Impact parameter dependence of color charge correlations in the proton

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Abstract

The impact parameter dependence of color charge correlators in the proton is obtained from the light front formalism in light cone gauge. We include NLO corrections due to the $|qqqg\rangle$ Fock state via light-cone perturbation theory. Near the center of the proton, the b-dependence of the correlations is very different from a "transverse profile function". The resulting t-dependence of exclusive J/Ψ photoproduction transitions from exponential to power law at $|t| \approx 1 \text{ GeV}^2$. This prediction could be tested at upcoming DIS facilities or in nucleus-proton ultraperipheral collisions (UPCs).

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1 Introduction

The Hamiltonian light front formalism [1] in light cone gauge provides essential insight into correlations of color charges in the proton [2]. These can be expressed as matrix elements of nonperturbative boost-invariant light cone Fock-space wave functions of the QCD Hamiltonian, and related to physical observables such as the exclusive final states measured in DIS experiments. For example, at leading order the light-cone gauge color charge correlator is related to the average quark transverse momentum vector and to the Sivers asymmetry [3]. Furthermore, in the mixed transverse momentum – transverse coordinate space representation, the color charge correlator can be related to the Wigner distribution [4–6], and to various other
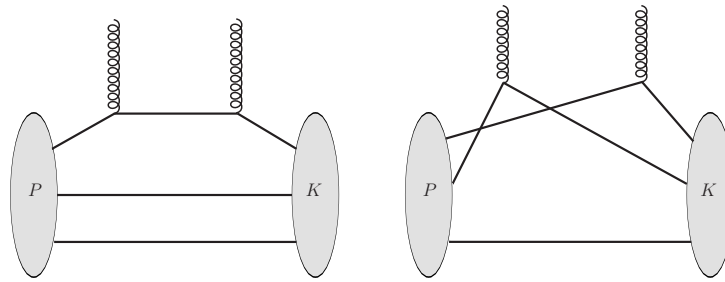
Figure 1: Handbag (left) and cat's ears (right) diagrams at LO. The former (latter) dominates for small (large) momentum transfer $|t|$.

generalized parton distribution functions. This detailed information about the partonic structure of the proton can be accessed experimentally e.g. in exclusive dijet or meson production, or in vector meson – lepton azimuthal correlations in DIS, as recently argued in refs. [7–10].

The notion of color charge density fluctuations in the transverse impact parameter plane emerges naturally in high-energy (small-x) scattering. The projectile charge traverses without recoil the (color) field produced coherently by all "valence" charges in the target, and the scattering amplitude follows from a correlator of path ordered exponentials of that field [11]. The scale separation in soft coherent fields sourced by "frozen" valence charges was introduced by McLerran and Venugopalan (MV) in Refs. [12,13]. Their model was devised for a very large nucleus and describes Gaussian fluctuations of classical color charge densities. However, when the density of valence charges in the target is not very large, one would rather take the two-dimensional color charge density as an operator acting on the light-cone wave function of the target [2].

In this contribution we focus on the non-trivial impact parameter dependence of color charge correlations in the proton, from which we predict a non-exponential, power-law tail of $d\sigma/dt$ for high-energy exclusive (coherent) J/Ψ photoproduction at large transverse momentum transfer $|t| \geq 1$ GeV^2.

2 The color charge correlator and coherent J/Ψ photoproduction at high momentum transfer

The central object of consideration is the two-point color charge correlator

$$\langle \rho^a(\vec{q}_1) \rho^b(\vec{q}_2) \rangle \equiv \delta^{ab} g^2 G_2(\vec{q}_1, \vec{q}_2) \,. \tag{1}$$

The notation $\langle \cdots \rangle$ denotes an expectation value between proton states, $\langle K|$ and $|P\rangle$, stripped of the delta-functions for conservation of transverse and light-cone momentum.

The insertion of the charge operators $\rho^a(\vec{q}_1), \rho^b(\vec{q}_2)$ between the incoming and scattered proton states corresponds to the attachment of two static gluon probes (with amputated propagators) to the color charges in the proton, in all possible ways. The two static gluons that probe the proton structure carry transverse momenta $\vec{q}_1$ and $\vec{q}_2$, and the total momentum transfer to the proton is

$$\vec{K} - \vec{P} = -(\vec{q}_1 + \vec{q}_2) \,. \tag{2}$$

From here on we choose $\vec{P} = 0$ for the incoming proton.

Figure 1 shows "handbag" and "cat's ears" diagrams at leading order (LO) $\mathcal{O}(g^2)$. The former are represented by one-body operators so that the entire momentum transfer $\vec{K}$ to the

proton flows into a single valence quark line. Therefore, for $\vec{q}_1 \to -\vec{q}_2$ (i.e. $\vec{K} \to 0$) there is maximal wave function overlap for the LO handbag diagram. This diagram is proportional to the electromagnetic (Dirac) form factor, i.e. to the distribution $\langle \rho(\vec{q}) \rangle$ of electric charge in the proton. See, for example, Eqs. (9,10) in ref. [14].

For large momentum transfer $|t| \simeq \vec{K}^2$, on the other hand, wave function overlap in the handbag diagram is highly suppressed. Here, the overlap of incoming and scattered proton is much greater for the cat's ears diagram because the momentum transfer is shared by two (or even three, at NLO) valence quarks. Since $\vec{K}$ is the Fourier conjugate to the two-dimensional (2D) transverse coordinate vector (impact parameter) $\vec{b}$, it follows that color charge correlators near the center of the proton are dominated by diagrams where the momentum transfer is shared by multiple partons (n-body operators).

NLO corrections due to the emission of a perturbative gluon by one of the quarks (plus the corresponding virtual corrections) have been computed in ref. [15]. These are suppressed by an additional factor of α_s but enhanced by a logarithm of the minimal light-cone momentum fraction x of the gluon [16–18]. Diagrams where a quark exchanges a gluon with itself across $\langle K| \to |P \rangle$ also involve a DGLAP [19–21] collinear logarithm. Numerically, the one-gluon emission corrections to the color charge correlator G_2 are small at $x \simeq 0.1$ but turn into the dominant contribution at $x \simeq 0.01$ [22].

To show the behavior of G_2 as a function of impact parameter we perform a Fourier transform w.r.t. the momentum transfer,

$$G_2(\vec{q}_{12}, \vec{b}) = \int \frac{\mathrm{d}^2 \vec{K}}{(2\pi)^2} \, e^{-i\vec{b}\cdot\vec{K}} \, G_2\left(\frac{\vec{q}_{12} - \vec{K}}{2}, -\frac{\vec{q}_{12} + \vec{K}}{2} \right) . \tag{3}$$

Here $\vec{q}_{12} = \vec{q}_1 - \vec{q}_2$ denotes the relative transverse momentum of the two gluon probes, the Fourier conjugate to the transverse distance $\vec{r}$ between the two gluons. For $\vec{q}_{12} = 0$, the integral of G_2 over the transverse impact parameter plane vanishes,

$$\int \mathrm{d}^2 \vec{b} \, G_2(\vec{q}_{12} = 0, \vec{b}) = 0 . \tag{4}$$

This follows from the fact that $G_2(\vec{q}_1, \vec{q}_2)$ satisfies a Ward identity and vanishes when either $\vec{q}_i \to 0$ [15, 23–25].

Figure 2 shows G_2 versus b. Near the center of the proton we find that $G_2 < 0$, i.e. "repulsive" two-body correlations dominate here. Also, comparing $x = 0.1$ to $x = 0.01$ we note that the NLO correction mainly affects G_2 at small b to strongly boost these negative two-body correlations, more so for smaller q_{12}. However, hand-bag type contributions become more prominent with increasing q_{12} or b (the standard GPD limit). Generically, the large-b tails of the two-point correlator G_2 exhibit a fall-off that resembles a transverse profile function.

We may compare the above result from LCPT to the widely used IPsat model [26,27]. Here, the color charge correlator is assumed to factorize into a function of $\vec{q}_{12}$ (or of $\vec{r}$, in coordinate space representation) times a Gaussian spatial profile function

$$T_p(\vec{b}) = \frac{1}{2\pi B} e^{-b^2/2B} . \tag{5}$$

A similar factorized form of G_2 was proposed in ref. [28]. In such models the correlator of course does not change sign as a function of b. Also, it does not satisfy the sum rule eq. (4).
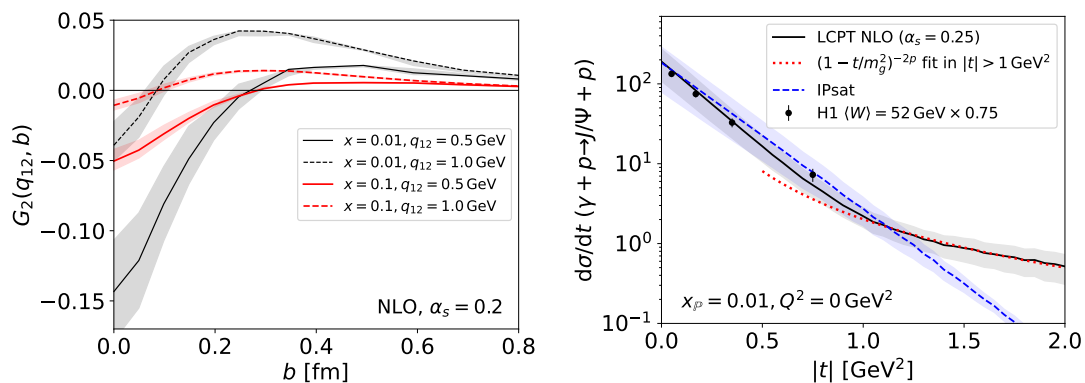
Figure 2: Left: Color charge correlator $G_2(\vec{q}_{12}, \vec{b})$ as a function of impact parameter for various relative gluon transverse momenta $\vec{q}_{12}$ [22]. The bands indicate the variation with the collinear cutoff, over the range $m = 0.1 \ldots 0.4$ GeV.
Right: cross section for exclusive J/Ψ photoproduction versus squared transverse momentum transfer. We compare predictions of our light-cone perturbation theory approach and of the IPsat model at $x_{\mathbb{P}} = 0.01$ (or $W \approx 30$ GeV). The bands reflect the uncertainty of the J/Ψ wave function [29]. For reference, we also show scaled data by the H1 collaboration [30] taken at higher energy.

From the color charge correlator we can compute the scattering amplitude of a quark-antiquark dipole as a function of its size and of impact parameter [2]:

$$
\begin{aligned}
N(\vec{r}, \vec{b}) = {} & -g^4 C_F \int \frac{\mathrm{d}^2\vec{K}\mathrm{d}^2\vec{q}}{(2\pi)^4} \frac{\cos(\vec{b}\cdot\vec{K})}{(\vec{q}-\frac{1}{2}\vec{K})^2\,(\vec{q}+\frac{1}{2}\vec{K})^2} \\
& \times \left(\cos(\vec{r}\cdot\vec{q}) - \cos\left(\frac{\vec{r}\cdot\vec{K}}{2}\right)\right) G_2\left(\vec{q}-\frac{1}{2}\vec{K}, -\vec{q}-\frac{1}{2}\vec{K}\right).
\end{aligned}
\tag{6}
$$

This expression applies in the two-gluon exchange approximation in the regime of weak scattering, $N(\vec{r}, \vec{b}) \ll 1$ since it does not resum the Glauber-Mueller multiple scattering series. To perform such resummation, the color charge correlator would have to be transformed from light cone to covariant gauge.

The dipole scattering amplitude exhibits an interesting dependence on the azimuthal angle between $\vec{r}$ and $\vec{b}$ which is discussed in more detail in ref. [22].

We now employ our color charge correlator $G_2(\vec{q}_1, \vec{q}_2)$ obtained from light-cone perturbation theory (LCPT) to compute the cross section for $\gamma + p \to J/\Psi + p$ as a function of transverse momentum transfer. Our main interest is in the regime of relatively high momentum transfer where, as explained above, the quark-antiquark dipole in the photon predominantly scatters from multiple partons in the proton.

The computation of $\mathrm{d}\sigma/\mathrm{d}t$ from the dipole scattering amplitude is described in detail in the literature, see for example refs. [29, 31, 32] (and ref. [33] for a first next-to-leading order calculation presented at this Workshop). Our result at $x_{\mathbb{P}} = 0.01$ is shown in Fig. 2. (For reference, we also show HERA data taken at higher energy, $x_{\mathbb{P}} \approx 0.0035$ shifted to match the normalization of our calculation. However, our main focus here is on the t-dependence.)

The IPsat model leads to an exponential drop off with $|t|$ which of course follows from the parameterized b-dependence eq. (5). The proton structure that emerges from our LCPT computation exhibits a similar exponential fall-off up to $|t| \approx 0.8$ GeV^2. However, this turns into a power-law dependence at greater momentum transfer. The cross section at large $|t|$

probes color charge correlations at small b, near the center of the proton. As shown in the previous section, there our correlation function $G_2(\vec{b}, \vec{q}_{12})$ is very different from a "profile function".

A description of $d\sigma/dt$ in terms of a dipole form factor has been proposed previously [34] in order to account for the non-exponential slope at small *transverse momentum* transfer, in particular for low energies near the kinematic production threshold. Instead, here we consider the high-energy limit far above threshold and high $|t| \simeq K_T^2 > 1$ GeV^2, where the cross section is *not* expressed in terms of a form factor but in terms of a two-point correlation function of color charge densities in the proton, as explained above.

3 Conclusion

In conclusion, we suggest that the impact parameter dependence of color charge correlations in the proton (at moderately small x) is very different from a "transverse profile function", especially at small b. In particular, the two-point correlation function is negative at $b \to 0$, and changes sign at intermediate impact parameters.

The non-trivial impact parameter dependence of color charge correlation functions leads to a t-dependence of the cross section for exclusive J/Ψ production which exhibits an exponential fall-off with $|t|$ at intermediate momentum transfer, which then changes into a power-law (approximately dipole-like) fall-off beyond $|t| \approx 1$ GeV^2.

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