Three-loop soft anomalous dimensions for top-quark processes

Nikolaos Kidonakis⋆

Kennesaw State University, Kennesaw, GA 30144, USA

⋆ nkidonak@kennesaw.edu

Proceedings for the XXVIII International Workshop on Deep-Inelastic Scattering and Related Subjects, Stony Brook University, New York, USA, 12-16 April 2021
doi:10.21468/SciPostPhysProc.8

Abstract

I present results for soft anomalous dimensions through three loops for several processes involving the production of top quarks. In particular, I discuss single-top and top-pair production. I also present some numerical results for double-differential distributions in $t\bar{t}$ production through approximate N^{3}LO.

Received 01-06-2021
Accepted 09-03-2022
Published 13-07-2022
doi:10.21468/SciPostPhysProc.8.104

1 Introduction

The inclusion of soft-gluon corrections in theoretical predictions for top-quark processes is required for better accuracy, and it involves calculations of soft anomalous dimensions. The first calculations at one loop were done in the mid 90's [1], but two-loop calculations appeared much later. The current state-of-the-art has been extended to three loops for some processes.

For partonic processes $f_1(p_1) + f_2(p_2) \to t(p_t) + X$, we define a kinematical threshold variable $s_4 = s + t + u - \sum_i m_i^2$ where $s = (p_1 + p_2)^2$, $t = (p_1 - p_t)^2$, and $u = (p_2 - p_t)^2$. At partonic threshold $s_4 \to 0$, and the soft-gluon corrections involve logarithms of the form $[\ln^k(s_4/m_t^2)/s_4]_+$ with $k \le 2n-1$ at perturbative order α_s^n.

We define transforms of the partonic cross section as $\hat{\sigma}(N) = \int (ds_4/s)\, e^{-Ns_4/s} \hat{\sigma}(s_4)$, with transform variable N. The factorized expression for the cross section is [1]

$$\sigma^{f_1 f_2 \to tX}(N) = H^{f_1 f_2 \to tX}\, S^{f_1 f_2 \to tX}\left(\frac{m_t}{N\mu_F}\right) \psi_1(N_1, \mu_F)\, \psi_2(N_2, \mu_F) \prod_i J_i(N, \mu_F)\,, \quad (1)$$

where $H^{f_1 f_2 \to tX}$ is an N-independent hard function, $S^{f_1 f_2 \to tX}$ is a soft function [1], while the ψ_i and J_i describe collinear emission from initial- and final-state particles [2].

The soft function $S^{f_1 f_2 \to tX}$ satisfies the renormalization group equation

$$\left(\mu_R \frac{\partial}{\partial \mu_R} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S^{f_1 f_2 \to tX} = -\Gamma_S^{\dagger\, f_1 f_2 \to tX} S^{f_1 f_2 \to tX} - S^{f_1 f_2 \to tX}\, \Gamma_S^{f_1 f_2 \to tX}\,, \quad (2)$$

where the soft anomalous dimension $\Gamma_S^{f_1f_2\to tX}$ controls the evolution of $S^{f_1f_2\to tX}$, which gives the exponentiation of logarithms of N in the resummed cross section. The resummation of these soft corrections at NNLL accuracy requires knowledge of two-loop soft anomalous dimensions while at N^{3}LL accuracy it requires three-loop soft anomalous dimensions. The resummed cross sections may be expanded at finite order and produce, upon inversion to momentum space, physical predictions.

A recent review of calculations of cusp and soft anomalous dimensions for many processes can be found in Ref. [3] (see also [4–6] for the cusp, [7–11] for single top, and [1,12] for $t\bar{t}$).

2 Cusp anomalous dimension

The cusp anomalous dimension, $\Gamma_{\rm cusp}$, involves two eikonal lines, and it is a basic ingredient in calculations of soft anomalous dimensions for partonic processes. For two lines with momenta p_i and p_j, the cusp angle is $\theta=\cosh^{-1}(p_i\cdot p_j/\sqrt{p_i^2p_j^2})$, and we write the perturbative series $\Gamma_{\rm cusp}=\sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n\Gamma_{\rm cusp}^{(n)}$. At one loop, $\Gamma_{\rm cusp}^{(1)}=C_F(\theta\coth\theta-1)$. In the case of two heavy quarks, this can be written in terms of the speed $\beta=\tanh(\theta/2)$ as

$$\Gamma_{\rm cusp}^{(1)\beta}=-C_F\left[\frac{(1+\beta^2)}{2\beta}\ln\frac{(1-\beta)}{(1+\beta)}+1\right]. \tag{3}$$

At two loops, we have [4]

$$\begin{aligned}\Gamma_{\rm cusp}^{(2)} &= K_2\,\Gamma_{\rm cusp}^{(1)}+\frac{1}{2}C_FC_A\left\{1+\zeta_2+\theta^2-\coth\theta\left[\zeta_2\theta+\theta^2+\frac{\theta^3}{3}+{\rm Li}_2\left(1-e^{-2\theta}\right)\right]\right.\\ &\left.\quad+\coth^2\theta\left[-\zeta_3+\zeta_2\theta+\frac{\theta^3}{3}+\theta\,{\rm Li}_2\left(e^{-2\theta}\right)+{\rm Li}_3\left(e^{-2\theta}\right)\right]\right\}, \end{aligned}\tag{4}$$

where $K_2=C_A\left(\frac{67}{36}-\frac{\zeta_2}{2}\right)-\frac{5}{18}n_f$. This can be written in terms of β and denoted as $\Gamma_{\rm cusp}^{(2)\beta}$.

The three-loop result [5,6] can be written as [3,6]

$$\Gamma_{\rm cusp}^{(3)}=K_3\Gamma_{\rm cusp}^{(1)}+2K_2\left(\Gamma_{\rm cusp}^{(2)}-K_2\,\Gamma_{\rm cusp}^{(1)}\right)+C^{(3)}, \tag{5}$$

where $K^{(3)}$ and $C^{(3)}$ have long expressions. Again, the result can be expressed in terms of β.

If eikonal line i represents a massive quark, with mass m_i, and eikonal line j a massless quark, then we find simpler expressions. At one loop, $\Gamma_{\rm cusp}^{(1)m_i}=C_F[\ln(2p_i\cdot p_j/(m_i\sqrt{s}))-1/2]$. At two loops [9,11], $\Gamma_{\rm cusp}^{(2)m_i}=K_2\,\Gamma_{\rm cusp}^{(1)m_i}+C_FC_A(1-\zeta_3)/4$. At three loops [11],

$$\Gamma_{\rm cusp}^{(3)m_i}=K_3\,\Gamma_{\rm cusp}^{(1)m_i}+\frac{1}{2}K_2C_FC_A(1-\zeta_3)+C_FC_A^2\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right). \tag{6}$$

If both eikonal lines are massless, then $\Gamma_{\rm cusp}^{\rm massless}=C_F\ln(2p_i\cdot p_j/s)\sum_{n=1}^{\infty}(\alpha_s/\pi)^nK_n$.

3 Single-top production

Next, we discuss various single-top production processes [7–11].

3.1 Single-top t-channel production

The soft anomalous dimension for t-channel single-top production, $\Gamma_S^{bq\to tq'}$, is a 2×2 matrix in color space. We use a t-channel singlet-octet color basis. The one-loop [7, 10, 11] and two-loop [10, 11] results are well known.

At three loops, we have

$$
\begin{aligned}
\Gamma_{S\,11}^{(3)bq\to tq'} &= K_3\,\Gamma_{S\,11}^{(1)bq\to tq'} + \frac{1}{2}K_2 C_F C_A(1-\zeta_3) + C_F C_A^2\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right)\\
\Gamma_{S\,12}^{(3)bq\to tq'} &= K_3\,\Gamma_{S\,12}^{(1)bq\to tq'} + X_{S\,12}^{(3)bq\to tq'}\,, \qquad \Gamma_{S\,21}^{(3)bq\to tq'} = K_3\,\Gamma_{S\,21}^{(1)bq\to tq'} + X_{S\,21}^{(3)bq\to tq'}\\
\Gamma_{S\,22}^{(3)bq\to tq'} &= K_3\,\Gamma_{S\,22}^{(1)bq\to tq'} + \frac{1}{2}K_2 C_F C_A(1-\zeta_3)\\
&\quad + C_F C_A^2\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right) + X_{S\,22}^{(3)bq\to tq'}\,. \qquad (7)
\end{aligned}
$$

The first element, i.e. the "11" element, of the matrix at three loops was calculated in [11]. Due to the relatively simple color structure of the hard matrix for this process, it is the only three-loop element that contributes to the N^{3}LO soft-gluon corrections. Here we have also provided three-loop results for the other three matrix elements up to unknown terms from four-parton correlations, which are denoted as $X_S^{(3)bq\to tq'}$ in the above equation.

3.2 Single-top s-channel production

We continue with results for the s-channel, for which $\Gamma_S^{q\bar{q}'\to t\bar{b}}$ is a 2×2 matrix, and we use an s-channel singlet-octet color basis. The one-loop [7, 8, 11] and two-loop [8, 11] results are, again, well known.

At three loops, we have

$$
\begin{aligned}
\Gamma_{S\,11}^{(3)q\bar{q}'\to t\bar{b}} &= K_3\,\Gamma_{S\,11}^{(1)q\bar{q}'\to t\bar{b}} + \frac{1}{2}K_2 C_F C_A(1-\zeta_3) + C_F C_A^2\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right)\\
\Gamma_{S\,12}^{(3)q\bar{q}'\to t\bar{b}} &= K_3\,\Gamma_{S\,12}^{(1)q\bar{q}'\to t\bar{b}} + X_{S\,12}^{(3)q\bar{q}'\to t\bar{b}}\,, \qquad \Gamma_{S\,21}^{(3)q\bar{q}'\to t\bar{b}} = K_3\,\Gamma_{S\,21}^{(1)q\bar{q}'\to t\bar{b}} + X_{S\,21}^{(3)q\bar{q}'\to t\bar{b}}\\
\Gamma_{S\,22}^{(3)q\bar{q}'\to t\bar{b}} &= K_3\,\Gamma_{S\,22}^{(1)q\bar{q}'\to t\bar{b}} + \frac{1}{2}K_2 C_F C_A(1-\zeta_3)\\
&\quad + C_F C_A^2\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right) + X_{S\,22}^{(3)q\bar{q}'\to t\bar{b}}\,. \qquad (8)
\end{aligned}
$$

Again, the "11" element of the matrix at three loops was calculated in [11] and is the only three-loop element to contribute to the N^{3}LO soft-gluon corrections. We have also provided in the above equation three-loop results for the other three matrix elements up to unknown terms from four-parton correlations, which are denoted as $X_S^{(3)q\bar{q}'\to t\bar{b}}$.

3.3 Associated tW production

The soft anomalous dimension for tW production has only one element (not a matrix). It is known at one loop [7, 9], two-loops [9], and three loops [11]. The three-loop result is [11]

$$
\Gamma_S^{(3)bg\to tW} = K_3\,\Gamma_S^{(1)bg\to tW} + \frac{1}{2}K_2 C_F C_A(1-\zeta_3) + C_F C_A^2\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right). \qquad (9)
$$

4 Top-antitop pair production

We continue with top-antitop pair production which can proceed via the $q\bar{q} \to t\bar{t}$ and the $gg \to t\bar{t}$ channels.

In the $q\bar{q} \to t\bar{t}$ channel, $\Gamma_S^{q\bar{q}\to t\bar{t}}$ is a 2×2 matrix, and we use an s-channel singlet-octet color basis. Here we will concentrate on the "22" matrix element which at one-loop contributes already to the soft-gluon corrections at NLO. At one loop, this element is [1, 12]

$$\Gamma_{22}^{(1)q\bar{q}\to t\bar{t}} = \left(1 - \frac{C_A}{2C_F}\right)\Gamma_{\rm cusp}^{(1)\beta} + 4C_F \ln\left(\frac{t-m_t^2}{u-m_t^2}\right) - \frac{C_A}{2}\left[1 + \ln\left(\frac{s m_t^2 (t-m_t^2)^2}{(u-m_t^2)^4}\right)\right], \quad (10)$$

while at two loops it is [3, 12]

$$\Gamma_{22}^{(2)q\bar{q}\to t\bar{t}} = K_2 \Gamma_{22}^{(1)q\bar{q}\to t\bar{t}} + \left(1 - \frac{C_A}{2C_F}\right)\left(\Gamma_{\rm cusp}^{(2)\beta} - K_2 \Gamma_{\rm cusp}^{(1)\beta}\right) + \frac{C_A^2}{4}(1-\zeta_3). \quad (11)$$

At three loops, we find the following expression:

$$\begin{aligned}\Gamma_{S\,22}^{(3)q\bar{q}\to t\bar{t}} &= K_3\, \Gamma_{S\,22}^{(1)q\bar{q}\to t\bar{t}} + \left(1 - \frac{C_A}{2C_F}\right)\left(\Gamma_{\rm cusp}^{(3)\beta} - K_3 \Gamma_{\rm cusp}^{(1)\beta}\right) + \frac{K_2}{2} C_A^2 (1-\zeta_3) \\ &\quad + C_A^3\left(-\frac{1}{4} + \frac{3}{8}\zeta_2 - \frac{\zeta_3}{8} - \frac{3}{8}\zeta_2\zeta_3 + \frac{9}{16}\zeta_5\right) + X_{S\,22}^{(3)q\bar{q}\to t\bar{t}}, \end{aligned} \quad (12)$$

where $X_{S\,22}^{(3)q\bar{q}\to t\bar{t}}$ denotes unknown three-loop contributions from four-parton correlations. The other three-loop matrix elements are not fully known either but have analogous structure to that at two loops (see also [3]).

In the $gg \to t\bar{t}$ channel, $\Gamma_{S\,22}^{gg\to t\bar{t}}$ is a 3×3 matrix, and we use the color basis $c_1 = \delta^{ab}\delta_{12}$, $c_2 = d^{abc} T_{12}^c$, $c_3 = i f^{abc} T_{12}^c$. At one loop for $gg \to t\bar{t}$, the "22" matrix element is [1, 12]

$$\Gamma_{S\,22}^{(1)gg\to t\bar{t}} = \left(1 - \frac{C_A}{2C_F}\right)\Gamma_{\rm cusp}^{(1)\beta} + \frac{C_A}{2}\left[\ln\left(\frac{(t-m_t^2)(u-m_t^2)}{s\, m_t^2}\right) - 1\right], \quad (13)$$

while at two loops it is [3, 12]

$$\Gamma_{S\,22}^{(2)gg\to t\bar{t}} = K_2\, \Gamma_{S\,22}^{(1)gg\to t\bar{t}} + \left(1 - \frac{C_A}{2C_F}\right)\left(\Gamma_{\rm cusp}^{(2)\beta} - K_2 \Gamma_{\rm cusp}^{(1)\beta}\right) + \frac{C_A^2}{4}(1-\zeta_3). \quad (14)$$

At three loops, we find the expression

$$\begin{aligned}\Gamma_{S\,22}^{(3)gg\to t\bar{t}} &= K_3\, \Gamma_{S\,22}^{(1)gg\to t\bar{t}} + \left(1 - \frac{C_A}{2C_F}\right)\left(\Gamma_{\rm cusp}^{(3)\beta} - K_3 \Gamma_{\rm cusp}^{(1)\beta}\right) + \frac{K_2}{2} C_A^2 (1-\zeta_3) \\ &\quad + C_A^3\left(-\frac{1}{4} + \frac{3}{8}\zeta_2 - \frac{\zeta_3}{8} - \frac{3}{8}\zeta_2\zeta_3 + \frac{9}{16}\zeta_5\right) + X_{S\,22}^{(3)gg\to t\bar{t}}, \end{aligned} \quad (15)$$

where $X_{S\,22}^{(3)gg\to t\bar{t}}$ denotes unknown three-loop contributions from four-parton correlations.

As an application of the soft-gluon formalism at NNLL accuracy, in Figure 1 we show top-quark double-differential distributions in p_T and rapidity with soft-gluon corrections through approximate N^{3}LO [13]. The theoretical predictions describe very well the CMS data at 13 TeV [14].

5 Conclusion

I have presented results for soft anomalous dimensions at one, two, and three loops. The cusp anomalous dimension was discussed first followed by results for the soft anomalous dimensions in single-top production and in top-antitop pair production.

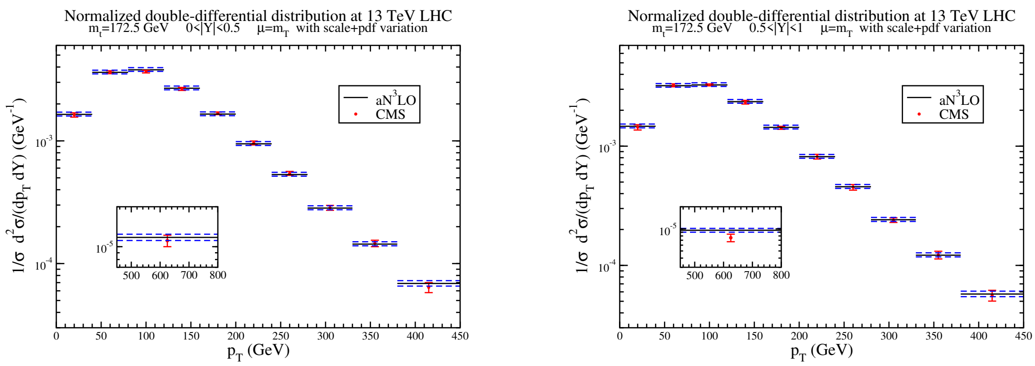

Figure 1: Top-quark double-differential distributions.

Funding information This material is based upon work supported by the National Science Foundation under Grant No. PHY 1820795.

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