Proton gravitational form factors
in a light-front quark-diquark model

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Abstract

We present a recent calculation of the gravitational form factors (GFFs) of proton using the light-front quark-diquark model constructed by the soft-wall AdS/QCD [1]. The four GFFs $A(Q^2)$, $B(Q^2)$, $C(Q^2)$ and $\bar{C}(Q^2)$ are calculated in this model. We also show the pressure and shear distributions of quarks inside the proton. The GFFs, $A(Q^2)$ and $B(Q^2)$ are found to be consistent with the lattice QCD, while the qualitative behavior of the D-term form factor is in agreement with the extracted data from the deeply virtual Compton scattering (DVCS) experiments at JLab, the lattice QCD, and the predictions of different phenomenological models.

1 Introduction

Understanding the parton dynamics inside the nucleon by studying the mechanical properties of the nucleon like the mass, spin and pressure distribution has recently received widespread attention following the first experimental evidence of the pressure distribution of quarks inside the nucleon [2]. The matrix elements of the energy-momentum tensor encode the information about the gravitational form factors. The gravitational form factors (GFFs) can be assessed experimentally in hard exclusive processes like deeply virtual Compton scattering (DVCS)

through generalized parton distributions (GPDs) [2]. The symmetric energy-momentum tensor for a spin-half particle is parameterized in terms of four GFFs. The GFFs $A(Q^2)$ and $B(Q^2)$ are related to the mass and total angular momentum of the proton [3] . The GFFs A and B are constrained at $Q^2 = 0$ as they are related to the generators of the Poincare group. But the GFF $C(Q^2)$ (also called the D-term) is unconstrained at $Q^2 = 0$. The D-term is connected to the internal properties of the nucleon like the pressure and stress distributions [4, 5].

2 Light-front quark-diquark model

In the light-front quark-diquark model the three valence quarks inside the proton are reduced to a quark (fermion) and composite state of a diquark (boson). Here we assume that a diquark has spin zero. The light-front wave functions $\psi_{\lambda_q q}^{\lambda_N}(x, \mathbf{k}_\perp)$ are modeled from the solution of soft-wall AdS/QCD. [6]. The generic ansatz for different spin configurations of nucleon helicities $\lambda_N = \pm \frac{1}{2}$ and for quark helicities $\lambda_q = \pm \frac{1}{2}$ of these light-front wave functions can be written compactly as follows:

$$\psi_{\lambda_q q}^{\lambda_N}(x, \mathbf{k}_\perp) = \delta_{\lambda_N, \lambda_q} \varphi_q^{(1)}(x, \mathbf{k}_\perp) - \delta_{\lambda_N, -\lambda_q} \varphi_q^{(2)}(x, \mathbf{k}_\perp) \frac{\mathbf{k}_\perp}{xM} e^{i \text{Sign}(\lambda_N)\theta_{\mathbf{k}_\perp}} , \tag{1}$$

where, the wave functions $\varphi_q^{(i=1,2)}(x, \mathbf{k}_\perp)$ are the modified form of the soft-wall AdS/QCD prediction constructed by introducing the parameters $a_q^{(i)}$ and $b_q^{(i)}$ for quark q [6, 7],

$$\varphi_q^{(i)}(x, \mathbf{k}_\perp) = N_q^{(i)} \frac{4\pi}{\kappa} \sqrt{\frac{\log(1/x)}{1-x}} x^{a_q^{(i)}} (1-x)^{b_q^{(i)}} \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2} \frac{\log(1/x)}{(1-x)^2} \right]. \tag{2}$$

We have taken the quark to be massless and all parameters are set in accordance with the electromagnetic properties of the nucleons. The parameters used in this model are taken from [8].

3 Gravitational Form Factors

The symmetric energy-momentum tensor $T^{\mu\nu}$ for a spin half system can be parameterized in terms of four GFFs [9, 10]

$$\begin{aligned}
\langle P', S' | T_i^{\mu\nu}(0) | P, S \rangle &= \bar{U}(P', S') \bigg[-B_i(q^2) \frac{\bar{P}^\mu \bar{P}^\nu}{M} + (A_i(q^2) + B_i(q^2)) \frac{1}{2} (\gamma^\mu \bar{P}^\nu + \gamma^\nu \bar{P}^\mu) \\
&+ C_i(q^2) \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2) M g^{\mu\nu} \bigg] U(P, S),
\end{aligned} \tag{3}$$

where, $\bar{P}^\mu = \frac{1}{2}(P' + P)^\mu$, $q^\mu = (P' - P)^\mu$, $U(P, S)$ is the spinor, $Q^2 = -q^2$ and M is the system mass. The GFFs $A_i(Q^2)$ and $B_i(Q^2)$ are calculated using the T_i^{++} component. The GFFs $C_i(Q^2)$ are $\bar{C}_i(Q^2)$ are calculated using the transverse components $T_i^{\perp\perp}$.

The distributions of pressure and shear forces inside the nucleon are given by

$$p(b) = \frac{1}{6M_n} \frac{1}{b^2} \frac{d}{db} b^2 \frac{d}{db} \tilde{D}(b), \quad s(b) = -\frac{1}{4M_n} b \frac{d}{db} \frac{1}{b} \frac{d}{db} \tilde{D}(b), \tag{4}$$

where $\tilde{D}(b)$ represents the Fourier transform of GFF $D(Q^2) = 4C(Q^2)$ and $b = |\vec{b}_\perp|$ is the impact parameter.
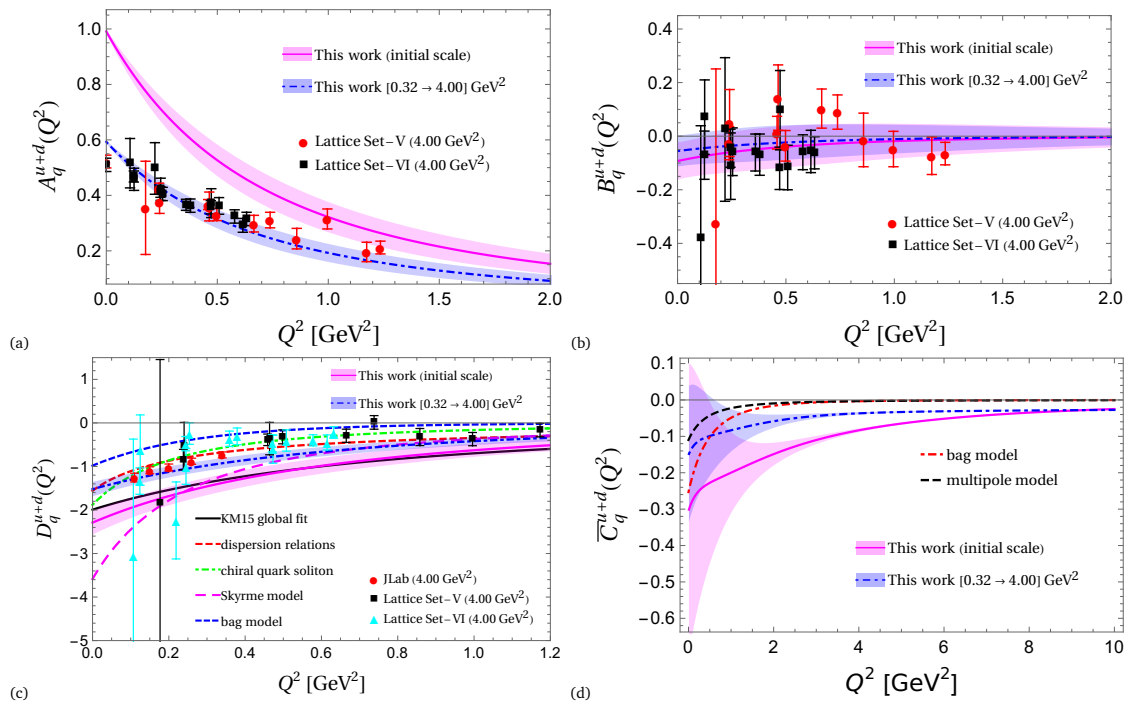
Figure 1: The plots of the four GFFs as functions of Q^2. The solid magenta lines with magenta bands are the results at the initial scale, while the dash-dotted blue lines with blue bands represent the results at $\mu^2 = 4$ GeV^2 evolved from the initial scale $\mu_0^2 = 0.32$ GeV^2.

In Fig. 1 (a) and (b), we show the GFFs $A^{u+d}(Q^2)$ and $B^{u+d}(Q^2)$ which are compared with the lattice QCD prediction [11]. We show the results at the initial scale of $\mu_0^2 = 0.32$ GeV^2 and at the evolved final scale of $\mu^2 = 4$ GeV^2. The evolution is done using the higher order perturbative parton evolution toolkit (HOPPET) [12] following Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations of QCD with next-to-next-to-leading order. We observe that after QCD evolution, $A^{u+d}(Q^2)$ and $B^{u+d}(Q^2)$ are consistent with the lattice QCD results. In Fig. 1 (c) we show our result for the GFF $4C^{u+d}(Q^2) = D^{u+d}(Q^2)$. The evolved value at zero momentum transfer is $D^{u+d}(0) = -1.521$. The fitted function $\frac{-1.521}{(1+0.531Q^2)^{3.026}}$ reproduces the evolved result for $D^{u+d}(Q^2)$. We find that the qualitative behavior of our result is compatible with lattice QCD [11] and the experimental data from JLab [2] as well as other theoretical predictions from the KM15 global fit [13], dispersion relation [14], χQSM [15], Skyrme model [16], and bag model [17]. In Fig. 1 (d) our result for the GFF $\bar{C}^{u+d}(Q^2)$ is shown. The error bands reflect a 10% uncertainty in the model parameters. In Fig. 2 (a) and (b) we show the profile for the pressure and shear distributions of the quark inside the nucleon in the impact parameter space. Our results are compared with the distribution evaluated in leading order light-cone sum rule [18] and the distribution obtained from the fitting functions of the experimental data for $D(Q^2)$ at JLab [2]. The pressure distribution satisfies the von Laue stability condition [19] showing a positive core and a negative tail. The shear distribution for stable hydrostatic systems are positive and has connection to surface tension and surface energy [4]. We observe that our shear distribution is also positive and the qualitative behavior is in accord with other approaches [15, 16, 18].
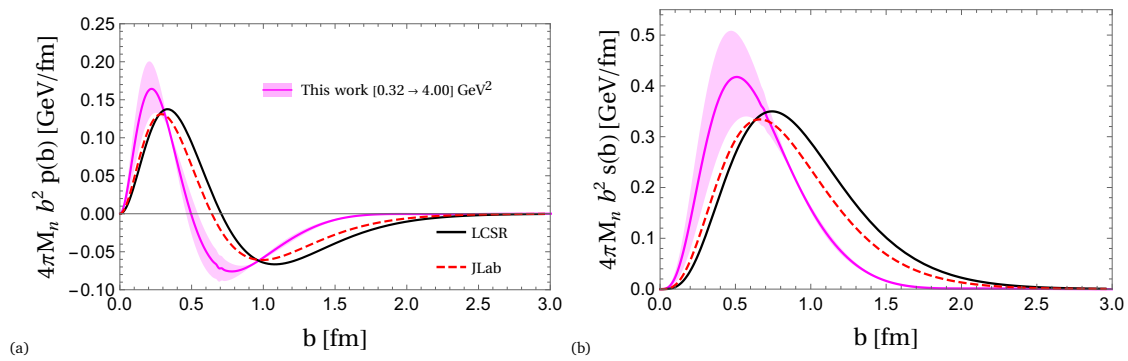
Figure 2: Plots of (a) the pressure distribution $4\pi M_n b^2 p(b)$, and (b) the shear distribution $4\pi M_n b^2 s(b)$ as a function of b.

4 Conclusion

We calculated the four GFFs of a spin half nucleon in the light-front scalar quark-diquark model. The D-term was extracted using the perpendicular component of the energy momentum tensor. Our results for $A(Q^2)$ and $B(Q^2)$ were found to be consistent with lattice QCD predictions. The evolved result for the D-term is in accord with the experimental data extracted from DVCS process at JLab. The value of $\bar{C}(Q^2)$ is found to be negative at zero momentum transfer. Our pressure $p(b)$ and shear force $s(b)$ distributions are found to be consistent with the experimental observation and other theoretical predictions.

Acknowledgements

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