

Drell-Yan transverse spectra at the LHC: a comparison of parton branching and analytical resummation approaches

Aron Mees van Kampen*

Elementaire deeltjesfysica, University of Antwerp

* AronMees.vanKampen@uantwerpen.be



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Abstract

I report on precision comparisons of theoretical predictions for Drell-Yan (DY) transverse momentum spectra from two approaches: the analytical resummation approach based on Collins-Soper-Sterman (CSS) formalism and the parton branching (PB) approach to the evolution of transverse momentum dependent (TMD) parton distributions.



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1 Introduction

Extensions of the QCD collinear factorization theorem [1] involving TMD parton distributions [2] are relevant to perform perturbative resummations to all orders in the strong coupling in kinematic regions of interest to collider physics, e.g. the Sudakov (or low- p_T) region [3–5] and high-energy (or low- x) region [6–8].

In the last few years, precision measurements of transverse momentum spectra in DY lepton-pair production have been carried out at the Large Hadron Collider (LHC) [9–12], which call for detailed studies of TMD resummations in the region of low vector-boson transverse momentum p_T .

In this article I report on a comparison of theoretical predictions for DY p_T spectra based on two different approaches: the analytical resummation approach based on the CSS formalism [3, 4] and the PB approach [13, 14] to the evolution of TMD distributions. I start in Sec. 2 by briefly recalling the basic elements of the two methods. In Sec. 3 I compare them at analytical level. In Sec. 4 I present numerical comparisons. I give conclusions in Sec. 5.

2 Theoretical methods

2.1 CSS method

The CSS method [3, 4] decomposes the DY differential cross section into the sum of the resummed term (dominant for $p_T \ll Q$, where Q is the vector boson invariant mass) and the finite term (important for p_T of order Q). The resummed term is factorized in impact parameter space into a hard-scattering coefficient function and TMD distributions, which in turn are expressed as convolutions of Sudakov form factors, collinear evolution coefficients and parton distributions.

The resummed and finite terms require an appropriate *matching* procedure in order to avoid double counting. The matching can be done by subtracting counterterms from the resummed cross section [15–17].

Nonperturbative effects are incorporated in the formalism through a Gaussian smearing factor in impact parameter space, as well as nonperturbative contributions to Sudakov evolution [18–21].

2.2 PB method

The parton branching (PB) method [13, 14] formulates the evolution of TMD distributions in terms of Sudakov form factors and real emission splitting functions using the *unitarity* picture of parton interactions commonly employed in showering Monte Carlo algorithms [22, 23]. The resolvable and non-resolvable regions of the branching phase space are separated by the soft-gluon resolution scale z_M (in general, branching scale dependent [24]). Soft-gluon angular ordering is implemented in the TMD evolution so that color coherence effects are taken into account and the collinear limit agrees with the coherent branching approach of Refs. [23, 25].

The matching with finite-order hard-scattering matrix elements is not done additively as in the CSS framework, but rather by using the showering operator method with subtracted matrix elements [26–28].

Nonperturbative TMD effects are encapsured in the TMD distribution at the starting evolution scale $\mu_0 = \mathcal{O}(1 \text{ GeV})$, which is parametrized (for instance, as a transverse momentum Gaussian times a longitudinal momentum distribution in a simplest parametrization) and determined from fits to experimental data [29].

3 Analytical comparison

The Sudakov form factor in CSS is written as

$$S(Q, b) = \exp \left(- \int_{b_0/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[\ln \left(\frac{Q^2}{\mu^2} \right) A(\alpha_s(\mu^2)) + B(\alpha_s(\mu^2)) \right] \right), \quad (1)$$

where b is the impact parameter, and the functions A and B are perturbatively calculable as power series expansions in α_s , $A = \sum_n A^{(n)} \alpha_s^n$, $B = \sum_n B^{(n)} \alpha_s^n$, and control the resummation of double-logarithmic and single-logarithmic perturbative corrections respectively, with $A^{(1)}$ corresponding to leading logarithms (LL), $A^{(2)}$ and $B^{(1)}$ corresponding to next-to-leading logarithms (NLL), and so on.

The Sudakov form factor in PB is written as

$$\Delta_a(\mu^2, \mu_0^2) = \exp \left(- \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz z P_{ba}^{(R)}(\alpha_s(q_t), z) \right), \quad (2)$$

where z_M is the soft-gluon resolution scale and $P_{ba}^{(R)}$ are the real emission splitting functions, perturbatively calculable as power series expansions in α_s . By rewriting the PB Sudakov using the momentum sum rule, virtual splitting functions and the angular ordering condition, we can compare the CSS perturbative coefficients $A^{(i)}$ with the PB coefficients $k^{(i-1)}$ of Ref. [13], and the CSS perturbative coefficients $B^{(i)}$ with the PB coefficients $d^{(i-1)}$ of Ref. [13].

At LL and NLL level, all coefficients are in agreement between CSS and PB [30]. At next-to-next-to-leading logarithms (NNLL), differences between coefficients both at the single and double logarithmic level are observed which can be understood [30] either as a result of renormalization group transformations and resummation scheme dependence [15, 16] (in the single-log coefficients) or as an effect of soft-gluon effective coupling [31–33] (in the double-log coefficients). This gives the possibility of future systematic comparisons of precision predictions in the phase space regions influenced by Sudakov resummation.

4 Numerical comparison

First numerical comparisons have been performed for the DY transverse momentum spectrum based on the implementation of the CSS method in the RESOLVE [34] event generator and on the implementation of the PB method in the CASCADE [35, 36] event generator, supplemented by the uPDFevolv program [37] for TMD evolution.

RESOLVE implements the resummed part of the cross section as described by CSS up to NNLL accuracy [34, 38]. Matching of the resummed part to finite-order terms is not included. The result for the p_T spectrum is shown in the left plot of Fig. 1.

The calculation with CASCADE implements the PB TMD, and performs the matching with NLO hard-scattering events from MADGRAPH_AMC@NLO [39], in combination with the TMD. For this calculation the PB-NLO-HERAI+II-2018-set2 TMD PDF [29] is extracted from TMDLIB [40, 41] and used by CASCADE. The matching of the NLO events to the TMD evolution is done as in Ref. [28] by using HERWIG6 subtraction terms [42] and a matching scale to avoid double counting. The result with PB is shown in the right plot of Fig. 1.

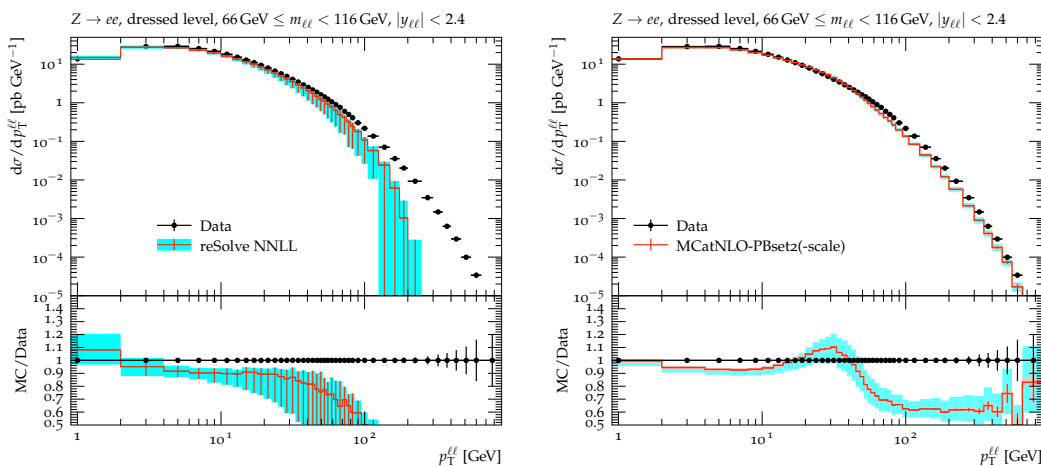


Figure 1: Inclusive Z boson p_T spectrum by reSolve/CSS (left) and Cascade/PB (right). Theoretical uncertainty bands are shown in blue.

From the plots, a few observations can be made. Firstly, both approaches have an accurate prediction in the low- p_T region up to $p_T \sim 20$ GeV. The predictions fail to describe the high- p_T region for different reasons. In RESOLVE it is due to absence of finite-order terms. The PB TMD is matched to NLO hard-scattering events for inclusive Z production. The deficit at high- p_T is

due to missing higher orders or higher jet multiplicities. One way to increase accuracy for the high- p_T tail is to include TMD *merging* with higher jet multiplicities, as in Ref. [43].

The second observation is that the theory uncertainties are calculated differently. In RE-SOLVE they are obtained by varying three scales: the renormalization (μ_R), factorization (μ_F) and resummation (μ_S) scales. Since the resummation scale is mostly important for matching to finite terms, the variation of μ_S gives a distorted view of the uncertainty at the intermediate p_T region. The PB uncertainties in Fig. 1 include only two scale variations, that of μ_R and μ_F . The matching scale has not been varied yet. Besides these uncertainties, there are also TMD uncertainties. In this kinematic range of energy, mass and transverse momentum, these are much smaller than the scale variations.

Finally, the nonperturbative contributions are different. In the CSS implementation of re-Solve, the nonperturbative factor is an overall Gaussian smearing factor of the form $\exp(-g b^2)$ [34]. In PB, the starting TMD contains all the nonperturbative parts. The intrinsic transverse momentum (e.g. a Gaussian $\exp(-k_T^2/2\sigma^2)$) gets smeared out by the TMD evolution [28,29].

5 Conclusion

Two formalisms that include TMD physics, CSS and PB, have been systematically compared with respect to Sudakov resummation for DY p_T spectra. At analytical level, CSS and PB coincide at LL and NLL order and differences at higher order coefficients are being tracked down. Numerically, both approaches are describing the low- p_T region of the DY p_T spectrum. Differences are observed in the estimation of theory uncertainties and the parametrization of non-perturbative effects. The features that emerged are potentially important for high-precision DY phenomenology and are well-suited within benchmark exercises for the LHC and future colliders.

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References

- [1] J. C. Collins, D. E. Soper and G. Sterman, *Factorization of Hard Processes in QCD*, in *Perturbative QCD*, World Scientific (1989), doi:[10.1142/9789814503266_0001](https://doi.org/10.1142/9789814503266_0001), [Adv. Ser. Direct. High Energy Phys. **5**, 1 (1989)].
- [2] R. Angeles-Martinez et al., *Transverse Momentum Dependent (TMD) Parton Distribution Functions: Status and Prospects*, Acta Phys. Pol. B **46**, 2501 (2015), doi:[10.5506/APhysPolB.46.2501](https://doi.org/10.5506/APhysPolB.46.2501).
- [3] J. C. Collins and D. E. Soper, *Back-to-back jets: Fourier transform from b to k_T* , Nucl. Phys. B **197**, 446 (1982), doi:[10.1016/0550-3213\(82\)90453-9](https://doi.org/10.1016/0550-3213(82)90453-9).
- [4] J. C. Collins, D. E. Soper and G. Sterman, *Transverse momentum distribution in Drell-Yan pair and W and Z boson production*, Nucl. Phys. B **250**, 199 (1985), doi:[10.1016/0550-3213\(85\)90479-1](https://doi.org/10.1016/0550-3213(85)90479-1).

- [5] J. Collins, *Foundations of perturbative QCD*, vol. 32, Cambridge University Press, ISBN 978-1-107-64525-7, 978-1-107-64525-7, 978-0-521-85533-4, 978-1-139-09782-6 (2013).
- [6] S. Catani, M. Ciafaloni and F. Hautmann, *Gluon contributions to small χ heavy flavour production*, Phys. Lett. B **242**, 97 (1990), doi:[10.1016/0370-2693\(90\)91601-7](https://doi.org/10.1016/0370-2693(90)91601-7).
- [7] S. Catani, M. Ciafaloni and F. Hautmann, *High energy factorization and small- x heavy flavour production*, Nucl. Phys. B **366**, 135 (1991), doi:[10.1016/0550-3213\(91\)90055-3](https://doi.org/10.1016/0550-3213(91)90055-3).
- [8] S. Catani and F. Hautmann, *High-energy factorization and small- x deep inelastic scattering beyond leading order*, Nucl. Phys. B **427**, 475 (1994), doi:[10.1016/0550-3213\(94\)90636-X](https://doi.org/10.1016/0550-3213(94)90636-X).
- [9] G. Aad et al., *Measurement of the transverse momentum and ϕ_η^* distributions of Drell–Yan lepton pairs in proton–proton collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, Eur. Phys. J. C **76**, 291 (2016), doi:[10.1140/epjc/s10052-016-4070-4](https://doi.org/10.1140/epjc/s10052-016-4070-4).
- [10] V. Khachatryan et al., *Measurement of the transverse momentum spectra of weak vector bosons produced in proton-proton collisions at $\sqrt{s} = 8$ TeV*, J. High Energy Phys. **02**, 096 (2017), doi:[10.1007/JHEP02\(2017\)096](https://doi.org/10.1007/JHEP02(2017)096).
- [11] G. Aad et al., *Measurement of the transverse momentum distribution of Drell–Yan lepton pairs in proton–proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, Eur. Phys. J. C **80**, 616 (2020), doi:[10.1140/epjc/s10052-020-8001-z](https://doi.org/10.1140/epjc/s10052-020-8001-z).
- [12] A. M. Sirunyan et al., *Measurements of differential Z boson production cross sections in proton-proton collisions at $\sqrt{s} = 13$ TeV*, J. High Energy Phys. **12**, 061 (2019), doi:[10.1007/JHEP12\(2019\)061](https://doi.org/10.1007/JHEP12(2019)061).
- [13] F. Hautmann, H. Jung, A. Lelek, V. Radescu and R. Žlebčák, *Collinear and TMD quark and gluon densities from parton branching solution of QCD evolution equations*, J. High Energy Phys. **01**, 070 (2018), doi:[10.1007/JHEP01\(2018\)070](https://doi.org/10.1007/JHEP01(2018)070).
- [14] F. Hautmann, H. Jung, A. Lelek, V. Radescu and R. Žlebčák, *Soft-gluon resolution scale in QCD evolution equations*, Phys. Lett. B **772**, 446 (2017), doi:[10.1016/j.physletb.2017.07.005](https://doi.org/10.1016/j.physletb.2017.07.005).
- [15] S. Catani, D. de Florian and M. Grazzini, *Universality of non-leading logarithmic contributions in transverse-momentum distributions*, Nucl. Phys. B **596**, 299 (2001), doi:[10.1016/S0550-3213\(00\)00617-9](https://doi.org/10.1016/S0550-3213(00)00617-9).
- [16] D. de Florian and M. Grazzini, *Next-to-Next-to-Leading-Order Logarithmic Corrections at Small Transverse Momentum in Hadronic Collisions*, Phys. Rev. Lett. **85**, 4678 (2000), doi:[10.1103/PhysRevLett.85.4678](https://doi.org/10.1103/PhysRevLett.85.4678).
- [17] G. Bozzi, S. Catani, D. de Florian and M. Grazzini, *Transverse-momentum resummation and the spectrum of the Higgs boson at the LHC*, Nucl. Phys. B **737**, 73 (2006), doi:[10.1016/j.nuclphysb.2005.12.022](https://doi.org/10.1016/j.nuclphysb.2005.12.022).
- [18] G. A. Ladinsky and C.-P. Yuan, *Nonperturbative regime in QCD resummation for gauge boson production at hadron colliders*, Phys. Rev. D **50**, R4239 (1994), doi:[10.1103/PhysRevD.50.R4239](https://doi.org/10.1103/PhysRevD.50.R4239).

- [19] F. Landry, R. Brock, P. M. Nadolsky and C.-P. Yuan, *Fermilab Tevatron run-1 Z boson data and the Collins-Soper-Sterman resummation formalism*, Phys. Rev. D **67**, 073016 (2003), doi:[10.1103/PhysRevD.67.073016](https://doi.org/10.1103/PhysRevD.67.073016).
- [20] A. V. Konychev and P. M. Nadolsky, *Universality of the Collins–Soper–Sterman non-perturbative function in vector boson production*, Phys. Lett. B **633**, 710 (2006), doi:[10.1016/j.physletb.2005.12.063](https://doi.org/10.1016/j.physletb.2005.12.063).
- [21] F. Hautmann, I. Scimemi and A. Vladimirov, *Non-perturbative contributions to vector-boson transverse momentum spectra in hadronic collisions*, Phys. Lett. B **806**, 135478 (2020), doi:[10.1016/j.physletb.2020.135478](https://doi.org/10.1016/j.physletb.2020.135478).
- [22] H.-U. Bengtsson and T. Sjöstrand, *The Lund Monte Carlo for hadronic processes: PYTHIA version 4.8*, Comput. Phys. Commun. **46**, 43 (1987), doi:[10.1016/0010-4655\(87\)90036-1](https://doi.org/10.1016/0010-4655(87)90036-1).
- [23] G. Marchesini and B. R. Webber, *Monte Carlo simulation of general hard processes with coherent QCD radiation*, Nucl. Phys. B **310**, 461 (1988), doi:[10.1016/0550-3213\(88\)90089-2](https://doi.org/10.1016/0550-3213(88)90089-2).
- [24] F. Hautmann, L. Keersmaekers, A. Lelek and A. M. van Kampen, *Dynamical resolution scale in transverse momentum distributions at the LHC*, Nucl. Phys. B **949**, 114795 (2019), doi:[10.1016/j.nuclphysb.2019.114795](https://doi.org/10.1016/j.nuclphysb.2019.114795).
- [25] S. Catani, B. R. Webber and G. Marchesini, *QCD coherent branching and semi-inclusive processes at large χ* , Nucl. Phys. B **349**, 635 (1991), doi:[10.1016/0550-3213\(91\)90390-J](https://doi.org/10.1016/0550-3213(91)90390-J).
- [26] J. C. Collins and F. Hautmann, *Soft gluons and gauge-invariant subtractions in NLO parton-shower Monte Carlo event generators*, J. High Energy Phys. **03**, 016 (2001), doi:[10.1088/1126-6708/2001/03/016](https://doi.org/10.1088/1126-6708/2001/03/016).
- [27] A. Bermudez Martinez et al., *The transverse momentum spectrum of low mass Drell–Yan production at next-to-leading order in the parton branching method*, Eur. Phys. J. C **80**, 598 (2020), doi:[10.1140/epjc/s10052-020-8136-y](https://doi.org/10.1140/epjc/s10052-020-8136-y).
- [28] A. Bermudez Martinez et al., *Production of Z bosons in the parton branching method*, Phys. Rev. D **100**, 074027 (2019), doi:[10.1103/PhysRevD.100.074027](https://doi.org/10.1103/PhysRevD.100.074027).
- [29] A. Bermudez Martinez, P. Connor, H. Jung, A. Lelek, R. Žlebčák, F. Hautmann and V. Radescu, *Collinear and TMD parton densities from fits to precision DIS measurements in the parton branching method*, Phys. Rev. D **99**, 074008 (2019), doi:[10.1103/PhysRevD.99.074008](https://doi.org/10.1103/PhysRevD.99.074008).
- [30] A. M. van Kampen et al., *Work in preparation* .
- [31] S. Catani, D. de Florian and M. Grazzini, *Soft-gluon effective coupling and cusp anomalous dimension*, Eur. Phys. J. C **79**, 685 (2019), doi:[10.1140/epjc/s10052-019-7174-9](https://doi.org/10.1140/epjc/s10052-019-7174-9).
- [32] A. Banfi, B. Kamal El-Menoufi and P. F. Monni, *The Sudakov radiator for jet observables and the soft physical coupling*, J. High Energy Phys. **01**, 083 (2019), doi:[10.1007/JHEP01\(2019\)083](https://doi.org/10.1007/JHEP01(2019)083).
- [33] T. Becher and M. Neubert, *Drell–Yan production at small q_T , transverse parton distributions and the collinear anomaly*, Eur. Phys. J. C **71**, 1665 (2011), doi:[10.1140/epjc/s10052-011-1665-7](https://doi.org/10.1140/epjc/s10052-011-1665-7).

- [34] F. Coradeschi and T. Cridge, *reSolve – A transverse momentum resummation tool*, *Comput. Phys. Commun.* **238**, 262 (2019), doi:[10.1016/j.cpc.2018.11.024](https://doi.org/10.1016/j.cpc.2018.11.024).
- [35] S. Baranov et al., *CASCADE3 A Monte Carlo event generator based on TMDs*, *Eur. Phys. J. C* **81**, 425 (2021), doi:[10.1140/epjc/s10052-021-09203-8](https://doi.org/10.1140/epjc/s10052-021-09203-8).
- [36] H. Jung et al., *The CCFM Monte Carlo generator CASCADE Version 2.2.03*, *Eur. Phys. J. C* **70**, 1237 (2010), doi:[10.1140/epjc/s10052-010-1507-z](https://doi.org/10.1140/epjc/s10052-010-1507-z).
- [37] F. Hautmann, H. Jung and S. Taheri Monfared, *The CCFM uPDF evolution uPDFevolv Version 1.0.00*, *Eur. Phys. J. C* **74**, 3082 (2014), doi:[10.1140/epjc/s10052-014-3082-1](https://doi.org/10.1140/epjc/s10052-014-3082-1).
- [38] E. Accomando, F. Coradeschi, T. Cridge, J. Fiaschi, F. Hautmann, S. Moretti, C. Shepherd-Themistocleous and C. Voisey, *Production of Z'-boson resonances with large width at the LHC*, *Phys. Lett. B* **803**, 135293 (2020), doi:[10.1016/j.physletb.2020.135293](https://doi.org/10.1016/j.physletb.2020.135293).
- [39] J. Alwall et al., *The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations*, *J. High Energy Phys.* **07**, 079 (2014), doi:[10.1007/JHEP07\(2014\)079](https://doi.org/10.1007/JHEP07(2014)079).
- [40] N. A. Abdulov et al., *TMDlib2 and TMDplotter: a platform for 3D hadron structure studies*, *Eur. Phys. J. C* **81**, 752 (2021), doi:[10.1140/epjc/s10052-021-09508-8](https://doi.org/10.1140/epjc/s10052-021-09508-8).
- [41] F. Hautmann, H. Jung, M. Krämer, P. J. Mulders, E. R. Nocera, T. C. Rogers and A. Signori, *TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions*, *Eur. Phys. J. C* **74**, 3220 (2014), doi:[10.1140/epjc/s10052-014-3220-9](https://doi.org/10.1140/epjc/s10052-014-3220-9).
- [42] G. Corcella, I. Knowles, G. Marchesini, S. Moretti, K. Odagiri, P. Richardson, M. Seymour and B. Webber, *HERWIG 6.5 Release Note*, [arXiv:hep-ph/0210213](https://arxiv.org/abs/hep-ph/0210213).
- [43] A. Bermudez Martinez, F. Hautmann and M. L. Mangano, *TMD evolution and multi-jet merging*, *Phys. Lett. B* **822**, 136700 (2021), doi:[10.1016/j.physletb.2021.136700](https://doi.org/10.1016/j.physletb.2021.136700).