

# Higher-order corrections to exclusive heavy vector meson production

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## Abstract

We present results for higher-order corrections to exclusive  $J/\psi$  production. This includes the first relativistic correction of order  $v^2$  in quark velocity, and next-to-leading order corrections in  $\alpha_s$  for longitudinally polarized production. The relativistic corrections are found to be important for a good description of the HERA data, especially at small values of the photon virtuality. The next-to-leading order results for longitudinal production are evaluated numerically. We also demonstrate how the vector meson production provides complementary information to the structure functions for extracting the initial condition for the small- $x$  evolution of the dipole-proton scattering amplitude.



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## 1 Introduction

Exclusive vector meson production is a powerful tool for determining the small- $x$  gluon structure of the proton. There is already plenty of data for this process from HERA, and in the future this will be supplemented by measurements at the LHC and the future Electron-Ion Collider (EIC) [1]. Thus there is a need for a precise theoretical description of the process. Currently, there is a major source of model uncertainty coming from the vector meson wave function which is often taken from phenomenological models or assumed to be fully nonrelativistic. In this Contribution, we will consider the first relativistic corrections to the heavy vector meson wave function, which will quantify model uncertainties of the wave function. We will also present results of the next-to-leading order calculation for exclusive production of longitudinally polarized heavy vector mesons, which is important for developing a precise theoretical description of the process.

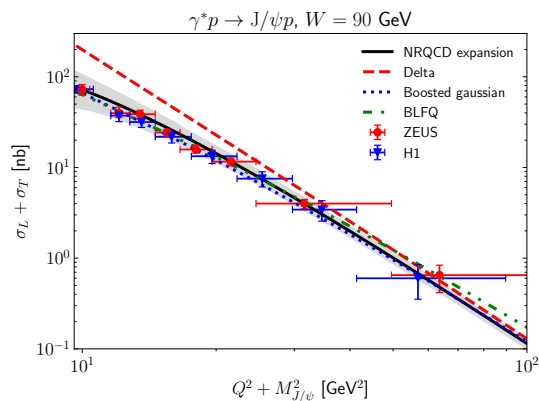


Figure 1: Total exclusive  $J/\psi$  production cross section as a function of the photon virtuality [6].

## 2 Relativistic corrections of order $v^2$

At high energies exclusive vector meson production can be conveniently described in the dipole picture where an energetic virtual photon scatters off the target proton or nucleus and forms the vector meson. This allows us to describe the process in a factorized form where the scattering amplitude reads [2]

$$\mathcal{A}^\lambda = 2i \int d^2\mathbf{b} d^2\mathbf{r} \frac{dz}{4\pi} e^{-i(\mathbf{b} + (\frac{1}{2}-z)\mathbf{r}) \cdot \Delta} \Psi_\gamma^{\lambda*}(\mathbf{r}, Q^2, z) \Psi_V(\mathbf{r}, z) N(\mathbf{r}, \mathbf{b}, x_{\mathbb{P}}) \quad (1)$$

for a photon with polarization  $\lambda$ . Here the process is described in a mixed coordinate space where  $\mathbf{r}$  is the transverse size of the quark-antiquark dipole and  $z$  is the longitudinal momentum fraction carried by the quark. The necessary parts for the calculation are the virtual photon wave function  $\Psi_\gamma^\lambda$ , the meson wave function  $\Psi_V$ , and the dipole amplitude  $N$ . The photon wave function can be calculated directly by using light-front perturbation theory [3]. For the dipole amplitude we use here the IPSat parametrization [4] with the fit from Ref. [5]. This particular choice for the dipole amplitude is dependent on the impact parameter  $\mathbf{b}$  which allows us to study the dependence of the scattering amplitude on the momentum transfer  $t \approx -\Delta^2$ .

The final part needed for the scattering amplitude, the meson light front wave function, is nonperturbative and therefore introduces an additional model dependence. In Ref. [6], we propose a new approach for the meson wave function based on Non-Relativistic QCD (NRQCD), which includes relativistic corrections of order  $v^2$  in the quark's velocity  $v$ . In this approach, we start from the rest frame of the meson where the value of the wave function and its derivatives can be written in terms of the NRQCD long-distance matrix elements. This procedure allows us to include relativistic corrections of order  $v^2$  to the meson's rest frame wave function in terms of two nonperturbative constants, related to the NRQCD matrix elements, that have been determined for  $J/\psi$  from the decay constants of charmonia in Ref. [7]. The meson rest frame wave function is then related to the light front wave function by the so-called Melosh rotation [8] which acts as a transformation between two different spinor bases.

To quantify the effects of the relativistic corrections, we show the total  $J/\psi$  production cross section as a function of the photon virtuality  $Q^2$  in Fig. 1. Here we compare results from four different meson wave functions: the fully nonrelativistic (*Delta*), one with relativistic corrections of order  $v^2$  (*NRQCD expansion*), and two phenomenological wave functions from Refs. [2, 9]. These are compared to the HERA data from Refs. [10, 11]. We see that the relativistic corrections have the greatest impact at small values of the photon virtuality,

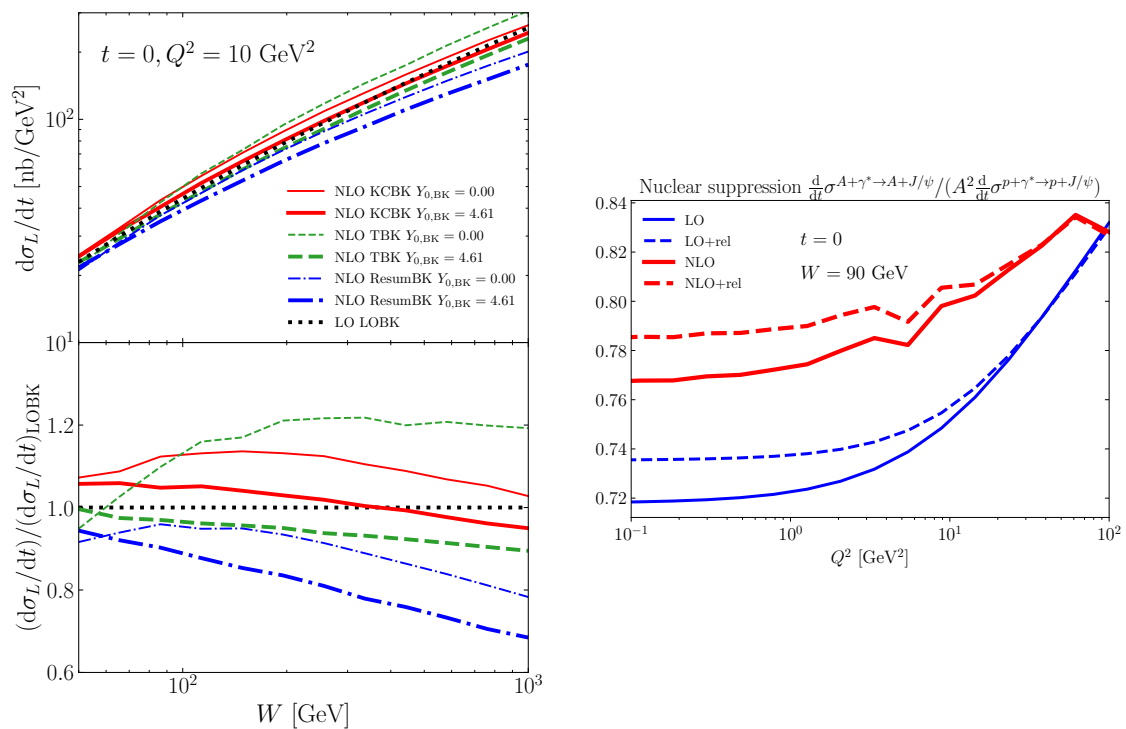


Figure 2: Left: Diffractive cross section for longitudinal  $J/\psi$  production as a function of the center-of-mass energy [14]. *LO BK* corresponds to a leading-order result with a leading-order dipole amplitude fit. The other curves show the NLO results with different NLO dipole amplitude fits. In the lower plot, the curves have been normalized by the leading-order result.

Right: Nuclear suppression for longitudinal  $J/\psi$  production. We compare the LO and NLO results at the nonrelativistic limit and with the relativistic corrections of order  $v^2$ .

whereas they become negligible at large  $Q^2$ . The relativistic corrections are also crucial for a good description of the HERA data as the fully nonrelativistic wave function results in an overestimation of the total cross section at small  $Q^2$ .

### 3 Next-to-leading order corrections at the nonrelativistic limit

Of comparative importance are the next-to-leading order corrections to the vector meson production. The significance of the relativistic and  $\alpha_s$  corrections can be estimated by NRQCD which tells us that numerically  $v^2 \sim \alpha_s$  in the case of  $J/\psi$  [7]. This means that we can do an expansion in both the velocity and  $\alpha_s$ , which indicates that the first NLO correction should be calculated at the nonrelativistic limit [12]. The calculation of the NLO corrections requires that we know the virtual photon and meson wave functions at the corresponding accuracy. In recent years, there has been major progress in calculating these: the NLO corrections to the vector meson wave function [12] and to the longitudinal photon wave function with massive quarks [13] have recently become available.

In Ref. [14], we combined these NLO wave functions to calculate exclusive  $J/\psi$  production at NLO in the longitudinal polarization case. This calculation demonstrates the cancellation of all UV and IR divergences present in the wave functions, and shows explicitly the appearance

of the Balitsky-Kovchegov (BK) equation for the energy dependence of the dipole amplitude. In Fig. 2, we show the numerical results for the NLO calculation of  $J/\psi$  production. In the left plot, the results for diffractive cross sections from the LO and NLO calculations are shown. The dipole amplitudes used in these calculations are taken from [15] in the LO case and [16] in the NLO case, and in both cases they have been fitted to the HERA structure function data. For the NLO case, we have calculated the results using various different parametrizations for the dipole amplitude satisfying modified versions of the BK equation, abbreviated here as *KCBK*, *TBK*, and *ResumBK*. This figure shows that there is more variation between the different NLO results than between the LO and NLO results. This tells us that vector meson production provides complementary information to structure function analyses, since all of these dipole amplitudes have been successfully fitted to the structure function data.

In the right plot, we show a ratio of the cross sections for the nuclear and proton targets. This ratio measures the nuclear suppression as without nonlinear effects the ratio would be identically one. We have compared the results from the LO and NLO cases, with and without relativistic corrections of order  $v^2$ . The effects of the NLO and relativistic corrections seem to be small, although they both reduce the nuclear suppression slightly. In Ref. [6], the effects of the relativistic corrections to nuclear suppression were found to be larger. The differences could be due to the fact that here we are only considering the longitudinal diffractive cross section at  $t = 0$  as opposed to the total cross section, and a different dipole amplitude was used in the two calculations.

## 4 Conclusion

We have shown how the first relativistic correction and the NLO correction in the longitudinal polarization case affect exclusive production of  $J/\psi$ . The relativistic corrections are important at small  $Q^2$  for a good agreement with the HERA data. The NLO corrections affect the cross section only modestly, and there is more variation between the results using different NLO fitted dipole amplitudes than between the LO and NLO results.

In the future, we are going to include next-to-leading order calculations for transversely polarized virtual photons. This will enable us to make comparisons to the vector meson production data from HERA [11, 17, 18] and the LHC [19, 20], while also making precise predictions for the future EIC.

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## References

- [1] R. Abdul Khalek et al., *Science Requirements and Detector Concepts for the Electron-Ion Collider: EIC Yellow Report*, [arXiv:2103.05419](https://arxiv.org/abs/2103.05419).
- [2] H. Kowalski, L. Motyka and G. Watt, *Exclusive diffractive processes at HERA within the dipole picture*, *Phys. Rev. D* **74**, 074016 (2006), doi:[10.1103/PhysRevD.74.074016](https://doi.org/10.1103/PhysRevD.74.074016).

- [3] H. G. Dosch, T. Gousset, G. Kulzinger and H. J. Pirner, *Vector meson lepton production and nonperturbative gluon fluctuations in QCD*, Phys. Rev. D **55**, 2602 (1997), doi:[10.1103/PhysRevD.55.2602](https://doi.org/10.1103/PhysRevD.55.2602).
- [4] H. Kowalski and D. Teaney, *Impact parameter dipole saturation model*, Phys. Rev. D **68**, 114005 (2003), doi:[10.1103/PhysRevD.68.114005](https://doi.org/10.1103/PhysRevD.68.114005).
- [5] H. Mäntysaari and P. Zurita, *In depth analysis of the combined HERA data in the dipole models with and without saturation*, Phys. Rev. D **98**, 036002 (2018), doi:[10.1103/PhysRevD.98.036002](https://doi.org/10.1103/PhysRevD.98.036002).
- [6] T. Lappi, H. Mäntysaari and J. Penttala, *Relativistic corrections to the vector meson light front wave function*, Phys. Rev. D **102**, 054020 (2020), doi:[10.1103/PhysRevD.102.054020](https://doi.org/10.1103/PhysRevD.102.054020).
- [7] G. T. Bodwin, H. Sok Chung, D. Kang, J. Lee and C. Yu, *Improved determination of color-singlet nonrelativistic QCD matrix elements for S-wave charmonium*, Phys. Rev. D **77**, 094017 (2008), doi:[10.1103/PhysRevD.77.094017](https://doi.org/10.1103/PhysRevD.77.094017).
- [8] H. J. Melosh, *Quarks: Currents and constituents*, Phys. Rev. D **9**, 1095 (1974), doi:[10.1103/PhysRevD.9.1095](https://doi.org/10.1103/PhysRevD.9.1095).
- [9] H. Li, *Z production in pPb collisions and charmonium production in PbPb ultra-peripheral collisions at LHCb*, Nucl. Phys. A **1005**, 121902 (2021), doi:[10.1016/j.nuclphysa.2020.121902](https://doi.org/10.1016/j.nuclphysa.2020.121902).
- [10] A. Aktas et al., *Elastic J/ψ production at HERA*, Eur. Phys. J. C **46**, 585 (2006), doi:[10.1140/epjc/s2006-02519-5](https://doi.org/10.1140/epjc/s2006-02519-5).
- [11] S. Chekanov et al., *Exclusive electroproduction of J/ψ mesons at HERA*, Nucl. Phys. B **695**, 3 (2004), doi:[10.1016/j.nuclphysb.2004.06.034](https://doi.org/10.1016/j.nuclphysb.2004.06.034).
- [12] M. Á. Escobedo and T. Lappi, *Dipole picture and the nonrelativistic expansion*, Phys. Rev. D **101**, 034030 (2020), doi:[10.1103/PhysRevD.101.034030](https://doi.org/10.1103/PhysRevD.101.034030).
- [13] G. Beuf, T. Lappi and R. Paatelainen, *Massive quarks in NLO dipole factorization for DIS: Longitudinal photon*, Phys. Rev. D **104**, 056032 (2021), doi:[10.1103/PhysRevD.104.056032](https://doi.org/10.1103/PhysRevD.104.056032).
- [14] H. Mäntysaari and J. Penttala, *Exclusive heavy vector meson production at next-to-leading order in the dipole picture*, Phys. Lett. B **823**, 136723 (2021), doi:[10.1016/j.physletb.2021.136723](https://doi.org/10.1016/j.physletb.2021.136723).
- [15] T. Lappi and H. Mäntysaari, *Single inclusive particle production at high energy from HERA data to proton-nucleus collisions*, Phys. Rev. D **88**, 114020 (2013), doi:[10.1103/PhysRevD.88.114020](https://doi.org/10.1103/PhysRevD.88.114020).
- [16] G. Beuf, H. Hänninen, T. Lappi and H. Mäntysaari, *Color glass condensate at next-to-leading order meets HERA data*, Phys. Rev. D **102**, 074028 (2020), doi:[10.1103/PhysRevD.102.074028](https://doi.org/10.1103/PhysRevD.102.074028).
- [17] C. Alexa et al., *Elastic and proton-dissociative photoproduction of J/ψ mesons at HERA*, Eur. Phys. J. C **73**, 2466 (2013), doi:[10.1140/epjc/s10052-013-2466-y](https://doi.org/10.1140/epjc/s10052-013-2466-y).
- [18] S. Chekanov et al., *Exclusive photoproduction of J/ψ mesons at HERA*, Eur. Phys. J. C **24**, 345 (2002), doi:[10.1007/s10052-002-0953-7](https://doi.org/10.1007/s10052-002-0953-7).

- [19] R. Aaij et al., *Updated measurements of exclusive  $J/\psi$  and  $\psi(2S)$  production cross-sections in  $pp$  collisions at  $\sqrt{s} = 7$  TeV*, J. Phys. G: Nucl. Part. Phys. **41**, 055002 (2014), doi:[10.1088/0954-3899/41/5/055002](https://doi.org/10.1088/0954-3899/41/5/055002).
- [20] S. Acharya et al., *Energy dependence of exclusive  $J/\psi$  photoproduction off protons in ultra-peripheral  $p$ -Pb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV*, Eur. Phys. J. C **79**, 402 (2019), doi:[10.1140/epjc/s10052-019-6816-2](https://doi.org/10.1140/epjc/s10052-019-6816-2).