Energy-momentum tensor form factors in QED

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Abstract

We present the results for the Energy-Momentum Tensor (EMT) form factor $D(t)$ at one-loop accuracy in quantum electrodynamics for an electron state. We report the the results in the case of both zero and nonzero photon mass. Moreover, individual electron and photon contributions to the EMT are investigated.

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1 Introduction

The gravitational form factors (GFFs) parametrize the Energy-Momentum Tensor (EMT) matrix elements, and contain information on the distributions of energy, momentum, angular momentum and internal pressure/shear forces of a system. The latter are encoded in the $D(t)$ form factor, being t the momentum transfer (squared). Hence the "mechanical properties" of the system are related to the D form factor. Its study opened a new avenue to unravel the internal structure of hadronic bound states [1–5]. The $D(t)$ form factor is also important in the case of the electron, where a non-vanishing result is generated by loop corrections in quantum electrodynamica (QED) [6,7]. It is found that $\lim_{t\to 0} D(t)$ ($D(0)$ is usually referred to as the D-term [8]) is divergent. This observation is related to the long-ranged nature of the electromagnetic interaction. In this work, we review the results of Ref. [9], where we extended the one-loop calculations of $D(t)$ in QED presented in Refs. [6,7] by keeping separate the contributions to the EMT coming from the electron and the photon. We also extend the discussion to the case of a massive photon. In this case we find a finite D-term. We finally present the results for the distribution of pressure and shear forces in the electron.

2 Basic definitions

We work with the symmetric EMT for QED, which reads

$$T^{\mu\nu}_{\rm QED} = T^{\mu\nu}_{e} + T'^{\mu\nu}_{\gamma}\,, \text{ with} \tag{1}$$

$$T^{\mu\nu}_{e} = Z_2\,\bar\psi\,\frac{i}{4}\,\gamma^{\{\mu}\overleftrightarrow{\partial}^{\nu\}}\psi - \frac{Z_2}{2}\,\mu^{2\varepsilon_{\rm UV}}\,e\,\bar\psi\,\gamma^{\{\mu}A^{\nu\}}\psi\,, \tag{2}$$

$$T'^{\mu\nu}_{\gamma} = -Z_3\,F^{\mu\alpha}F^{\nu}_{\ \alpha} + Z_3\,\frac{g^{\mu\nu}}{4}F^{\alpha\beta}F_{\alpha\beta}\,, \tag{3}$$

where the subscripts e and γ refer to the electron and photon contributions, respectively, and $a^{\{\mu}b^{\nu\}} \equiv a^\mu b^\nu + a^\nu b^\mu$. In Eqs. (2) and (3), it is understood the standard Lagrangian renormalization. We worked in dimensional regularization, with mass scale μ.

To include a massive photon mass (with mass m_γ), one must modify $T^{\mu\nu}_{\rm QED}$ in Eq. (1). Among the massive-photon extensions of QED, the two most prominent are the Stueckelberg Lagrangian and the Abelian Higgs model (see, e.g., Ref. [10]). The two approaches can be proven to be equivalent at $O(\alpha)$ for the EMT electron matrix elements. Both approaches generate a matrix element of the form

$$\langle e(p',s')|T^{\mu\nu}_{\rm QED} + m_\gamma^2\left(A^\mu A^\nu - \frac{g^{\mu\nu}}{2}A^2\right)|e(p,s)\rangle + O(\alpha^2)\,. \tag{4}$$

This argument can be generalized to show that <u>any</u> massive-photon QED extension generates the matrix element of Eq. (4) at $O(\alpha)$.

We will employ the standard parametrization for the EMT matrix elements (see, e.g. Ref. [11–13]):

$$\langle e(p',s')|\,T^{\mu\nu}_{i}\,|e(p,s)\rangle$$
$$= \bar u(p',s')\left(A_i(\tau^2)\frac{P^\mu P^\nu}{m_e} + J_i(\tau^2)\frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2m_e} + D_i(\tau^2)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4m_e} + \bar C_i(\tau^2)m_e\,g^{\mu\nu}\right)u(p,s)\,, \tag{5}$$

where $i = e,\gamma$ and m_e is the electron mass. We introduced the natural variables $P = \frac{1}{2}(p+p')$, $\Delta = p'-p$, $t = \Delta^2$ and $\tau^2 = -t/m_e^2$. In the following, we discuss the form factors $D_i(\tau^2)$ and $\bar C_i(\tau^2)$ in Eq. (5) in QED at $O(\alpha)$.

3 Results

The structure of one-loop results for $D(\tau^2)$ is as follows [9]

$$D_i(\tau^2,\lambda^2) = \int_0^1 dx\int_0^{1-x} dy\,\frac{f_i(x,y)}{\tau^2 + a_i(x,y,\lambda^2)}\,, \tag{6}$$

where $\lambda = m_\gamma/m_e$, and

$$f_e(x,y) = \frac{\alpha}{\pi}\frac{(x-2)(1-x-2y)^2}{y(1-x-y)}\,,\quad f_\gamma(x,y) = \frac{\alpha}{\pi}\frac{1-x-(1+x)(1-x-2y)^2}{y(1-x-y)}\,, \tag{7}$$

$$a_e(x,y,\lambda^2) = \frac{(1-x)^2 + x\lambda^2}{y(1-x-y)}\,,\qquad a_\gamma(x,y,\lambda^2) = \frac{x^2 + (1-x)\lambda^2}{y(1-x-y)}\,. \tag{8}$$

From a general point of view, no constraint has been derived on the sign and value of the D-term. There are, although, some speculations that a bound state exhibits a negative D-term [2].

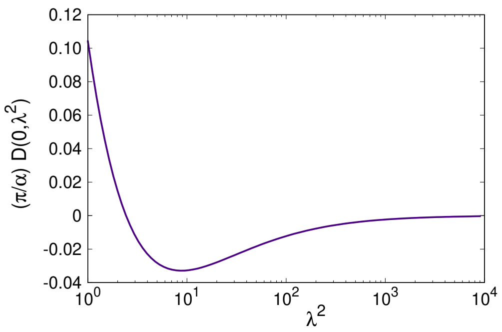

Figure 1: The λ^2 dependence of $D(t=0,\lambda^2)$. The minimum is at $\lambda^2_{\text{min}} \simeq 8.5$.

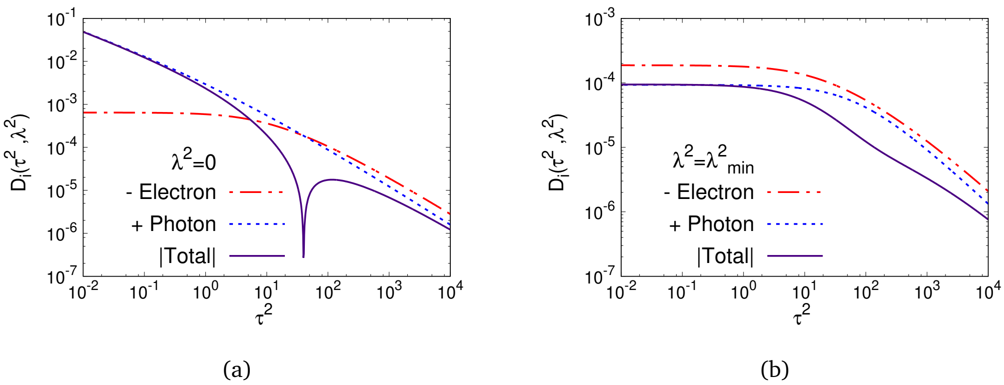

Figure 2: The form factor $D(\tau^2,\lambda^2)$ as a function of τ^2, for $\lambda^2=0$ (left) and for $\lambda^2=\lambda^2_{\text{min}}$ (right). The blue dotted (red dash-dotted) curves correspond to the photon (electron) contributions. The absolute value of the total form factor is shown as the solid purple curves.

In contrast, we find that the D-term for electron matrix elements is positive and infinite if the photon is assumed massless. More specifically, the contribution from the electron is finite and negative, whereas the photon contribution is divergent for vanishing m_γ and positive and finite for non-zero m_γ:

$$D_\gamma(\tau^2 \ll 1, \lambda^2=0) \simeq \frac{\alpha\pi}{4\sqrt{\tau^2}}, \quad D_\gamma(\tau^2=0,\lambda^2 \ll 1) \simeq \frac{\alpha}{3\sqrt{\lambda^2}}, \quad D_e(\tau^2=0,\lambda^2=0) = -\frac{5\alpha}{18\pi}. \tag{9}$$

Summing the two contributions in Eq. (9) one reproduces the result for $\tau^2 \to 0$ presented in Refs. [6,7].

Fig. 1 presents the dependence of the D-term on λ^2. For almost massless photons, a large and positive D-term is observed. This is bounded to the long-range nature of the electromagnetic interaction. Since any loop contribution, in the limit of large photon mass, is proportional to $1/\lambda^2$, we expect that the coupling of the physical electron to the EMT to be contact-like. This entails that the loop contributions behaves like the tree-level contributions, for which the D-term vanishes. For intermediate values of λ^2, the interaction becomes short-ranged, but not point-like. This produce a response of the physical electron when coupled with the EMT that mimics the behavior of a bound state. The similarity in the behavior of the D-term is not sufficient to claim that, for some particular value of the photon mass, the electron-photon system becomes bounded.

The form factor $D(\tau^2,\lambda^2)$ as a functions of τ^2 is given in Fig. 2. The case of a massless photon is shown in panel (a), and the case $\lambda^2 = \lambda^2_{\rm min}$ in panel (b), with the separate contributions to the EMT from the electron and the photon. For both values of λ, the photon (electron) contribution is always positive (negative). For massless photons, a clear change in the sign of the total form factor happens when the negative electron contribution starts to dominate over the falling positive photon contribution. The asymptotic expression in τ^2 for the total form factor is given by [6]

$$D(\tau^2 \gg 1, \lambda^2 = 0) = \frac{\alpha}{\pi}\,\frac{4 - \ln \tau^2}{\tau^2}\,. \tag{10}$$

For the asymptotic behavior of the ratio of the electron and photon contributions we find

$$\lim_{\tau^2 \to \infty} \frac{D_e(\tau^2, \lambda^2 = 0)}{D_\gamma(\tau^2, \lambda^2 = 0)} = -\frac{5}{2}\,. \tag{11}$$

This observation allows us to conclude that the photon and electron contributions have the same asymptotic behavior and that the total form factor $D(\tau^2)$ approaches zero from negative values. For $\lambda^2 = \lambda^2_{\rm min}$, the form factor is negative for all values of τ^2, indicating that the electron contribution dominates over the photon one. Again, for intermediate values of the photon mass, a clear similarity between the physical electron case and an hadron is established [2].

Via a Fourier transformation of $D_i(\tau^2, \lambda^2)$ and $\bar{C}_i(\tau^2, \lambda^2)$ [1,2] one obtains the pressure/shear distributions in position space. Assuming $\Delta = (0, \boldsymbol{\Delta})$ and using the dimensionless variable $\boldsymbol{\rho} = \boldsymbol{r}\, m_e$, we find:

$$\hat{p}_i(\rho, \lambda^2) = \frac{p_i(\rho, \lambda^2)}{m_e^4} = \frac{1}{6}\frac{d^2}{d\rho^2}\hat{D}_i(\rho, \lambda^2) + \frac{1}{3\rho}\frac{d}{d\rho}\hat{D}_i(\rho, \lambda^2) - \hat{\bar{C}}_i(\rho, \lambda^2)\,, \tag{12}$$

$$\hat{s}_i(\rho, \lambda^2) = \frac{s_i(\rho, \lambda^2)}{m_e^4} = -\frac{1}{4}\frac{d^2}{d\rho^2}\hat{D}_i(\rho, \lambda^2) + \frac{1}{4\rho}\frac{d}{d\rho}\hat{D}_i(\rho, \lambda^2)\,. \tag{13}$$

The contributions from $\hat{\bar{C}}_i(\rho, \lambda^2)$ are proportional to $\delta(\rho)/\rho^2$, since the form factors $\bar{C}_i(\tau^2, \lambda^2)$ do not depend on τ^2 (for any value of λ^2). We therefore arrive at the results

$$\hat{p}_{i,\text{fin.}}(\rho, \lambda^2) = \int_0^1 dx \int_0^{1-x} dy\; e^{-\rho\sqrt{a_i(x,y,\lambda^2)}} f_i(x,y) \frac{a_i(x,y,\lambda^2)}{24\pi\rho}\,, \tag{14}$$

$$\hat{p}_{i,D,\text{sing.}}(\rho) = \frac{\delta'(\rho)}{12\pi\rho} \int_0^1 dx \int_0^{1-x} dy\; f_i(x,y)\,, \tag{15}$$

$$\hat{s}_{i,\text{fin.}}(\rho, \lambda^2) = -\int_0^1 dx \int_0^{1-x} dy\; e^{-\rho\sqrt{a_i(x,y,\lambda^2)}} f_i(x,y) \frac{3 + 3\sqrt{a_i(x,y,\lambda^2)}\rho + a_i(x,y,\lambda^2)\rho^2}{16\pi\rho^3}\,, \tag{16}$$

$$\hat{s}_{i,\text{sing.}}(\rho) = \frac{3\delta(\rho) - \rho\delta'(\rho)}{8\pi\rho^2} \int_0^1 dx \int_0^{1-x} dy f_i(x,y)\,, \tag{17}$$

where $\hat{p}_{i,D}$ includes only the contribution from the $D_i(\tau^2)$ form factor. We have isolated the finite (fin.) and the singular (sing.) contributions. We note that both the $\bar{C}_i(\tau^2, \lambda^2)$ and $D_i(\tau^2, \lambda^2)$ contains singular terms at $\rho = 0$. From the expressions (14)-(17), we can also check the validity of the von Laue stability conditions [2,9].

4 Conclusions

We presented the one-loop calculation of the gravitational form factor $D(t)$ in QED for an electron state. We presented the separate results for the electron and photon contributions to the EMT. We stressed the observation of a divergent D-term, caused by the contribution of the photon, and how this is related to the long-range nature of the QED interaction. For a nonzero and sufficiently large photon mass, in contrast, one finds a negative $D(t)$ for all values of t and a finite result for $t = 0$. In this case, the behavior of electron-photon system resembles a bound-state one. However, the asymptotic behavior of $D(t)$ for large t and, therefore, the position-space behavior of the pressure and shear distributions, show significant differences compared to a true bound state. In the QED case both distributions have a delta function contribution at the center, which is not present for hadrons. Finally, we stress that the same large-t behavior shows up also in the one-loop QCD calculation for a quark state, since, at this order, the only difference from the QED results is a trivial color factor.

References

[1] M. V. Polyakov, *Generalized parton distributions and strong forces inside nucleons and nuclei*, Phys. Lett. B **555**, 57 (2003), doi:10.1016/S0370-2693(03)00036-4.

[2] M. V. Polyakov and P. Schweitzer, *Forces inside hadrons: Pressure, surface tension, mechanical radius, and all that*, Int. J. Mod. Phys. A **33**, 1830025 (2018), doi:10.1142/S0217751X18300259.

[3] C. Lorcé, H. Moutarde and A. P. Trawiński, *Revisiting the mechanical properties of the nucleon*, Eur. Phys. J. C **79**, 89 (2019), doi:10.1140/epjc/s10052-019-6572-3.

[4] A. Freese and G. A. Miller, *Forces within hadrons on the light front*, Phys. Rev. D **103**, 094023 (2021), doi:10.1103/PhysRevD.103.094023.

[5] C. Lorcé, *The light-front gauge-invariant energy-momentum tensor*, J. High Energy Phys. **08**, 045 (2015), doi:10.1007/JHEP08(2015)045.

[6] F. A. Berends and R. Gastmans, *Quantum electrodynamical corrections to graviton-matter vertices*, Ann. Phys. **98**, 225 (1976), doi:10.1016/0003-4916(76)90245-1.

[7] K. A. Milton, *Quantum-electrodynamic corrections to the gravitational interaction of the electron*, Phys. Rev. D **15**, 538 (1977), doi:10.1103/PhysRevD.15.538.

[8] M. V. Polyakov and H.-D. Son, *Nucleon gravitational form factors from instantons: forces between quark and gluon subsystems*, J. High Energy Phys. **09**, 156 (2018), doi:10.1007/JHEP09(2018)156.

[9] S. Rodini, A. Metz and B. Pasquini, *Mass sum rules of the electron in quantum electrodynamics*, J. High Energy Phys. **09**, 067 (2020), doi:10.1007/JHEP09(2020)067.

[10] H. Ruegg and M. Ruiz-Altaba, *The Stueckelberg field*, Int. J. Mod. Phys. A **19**, 3265 (2004), doi:10.1142/S0217751X04019755.

[11] I. Y. Kobzarev and L. B. Okun, *Gravitational interaction of fermions*, Zh. Eksp. Teor. Fiz. **43**, 1904 (1962).

[12] H. Pagels, *Energy-Momentum Structure Form Factors of Particles*, Phys. Rev. **144**, 1250 (1966), doi:10.1103/PhysRev.144.1250.

[13] X. Ji, *Gauge-Invariant Decomposition of Nucleon Spin*, Phys. Rev. Lett. **78**, 610 (1997), doi:10.1103/PhysRevLett.78.610.