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# Pion GPDs: Constraints, modelling and experimental access

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## Abstract

**The study of Generalised Parton Distributions (GPDs) is a hot-topic in hadron physics. Due to its connection with chiral symmetry breaking, assessing those of pions is of great interest. For this reason, we present a novel model-building strategy for pion GPDs capable of fulfilling all of the theoretical constraints required by QCD: support, polynomiality, positivity and soft-pion theorem. The methodology is illustrated with a simple model, and exploited afterwards for the calculation of pions' deeply virtual Compton Scattering (DVCS) Compton Form Factors (CFFs) at NLO. The results show gluon-content to play the dominant role in pions' response to DVCS at EIC kinematics.**

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## 1 Introduction

Describing hadronic structure in terms of quark and gluon degrees of freedom is one of the most intriguing problems in particle physics. In fact, plans for future experimental facilities dedicated to this subject exist [1–3], triggering a vast number of theoretical studies [4–6].

Among all hadrons, pions, as Nambu-Goldstone bosons of QCD's chiral symmetry breaking, are expected to play a crucial role in our grasp of the origin of mass and matter [7,8]. Understanding how quarks and gluons give rise to pions is therefore of uttermost importance; and, in this respect, gaining experimental insights into their structure would be of great interest.

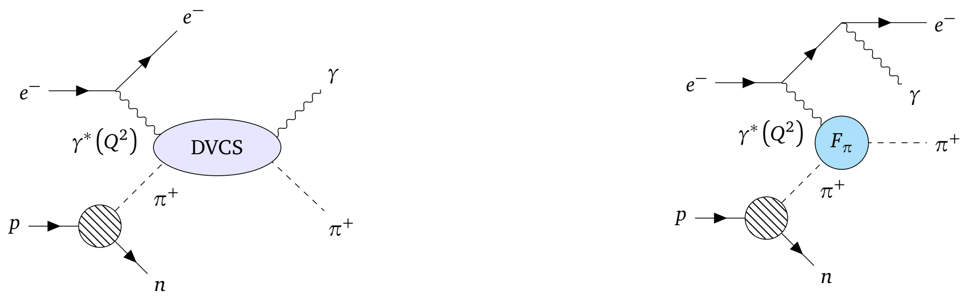
In this work we face the question of whether future electron-ion colliders may probe pion's "3D" structure. This problem lead us, first, to discuss the Sullivan process as a serious candidate for the assessment of pion's structure through generalised parton distributions (Sec. 2). Next, Sec. 3 presents a brief review on GPDs, their properties and their connection with hadronic structure. A novel strategy for modelling of pion GPDs is discussed and illustrated with a simple model in Sec. 4. Finally we exploit it for the calculation of Compton form factors (Sec. 5). We round off showing that gluon-content in the pion, indeed, dominates the behaviour of the corresponding CFFs and, therefore, the response to be observed at future colliders.

## 2 Sullivan process: one-pion-exchange approximation

Lepton-hadron Compton scattering, as interpreted in the Bjorken limit, has proved to be one of the most successful tools for the study of hadronic matter. Indeed, inclusive Deep Inelastic Scattering (DIS) turned out to provide access to the well-known Parton Distribution Functions (PDFs), thus drawing "one-dimensional" pictures of hadrons. Moreover, removing the inclusive constraint allows to further exploit DIS for the study of hadronic structure, *e.g.* by giving access to generalised parton distributions [9].

In this work, relying on a seminal paper by J.D. Sullivan [10], we consider the *one-pion-exchange* contribution to deep inelastic electron-proton scattering with $\pi n$ fixed final states. There, it was proved that under the assumption of small momentum transfers, $|t|$, between initial and final hadronic states, this contribution is favoured with respect to any other channel.

Notwithstanding, the low-$|t|$ kinematic domain may be highly contaminated by resonance contributions. Fortunately, a further cut on the system's invariant mass $W^2 \gtrsim 4$ GeV$^2$ [10,11] allows to minimize those contributions, taking us back to an interpretation of the *Sullivan process* in terms of direct photon-pion interaction (Fig. 1).



(a) Deeply virtual Compton scattering

(b) Bethe-Heitler

Figure 1: Feynman diagrams for the two contributions to the Sullivan process in the one-pion-exchange approximation.

Specifically, given a region in momentum transfer bound by pion's pole, $|t| \lesssim 0,6$ GeV$^2$ [10,11], and ensuring longitudinal polarisation of the incident photon [11,12], one is allowed to probe pion's structure through the Sullivan process. Two contributions can be distinguished: **1.** deeply virtual Compton scattering (DVCS), which can be realised through GPDs [9], and records the "3D" structure of pions (Sec. 3); and **2.**, the Bethe-Heitler contribution, which depends on pion's electromagnetic form factor.

High energy pion electro-production on a nucleon has already been exploited for the extraction of pion's electromagnetic form factor at large $Q^2$ [13] and plans have been made to exploit it again in present and future facilities [7]. It is therefore natural to ask whether it is, effectively, possible to probe pion GPDs through the DVCS contribution to the Sullivan process. In fact, this question was raised in the EIC Yellow report [1].

## 3 Generalised Parton Distributions

GPDs are defined from expectation values of non-local light-cone operators between hadronic states of different momentum and helicity [9,14,15], and depend on three kinematic variables: $x$, the fraction of the hadron's average light-cone momentum carried by the active parton; the skewness, $\xi$, representing the fraction of the hadron's average momentum transferred along the longitudinal direction. And $t_\pi$, the usual *Mandelstam* variable. Depending on the ratio

$|x|\,/\,|\xi|$, two kinematic regions can be defined: DGLAP ($|x| \geq |\xi|$) and ERBL ($|x| \leq |\xi|$). They are also renormalisation-scale-, $\mu$, dependent, and thus evolve with the energy-scale [9].

More importantly, GPDs benefit from a probabilistic interpretation in impact-parameter ($\vec{b}_\perp$) space, where, for vanishing skewness, they turn out to represent probability amplitudes for finding a parton at a given position in $\vec{b}_\perp$-plane, carrying a momentum fraction $x$ of the hadron's light-cone momentum [16]. Therefore, they constitute an outstanding tool for the analysis of hadron's structure.

Furthermore, QCD, the fundamental theory where the definition of GPDs is "embedded", imposes fundamental constraints[1] to be fulfilled by any candidate model:

- **Support:** causality and analyticity restrict GPDs' domain: $(x,\xi) \in [-1,1] \otimes [-1,1]$.
- **Polynomiality:** Lorentz invariance requires GPD's $m$Th-order Mellin moments to behave as polynomials of degree $m+1$ in the skewness.
- **Positivity:** positivity of Hilbert-space norm, realised through Cauchy-Schwartz inequality, defines bounds on GPDs within the DGLAP region.

  $$\left|H^q_{\pi^+}(x,\xi,t_\pi;\mu)\right| \leq \sqrt{q_{\pi^+}\left(\frac{x+\xi}{1+\xi};\mu\right) q_{\pi^+}\left(\frac{x-\xi}{1-\xi};\mu\right)} \quad , \quad |x| \geq |\xi| \tag{1}$$

  with $q_{\pi^+}(z;\mu)$ being the corresponding quark PDF.
- **Low-energy soft-pion theorem:** Axial-vector Ward-Takahashi identity [17] and PCAC [18] constraint isoscalar and isovector combinations of pion GPDs.

## 4 GPD modelling

The interest about pion GPDs is self-justified [16, 18, 19]. It has also become plain that any candidate-model for pion GPDs must meet certain theoretical constraints. However, meeting all of them by construction is still an unsolved problem. For that reason we present a novel model-building strategy for pion GPDs:

1. Start from a pion Light-Front Wave-Function (LFWF) which factorizes longitudinal and transverse degrees of freedom at a given factorisation scale: $\Psi^q_{\pi^+}(x,k_\perp;\mu_{\text{Ref.}}) = \mathcal{N}_\Psi \varphi_{q/\pi^+}(x;\mu_{\text{Ref.}})\,\phi_{q/\pi^+}(k_\perp;\mu_{\text{Ref.}})$ .
2. Employ the overlap representation [20, 21] to build a DGLAP GPD [22]:

   $$H^q_{\pi^+}(x,\xi,t_\pi;\mu_{\text{Ref.}})\Big|_{|x|\geq|\xi|} = \mathcal{N}_H \sqrt{q_{\pi^+}\left(\frac{x+\xi}{1+\xi};\mu_{\text{Ref.}}\right) q_{\pi^+}\left(\frac{x-\xi}{1-\xi};\mu_{\text{Ref.}}\right)}\,\Phi^q_{\pi^+}(x,\xi,t_\pi;\mu_{\text{Ref.}})\,. \tag{2}$$

   Since $\Phi(x,\xi,0;\mu) = 1$, as required by canonical normalization of the light-front wave-function [22], the resulting GPD model saturates the positivity constraint in Eq. [1].
3. Use the covariant extension [23] and the soft-pion theorem to determine the corresponding ERBL-GPD. The covariant extension guarantees fulfilment of polynomiality.

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[1] See [4, 5] and references therein.

This novel modelling strategy unambiguously defines pion GPD models fulfilling all of the theoretical constraints required by QCD and employs one single ingredient: the pion light-front wave-function.

Moreover, one can delve into the $(x, k_\perp)$-decoupling assumption through PTIRs for pion Bethe-Salpeter amplitudes [22, 24], showing that the isospin-symmetric limit leads to a factorised pion LFWF whose transverse-part takes a particularly simple dipole-like structure [22, 24, 25]. In that way, the modelling strategy here presented yields pion GPD models with an "in-built" momentum-transfer dependence, and thus simplifies even further: the only ingredient for the development of pion GPD models implementing all the necessary consequences of causality, analyticity and positivity together with Lorentz- and gauge-invariance is just a parametrization for the pion parton distribution function Eq. (2).

### 4.1 An algebraic model

As an illustration consider the following simple *ansatz* for the pion PDF [26]:

$$q_{\pi^+}(x;\mu_{\text{Ref.}}) = 30x^2(1-x)^2 \,. \tag{3}$$

The pion GPD model resulting from applying the above strategy to this PDF is shown in Fig. 2. As explained in Sec. 4, such model satisfies by construction all the properties required by QCD. Furthermore, as expected, it reduces to the initial PDF in the forward limit (blue line, Fig. 2) and also implements chiral symmetry through the soft-pion theorem; as seen from its $\xi = 1$ limit, which yields the pion's asymptotic parton distribution amplitude (red line, Fig. 2).

Lastly, it can be noticed from the "slice" $\xi = 1/2$ shown in Fig. 2, that the resulting model is continuous along the $x = \xi$ line, as demanded by factorisation theorems for DVCS amplitudes to remain finite [12]. Therefore, pion GPD models obtained through the novel strategy presented herein gather all the necessary properties for phenomenological analyses.

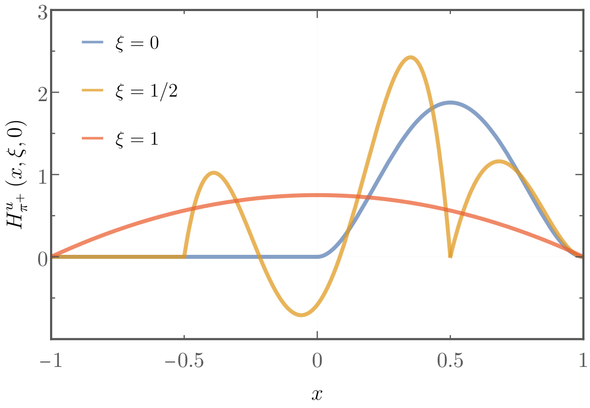

Figure 2: Pion GPD model obtained from the covariant extension of the algebraic model DGLAP-GPD yielded by the parametrisation of the pion-PDF given in Eq. [3].

## 5 DVCS Compton Form Factors (CFFs)

Once "theoretically-complete" GPD models can be built, it is natural to take advantage of them for phenomenological analyses. In particular, one could ask whether it would be possible to probe pion's structure through the Sullivan process (Sec. 2).

To this end, the crucial point is that DVCS amplitudes are written in terms of Compton form factors [9, 11]. And those, are defined as convolutions of the corresponding GPDs with perturbatively calculable kernels:

$$\mathcal{H}_{\pi^+}\left(\xi,t;Q^2\right)=\sum_{p=\{q\},g}\int_{-1}^{1}\frac{dx}{\xi}\mathcal{K}^p\left(\frac{x}{\xi},\frac{Q^2}{\mu_{\mathrm{F}}^2},\alpha_{\mathrm{S}}\left(\mu_{\mathrm{F}}^2\right)\right)H_{\pi^+}^p\left(x,\xi,t;\mu_F^2\right). \tag{4}$$

Comparison with experiment requires calculation of the corresponding Compton form factors at the relevant energy-scales. Therefore, solving QCD evolution equations for GPDs [9], taking them from the original scale, here set according to the prescriptions derived in [27] as $\mu_{\mathrm{Ref}} = 0.331$ GeV, to an experimentally-relevant one is a necessary step.

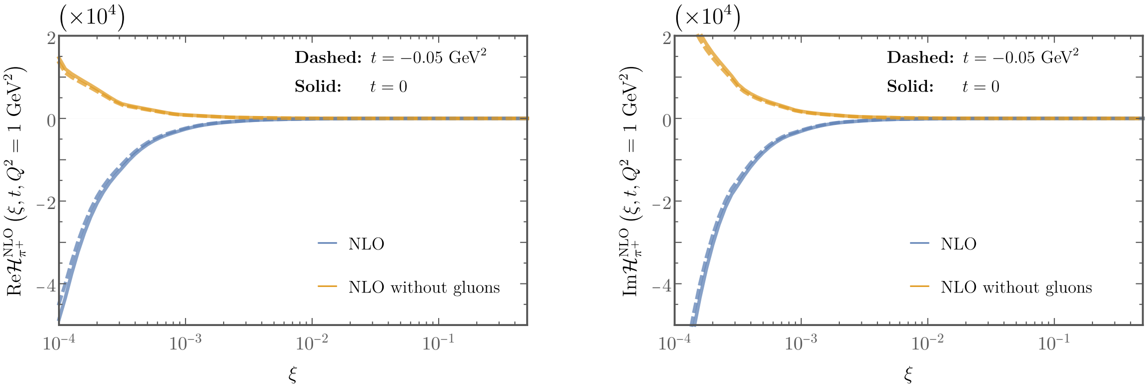

Figure 3: NLO DVCS Compton form factors yielded by the algebraic model GPD of Fig. 2 at a factorisation scale of $\mu_F^2 = 1$ GeV$^2$ (blue-line). Yellow plots correspond to the NLO calculation with the input gluon distribution set to zero.

`Apfel++` [28, 29] capitalises on the effect of scale evolution on our GPD model. `PARTONS` framework [30] is employed to compute NLO DVCS CFFs at the relevant scale (Fig. 3).

Resulting Compton form factors (Fig. 3) reveal gluon-content of the pion to make the dominant contribution to the behaviour of the corresponding NLO CFFs. It is therefore expected that gluons control the dynamics of pions in deep inelastic lepton-proton scattering in the one-pion-exchange approximation. Thus, gluons are expected to play a crucial role into the description of high-energy pion's structure; and to be a decisive piece for the analysis of future collider data: triggering the possibility of accessing pion GPDs at future EICs.

## 6 Conclusions

The Sullivan process is one of the most promising candidates to provide experimental access to pion GPDs at future experimental facilities. The development of pion GPD models fulfilling all of the theoretical constraints required by QCD is a crucial task for the description of DVCS data to be observed at foreseen electron-ion colliders.

To this end we have presented a novel model-building strategy for pion GPDs which, starting from pion PDF models derived from first-principles continuum calculations, and relying on the so-called covariant extension, is capable to match all the properties required by QCD. Those models are thus suited for phenomenological analyses.

We have validated and exploited the methodology with a simple algebraic pion GPD model. Using `Apfel++` and `PARTONS` computing packages, we have been able to compute NLO Compton form factors, which parametrize DVCS amplitudes. We have shown that gluon distributions within the pion play the dominant role into the description of Sullivan process' dynamics and thus, the description of pion's structure.

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