vector and helicity of the target. The quasi-PDFs depend on the third-component of the target momentum denoted by P^3.

The one-loop corrections in the Quark Target Model (QTM) in, for instance, the Feynman gauge can be summed up as,

$$g_{T,\mathrm{Q}}(x;P^3) = \delta(1-x) + g_{T,\mathrm{Q}}^{(1a)}(x;P^3) + g_{T,\mathrm{Q}}^{(1b)}(x;P^3) + g_{T,\mathrm{Q}}^{(1c)}(x;P^3) + g_{T,\mathrm{Q}}^{(1d)}(x;P^3)$$
$$+\,\delta(1-x)\Big(g_{T,\mathrm{Q}}^{(2a)} + g_{T,\mathrm{Q}}^{(2b)} + g_{T,\mathrm{Q}}^{(2c)} + g_{T,\mathrm{Q}}^{(2d)}\Big), \tag{5}$$

and likewise for the other PDFs. In Eq. (5), we denote the real diagrams as (1a)-(1d) and the virtual diagrams as (2a)-(2d) and we show the corresponding diagrams in Figs. 1 and 2, respectively. We regulate the ultra-violet (UV) divergences by means of Dimensional Regularization (DR) and the infra-red (IR) divergences by means of DR and nonzero quark mass (m_q). Below our focus will be on diagram (1a) only because this diagram gives rise to the zero-mode contributions. We will also focus on the quasi-PDFs only since the corresponding calculation for the light-cone PDFs is straightforward [2,3].

We begin with $g_{T,\mathrm{Q}}(x;P^3)$. In general, the contribution from diagram (1a) can be disentangled into a “singular" and a “canonical" part,

$$g_{T,\mathrm{Q}}^{(1a)}(x;p^3) = g_{T,\mathrm{Q(s)}}^{(1a)}(x;p^3) + g_{T,\mathrm{Q(c)}}^{(1a)}(x;p^3)\,, \tag{6}$$

where p^3 is the parton momentum. Our focus will be on the singular part which is the quasi-PDF analogue of the $\delta(x)$-singularities for the light-cone PDFs. One finds before carrying out the $k_\perp$ integral,

$$g_{T,\mathrm{Q(s)}}^{(1a)}(x;p^3) = 2i\alpha_s C_F \mu^{2\epsilon}(4-n)\int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}}\left(\frac{i}{4}\frac{p^3}{(k_\perp^2 + x^2 p_3^2 + m_q^2)^{3/2}}\right), \tag{7}$$

where, $\alpha_s = g^2/4\pi$ with g as the coupling for the quark-gluon vertex, $C_F = 4/3$ is the color factor, and $n = 4-2\epsilon$ where $\epsilon \to \epsilon_{\mathrm{UV}} > 0$ or $\epsilon \to \epsilon_{\mathrm{IR}} < 0$ depending upon whether DR is applied for the UV or IR. The expression in Eq. (7) is UV-finite and IR-finite with $m_q \neq 0$. Due to the overall factor of ϵ_{UV} (see the prefactor of $(4-n)$ in Eq. (7)), the singular part drops out from the calculation,

$$g_{T,\mathrm{Q(s)}}^{(1a)}(x;p^3)\Big|_{m_q} = 0 \quad -\infty < x < \infty\,. \tag{8}$$

On the other hand, if we set $m_q = 0$ in Eq. (7), then the $k_\perp$ integral exhibits an IR divergence at the point $x = 0$. To regulate this IR divergence, we apply DR for the $k_\perp$ integral around $x = 0$, say $-1 < x < 1$. This provides,

$$\mu^{2\epsilon_{\mathrm{IR}}}\int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}}\frac{p^3}{(k_\perp^2 + x^2 p_3^2)^{3/2}} = \frac{\mu^{2\epsilon_{\mathrm{IR}}}}{(p^3)^{2\epsilon_{\mathrm{IR}}}} 2^{-1+2\epsilon_{\mathrm{IR}}}\pi^{-3/2+\epsilon_{\mathrm{IR}}}\Gamma(1/2+\epsilon_{\mathrm{IR}})\frac{1}{|x|^{1+2\epsilon_{\mathrm{IR}}}}\,. \tag{9}$$

The term $1/|x|^{1+2\epsilon_{\mathrm{IR}}}$ is non-trivial — how do we expand in terms of ϵ_{IR} at the point $x = 0$? This non-triviality can be dealt with by realizing that the matching coefficients appear under an integral (see Eq. (13)). This allows us to reshuffle the $1/|x|^{1+2\epsilon_{\mathrm{IR}}}$ term as [3],

$$\frac{1}{|x|^{1+2\epsilon_{\mathrm{IR}}}} \approx -\frac{\delta(x)}{\epsilon_{\mathrm{IR}}} + \frac{1}{|x|_{+[0]}} + \mathcal{O}(\epsilon_{\mathrm{IR}}) \quad -1 < x < 1\,, \tag{10}$$

where the second term is a plus-function at $x = 0$. Using Eq. (10) in Eq. (7), we find that due to the overall factor of ϵ_{IR} a $\delta(x)$-function contributes in the case of DR when applied for the IR. A common mistake is to approximate $\epsilon_{\mathrm{IR}}/|x|^{1+2\epsilon_{\mathrm{IR}}} \approx \epsilon_{\mathrm{IR}}/|x| \approx 0$ for any x and hence to miss the $\delta(x)$-function. This leads to a regulator-dependence for the matching coefficient. On the other hand, it is known from the twist-2 case that the matching coefficients are independent of the IR scheme.

In Ref. [3], we have shown that unlike $g_T(x)$, both $h_L(x)$ and $e(x)$ contain zero modes regardless of the IR scheme. Notably, these zero-mode contributions contain prefactors that depend on the IR regulator. For example, for $m_q \neq 0$, $h_{L(s)}(x) = -e_{(s)}(x) \propto \delta(x)\ln \mu_{\mathrm{UV}}^2/m_q^2$. The corresponding quasi-PDFs appear as,

$$h_{L,\mathrm{Q(s)}}^{(1a)}(x;p^3)\Big|_{m_q} = -e_{\mathrm{Q(s)}}^{(1a)}(x;p^3)\Big|_{m_q} = -\frac{\alpha_s C_F}{2\pi}\frac{(1-\epsilon_{\mathrm{UV}})}{\sqrt{x^2+\eta^2}} \qquad -\infty < x < \infty\,, \tag{11}$$

where $\eta = m_q/p^3$. Clearly, the functional forms for the IR poles do not agree between $h_{L,\mathrm{Q(s)}}(x;p^3)$ ($e_{\mathrm{Q(s)}}(x;p^3)$) and $h_{L(\mathrm{s})}(x)$ ($e_{(\mathrm{s})}(x)$). A naive expansion in terms of η gives rise to an incorrect conclusion of a mismatch in the IR between the quasi-PDFs and the light-cone PDFs, thereby leading to the conclusion that there cannot be a matching. (This is because a naive expansion leads to $h_{L,\mathrm{Q(s)}}(x;p^3) = -e_{\mathrm{Q(s)}}(x;p^3) \propto 1/|x|$.) However, similar to the case of $g_{T,\mathrm{Q}}(x;p^3)$, we have to realize that we can reshuffle the expression in Eq. (11) since the matching coefficients appear under an integral. In this sense, one can derive [3],

$$\frac{\theta(1-x)\,\theta(1+x)}{\sqrt{x^2+\eta^2}} = \left[\frac{1}{|x|}\right]_{+[0]} + \delta(x)\left(\ln\frac{4}{\eta^2}\right) + \mathcal{O}(\eta^2)\,. \tag{12}$$

Using this, we find that there is an exact agreement in the IR-pole structure of $h_{L,\mathrm{Q(s)}}(x;p^3)$ ($e_{\mathrm{Q(s)}}(x;p^3)$) and $h_{L(\mathrm{s})}(x)$ ($e_{(\mathrm{s})}(x)$). Hence, matching is possible for both $h_L(x)$ and $e(x)$. However, one has to be very careful with the treatment of the point $x = 0$ for the quasi-PDFs.

3 Matching coefficients for $g_T(x)$, $e(x)$ and $h_L(x)$

The structure of the matching formula that we have used in our work is,

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu^2}{p_3^2}\right) q_{\mathrm{Q}}\left(\frac{x}{\xi}, \mu, P^3\right) + \mathcal{O}\left(\frac{1}{P_3^2}\right), \tag{13}$$

where, C is the matching coefficient, μ is the renormalization scale, and $p^3 = (x/\xi)P^3$ where P^3 is the hadron momentum. Note that Eq. (13) was first derived for the twist-2 PDFs. It is known that the twist-3 PDFs have mixing with quark-gluon-quark correlators that should be taken into consideration in the matching formula. Hence, the matching coefficients cannot be calculated by means of two-parton correlators as was done in Refs. [2,3]. A full matching formalism for $g_T(x)$ was developed in Ref. [4]. However, it requires the calculation of multi-parton correlators from lattice QCD, which at the present stage is not feasible. By taking these factors into account, we believe that our approach is justifiable, at least at the present stage. Also, our work in Refs. [2,3] has led to promising results for $g_T(x)$ [5] and $h_L(x)$ [6] for the first time from lattice QCD. (See also Ref. [7].) We now summarize our final results for the singular ($C^{(\mathrm{s})}$) and the canonical ($C^{(\mathrm{c})}$) parts of the matching coefficients for $g_T(x)$ and $h_L(x)$ in the so-called Modified $\overline{\mathrm{MS}}$ ($\mathrm{M}\overline{\mathrm{MS}}$) scheme. The result for $g_T(x)$ is [2],

$$C^{(\mathrm{s})}_{\mathrm{M}\overline{\mathrm{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} 0 & \xi > 1 \\ \delta(\xi) & 0 < \xi < 1 \\ 0 & \xi < 0\,, \end{cases} \tag{14}$$

$$C^{(\mathrm{c})}_{\mathrm{M}\overline{\mathrm{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ & \xi > 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)p_3^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0 < \xi < 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0\,. \end{cases} \tag{15}$$

The result for $h_L(x)$ is [3],

$$C^{(s)}_{\mathrm{M\overline{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} \delta(1-\xi)\left(\frac{1}{2} - \frac{1}{2}\ln\frac{\mu^2}{4p_3^2}\right) & \xi > 1 \\ -\delta(\xi)\left(\ln\frac{4p_3^2}{\mu^2} + 1\right) - \left[\frac{1}{|\xi|}\right]_{+[0]} & -1 < \xi < 1 \\ \delta(1+\xi)\left(\frac{1}{2} - \frac{1}{2}\ln\frac{\mu^2}{4p_3^2}\right) & \xi < -1\,, \end{cases} \tag{16}$$

$$C^{(c)}_{\mathrm{M\overline{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{2}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{1}{1-\xi} + \frac{1}{\xi}\right]_+ & \xi > 1 \\ \left[\frac{2}{1-\xi}\ln\frac{4\xi(1-\xi)p_3^2}{\mu^2} + 2(1-\xi) - \frac{1}{1-\xi}\right]_+ & 0 < \xi < 1 \\ \left[\frac{2}{1-\xi}\ln\frac{\xi-1}{\xi} - \frac{1}{1-\xi} + \frac{1}{1-\xi}\right]_+ & \xi < 0\,. \end{cases} \tag{17}$$

The result for $e(x)$ is very similar to that of $h_L(x)$ [3]. We work in the M$\overline{\mathrm{MS}}$ scheme because the matching coefficients in the usual $\overline{\mathrm{MS}}$ scheme lead to divergences in the norm. In the M$\overline{\mathrm{MS}}$ scheme, we design the matching coefficient such that the norm of the quasi-PDF is automatically equal to the norm of the matched (or light-cone) PDF. For the lattice QCD results, we use the M$\overline{\mathrm{MS}}$ scheme for the quasi-PDFs and the matched PDFs are always presented in the $\overline{\mathrm{MS}}$ scheme.

4 Conclusion

The main message of this work is that one has to be extremely careful with regard to the treatment of the singular terms for the quasi-PDFs. We have argued that since the matching coefficients appear under an integral, we can re-write the singular terms for the quasi-PDFs into a $\delta(x)$ term whose IR-pole structure exactly agrees with the light-cone result. Therefore, matching is possible for $g_T(x)$, $e(x)$ and $h_L(x)$. In Ref. [7], we have shown the first-ever result for the twist-3 GPD $\tilde{G}_2(x)$ using the matching results shown in this work.

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