Understanding the proton mass in QCD

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Abstract

Understanding the proton mass in quantum chromodynamics (QCD) is a very important and current topic in hadronic physics. The decomposition (sum rule) of the proton mass is not unique, and different sum rules, which are related to the QCD energy-momentum tensor, can be found in the literature. We review and compare these sum rules and identify open questions in this field.

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1 Introduction

Key questions in hadronic physics deal with the decomposition of global properties of the proton, such as its mass and spin, in terms of individual contributions from quarks and gluons. The focus of the present manuscript is on the understanding of the proton mass in QCD. Various sum rules of the proton mass have been proposed in the literature [1–7]. All of them are related to the QCD energy-momentum tensor (EMT) $T^{\mu\nu}$ — either its 00-component (energy component) or its trace, including the anomalous contribution [8,9].

To be now more specific, the following mass sum rules exist: (i) a decomposition by Ji into four terms [1,2], as well as a proposed (three-term) modification that we came up with recently in Refs. [6,7]; (ii) a two-term decomposition by Lorcé [3]; (iii) another two-term sum rule by Hatta, Rajan, Tanaka [4,5]. Here we briefly review these sum rules and also point out the most important similarities and differences.

The community largely agrees that, from a physics point of view, at most a few meaningful sum rules can be identified. On the other hand, different opinions/results have been put forward in relation to two crucial questions:

1. What is the correct form of the (renormalized) operators associated with the individual terms of the mass sum rules?

2. What is the physical meaning and significance of (some of) the terms of the various decompositions?

In this contribution, which is largely based on Refs. [6, 7], we address more details related to those questions. Comprehensive numerical results for the different mass sum rules of the proton can be found in Ref. [7].

2 QCD energy-momentum tensor and its renormalization

We begin our discussion by recalling the (symmetric and gauge invariant) EMT in QCD, which can be decomposed into a quark and gluon contribution according to

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}, \quad \text{with} \tag{1}$$

$$T_q^{\mu\nu} = \frac{i}{4}\,\overline{\psi}\,\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\,\psi\,, \qquad T_g^{\mu\nu} = -F^{\mu\alpha}F^{\nu}_{\ \alpha} + \frac{g^{\mu\nu}}{4}F^{\alpha\beta}F_{\alpha\beta}\,, \tag{2}$$

where in $T_q^{\mu\nu}$ a summation over quark flavors is understood, and $\overleftrightarrow{D}^{\mu} = \overrightarrow{\partial}^{\mu} - \overleftarrow{\partial}^{\mu} - 2igA_a^{\mu}T_a$ is the covariant derivative. For both $T_q^{\mu\nu}$ and $T_g^{\mu\nu}$, renormalization of the parameters of the QCD Lagrangian is implied. The (conserved) total EMT is not renormalized (beyond renormalization at the level of the Lagrange density), but the individual quark and gluon parts of the EMT must be renormalized.

The component T^{00} describes the energy density of a system, while the spatial terms T^{jj} on the main diagonal of the EMT are typically considered pressure contributions [10]. Note that the interpretation of T^{jj} for a system like the proton has given rise to controversial discussions about the "optimal" mass sum rule [3, 11]. We will return to this point below.

A very important quantity is the trace of the QCD EMT which is given by [8, 9]

$$T^{\mu}_{\ \mu} = (T_R)^{\mu}_{\ \mu} = (T^{\mu}_{\ \mu})_R = (m\overline{\psi}\psi)_R + \gamma_m\,(m\overline{\psi}\psi)_R + \frac{\beta}{2g}(F^{\alpha\beta}F_{\alpha\beta})_R\,, \tag{3}$$

with the anomalous dimension γ_m for the quark mass and the QCD beta function β. The subscript 'R' indicates renormalization. We point out that the operator $m\overline{\psi}\psi$ actually does not require additional renormalization. A classical treatment of the trace of the EMT in Eq. (1) would just provide $m\overline{\psi}\psi$. The additional terms on the r.h.s. of Eq. (3) are therefore quantum effects and denoted as trace anomaly.

Recently, the renormalization of the individual contributions $T_i^{\mu\nu}$ $(i = q, g)$ to the EMT was investigated in detail for the first time [4, 5]. In particular, it was found that the (renormalized) quark and gluon contributions to the trace read

$$(T_{q,R})^{\mu}_{\ \mu} = (1+y)(m\overline{\psi}\psi)_R + x\,(F^{\alpha\beta}F_{\alpha\beta})_R\,, \tag{4}$$

$$(T_{g,R})^{\mu}_{\ \mu} = (\gamma_m - y)(m\overline{\psi}\psi)_R + \left(\frac{\beta}{2g} - x\right)(F^{\alpha\beta}F_{\alpha\beta})_R\,. \tag{5}$$

The free parameters x and y are related to the finite terms of certain renormalization constants. They may be fixed in minimal-subtraction-type schemes [4, 5], but in the present context different schemes seem equally (if not more) natural. In fact, in Refs. [6, 7] we proposed the so-called D1 scheme and D2 scheme which are specified in the following manner:

- D1 scheme: $x = 0$, $y = \gamma_m$, implying that Eqs. (4) and (5) become diagonal.
- D2 scheme: $x = y = 0$, implying that the entire trace anomaly arises from the trace of the renormalized gluon part of the EMT.

3 Mass sum rules

In order to discuss the decompositions of the proton mass, we consider matrix elements of the EMT. For the full EMT one finds

$$\langle T^{\mu\nu} \rangle \equiv \langle P| T^{\mu\nu} |P\rangle = 2P^\mu P^\nu \,, \tag{6}$$

where $P^\mu = (P^0, \boldsymbol{P})$ (with $P^2 = M^2$) is the 4-momentum of the proton. Note that Eq. (6) holds in this form provided that the normalization $\langle P'|P\rangle = 2P^0(2\pi)^3\delta^{(3)}(\boldsymbol{P}' - \boldsymbol{P})$ is used. The matrix elements of the individual contributions to the EMT are given by

$$\langle T^{\mu\nu}_{i,R} \rangle = 2P^\mu P^\nu A_i(0) + 2M^2 g^{\mu\nu} \overline{C}_i(0) \,, \tag{7}$$

with the form factors $A_i(0)$ and $\overline{C}_i(0)$ evaluated at vanishing momentum transfer. Conservation of the total EMT provides the constraints

$$A_q(0) + A_g(0) = 1 \,, \qquad \overline{C}_q(0) + \overline{C}_g(0) = 0 \,, \tag{8}$$

which ensure that the sum of the quark and gluon contributions gives the r.h.s. of Eq. (6).

Equation (6) implies that the matrix elements of both the EMT trace and the component T^{00} are related to the proton mass according to

$$\mathcal{N} \langle T^\mu_{\ \mu} \rangle = M \,, \qquad \mathcal{N} \langle T^{00} \rangle = M \,, \tag{9}$$

where $\mathcal{N} = \frac{1}{2M}$ for the aforementioned normalization of $\langle P'|P\rangle$. (Note that throughout we consider the proton rest frame only.) Using the fact that T^{00} is the QCD Hamiltonian density $\mathcal{H}_{\text{QCD}}$, that is, $H_{\text{QCD}} = \int d^3\boldsymbol{x}\; T^{00}$, one finds the intuitive relation [1]

$$\frac{\langle H_{\text{QCD}} \rangle}{\langle P|P\rangle} = M \,, \tag{10}$$

which does not depend on the specific normalization of the proton state. We now have all the ingredients that are needed to discuss the various mass sum rules.

3.1 Two-term sum rule by Hatta, Rajan, Tanaka

The two-term sum rule proposed in Refs. [4, 5] is based on the relation between the trace of the EMT and the mass in (9), and the decomposition of the trace into quark and gluons parts,

$$M = \overline{M}_q + \overline{M}_g = \mathcal{N}\big(\langle (T_{q,R})^\mu_{\ \mu}\rangle + \langle (T_{g,R})^\mu_{\ \mu}\rangle\big) \,, \tag{11}$$

where the operators for the terms are given in Eqs. (4), (5). The two contributions of this sum rule can be readily related to the form factors of the EMT,

$$\overline{M}_i = M\big(A_i(0) + 4\overline{C}_i(0)\big) \,. \tag{12}$$

3.2 Two-term sum rule by Lorcé

The two-term sum rule of Ref. [3] is a decomposition of the energy density T^{00} into contributions from the partons,

$$M = U_q + U_g = \mathcal{N}\big(\langle T^{00}_{q,R}\rangle + \langle T^{00}_{g,R}\rangle\big) \,. \tag{13}$$

Expressed through the EMT form factors, the two terms read

$$U_i = M\big(A_i(0) + \overline{C}_i(0)\big) \,, \tag{14}$$

showing that obviously $\overline{M}_i \neq U_i$, where the difference lies in the contribution from the $\overline{C}_i$. To reveal the physics of this difference, we repeat that the T^{jj} may be interpreted as pressures. From Eq. (6) one finds $\langle T^{xx} \rangle = \langle T^{yy} \rangle = \langle T^{zz} \rangle = 0$ for a proton at rest, which is also intuitively clear since those (total) pressures must vanish in order to have a mechanically stable system. On the other hand, the individual pressure contributions $\langle T^{jj}_{i,R} \rangle$ do not vanish and, according to Eq. (7), are given by the form factors $\overline{C}_i$. From this perspective, the $\overline{M}_i$ are admixtures of energy contributions (proportional to $A_i(0)+\overline{C}_i(0)$) and pressure contributions. Proper expressions for the renormalized operators of the two terms in Eq. (13) have been obtained in Ref. [6],

$$T^{00}_{q,R} = (m\overline{\psi}\psi)_R + (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R\,, \tag{15}$$

$$T^{00}_{g,R} = \frac{1}{2}(E^2+B^2)_R\,. \tag{16}$$

The quark contribution to the total energy in Eq. (15) consists of the quark mass term, which has been studied extensively in various approaches, and the quark kinetic plus potential energy.

3.3 Sum rules with more than two terms

In the pioneering papers by Ji a four-term sum rule was proposed [1,2]. The focus there is on the Hamiltonian density T^{00}, with the decomposition of the EMT into a traceless part ($\overline{T}^{\mu\nu}$) and trace part ($\hat{T}^{\mu\nu} = \frac{1}{4} g^{\mu\nu} T^\alpha_{\ \alpha}$) as further key ingredient,

$$T^{\mu\nu} = \overline{T}^{\mu\nu} + \hat{T}^{\mu\nu}\,. \tag{17}$$

(By definition, the traceless term is then given by $\overline{T}^{\mu\nu} = T^{\mu\nu} - \hat{T}^{\mu\nu}$. Below we return to this term, which has played an important role in recent discussions about the mass sum rule.) Equation (17) is motivated by the fact that the two terms do not mix under renormalization. Further decomposing $\overline{T}^{00}$ and $\hat{T}^{00}$ into quark and gluon contributions ($\overline{T}^{00}_{i,R}$ and $\hat{T}^{00}_{i,R}$) then provides four terms. Minor re-shuffling, aimed at obtaining simple-looking operators like the ones in Eqs. (15), (16), leads to the final four-term decomposition of Ji [1,2].

In Refs. [6,7] we revisited this derivation in view of the recent developments concerning the renormalization of the EMT [4,5] discussed above. This naturally lead us to a three-term sum rule of the proton mass,

$$M = \mathcal{N}\big(\mathcal{H}_q + \mathcal{H}_m + \mathcal{H}_g\big), \tag{18}$$

with the energy densities

$$\mathcal{H}_q = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R\,, \tag{19}$$

$$\mathcal{H}_m = (m\overline{\psi}\psi)_R\,, \tag{20}$$

$$\mathcal{H}_g = \frac{1}{2}(E^2+B^2)_R\,. \tag{21}$$

Comparison with the operators in Eqs. (15) – (16) shows that this decomposition can be understood as a refinement of the two-term sum rule by Lorcé in that we consider the quark mass term as a separate contribution.

Let us now comment on the difference between Ji's four-term sum rule [1, 2] and our decomposition defined through Eqs. (19)–(21). When summing up the operators for all the terms in the two decompositions one finds a mismatch given by

$$\frac{1}{4}\left(\gamma_m\,(m\overline{\psi}\psi)_R + \frac{\beta}{2g}(F^{\alpha\beta}F_{\alpha\beta})_R\right), \tag{22}$$

which is just one quarter of the trace anomaly. The presentation in Refs. [1, 2] suggests that this mismatch is due to wrong expressions for the renormalized traceless operators $\overline{T}^{00}_{i,R}$. When constructing those operators one must subtract the proper trace terms in Eqs. (4), (5), which not only implies a dependence on the parameters x and y, but, in particular, the subtraction of a quarter of the EMT trace (including the anomalous terms) in the sum $\overline{T}^{00}_{q,R}+\overline{T}^{00}_{g,R}$. Apparently, in Refs. [1, 2] only the classical trace $m\overline{\psi}\psi$ was subtracted in the traceless operators, which would explain the mismatch. More recent work argues that the mismatch can be traced back to a different meaning of some of the renormalized operators [11–13]. Most importantly, it is pointed out that in the four-term decomposition of Refs. [1, 2] the operator $(E^2+B^2)_R$ contains "classical" terms only and that a quantum contribution, the so-called "quantum anomalous energy", must be added. While at first sight this could also cause the mismatch, we argue that this is not a viable explanation since the renormalized operator $(E^2+B^2)_R$ has a unique meaning. (The recent studies in Refs. [12, 13] do not provide any convincing argument against this point of view [14].)

Our criticism in Refs. [6, 7] of the four-term decomposition of Ji addresses primarily the operator expressions of that sum rule. Generally, a four-term sum rule, based on the $\overline{T}^{00}_{i,R}$ and $\hat{T}^{00}_{i,R}$, can certainly be considered. However, from a physics point of view, such a sum rule, like the two-term decomposition by Hatta, Rajan, Tanaka [4, 5], could also be criticized for mixing energy and pressure contributions as was emphasized in Ref. [3]. Furthermore, the physical significance of the aforementioned quantum anomalous energy, which could give support to a four-term decomposition, is presently not fully clear [14].

4 Conclusions

Recently, the decomposition of the proton mass in QCD into individual contributions from quarks and gluons has gained renewed interest. Somewhat differing opinions about this topic exist in the community, which is also reflected by the fact that different mass sum rules have been proposed. We believe that Refs. [4, 5] and our work in Refs. [6, 7] helped to clarify the structure of the renormalized operators that appear in the sum rules. Open questions remain about the interpretation and significance of different contributions. Specifically, do linear combinations of $T^{00}_{i,R}$ and $T^{jj}_{i,R}$, have a meaningful physical interpretation? Moreover, what physical significance can be assigned to the so-called quantum anomalous energy? Arguably, these two questions are presently the most important ones in this research area.

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