

# Production of three isolated photons in the high-energy factorization approach

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## Abstract

We study large- $p_T$  three-photon production at the LHC at the center-of-mass energy  $\sqrt{s} = 8$  TeV. We use the LO approximation of the parton Reggeization approach consistently merged with the real NLO corrections. For numerical calculations use the parton-level generator KaTie and modified KMR-type unintegrated parton distribution functions. We find good agreement between our predictions and data with the same accuracy as in the NNLO calculations based on the collinear parton model of QCD. At higher energies ( $\sqrt{s} = 13$  and 27 TeV) parton Reggeization approach predicts larger cross sections, up to  $\sim 10\%$  and  $\sim 20\%$ , respectively.



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## 1 Introduction

The recent experimental data for large- $p_T$  three-photon production at the energy 8 TeV [1] are extensively studied in the collinear parton model (CPM) within framework of perturbative approach of QCD beyond the leading-order (LO) accuracy in strong-coupling constant  $\alpha_s$ , i.e. at the next-to-leading-order (NLO) [2, 3] and even at next-to-next-to-leading-order (NNLO) [4, 5]. The high-order calculations for the three-photon production in CPM of QCD provide rather bad agreement with data at the level of NLO accuracy. Inclusion of the NNLO QCD corrections [4, 5] eliminates the existing discrepancy with respect to NLO QCD predictions. However, for three-photon production the agreement with data is not so good as for single or two-photon production and it is achieved when hard scale parameter  $\mu$  is taken very small [4, 5].

In CPM we neglect the transverse momenta of initial-state partons in hard-scattering amplitude that is correct assumption for the fully inclusive observables, such as  $p_T$  spectra of single prompt photons or jets, where their large transverse momentum defines single hard scale of the process,  $\mu \sim p_T$ . The multi-photon large- $p_T$  production is multi-scale hard process

Table 1: Predictions for  $p + p \rightarrow \gamma\gamma\gamma + X$  total cross section at  $\sqrt{s} = 8$  TeV for the different choice of factorization/renormalization scale ( $\mu = \mu_F = \mu_R$ ), errors indicate variation by factor two around the middle values which are listed in first column.

Hard scale, $\mu$	$\sigma_{\text{LO}}$ [fb]	$\sigma_{\text{NLO}}$ [fb]
$M_{3\gamma}/2$	$31.07^{+8.87}_{-6.76}$	$69.22^{+4.05}_{-1.07}$
$p_{T,3\gamma}/2$	$29.72^{+9.22}_{-6.72}$	$69.76^{+4.29}_{-1.85}$
$E_{T,3\gamma}/2$	$32.50^{+9.80}_{-2.65}$	$71.00^{+4.93}_{-2.65}$

Table 2: Predictions for  $p + p \rightarrow \gamma\gamma\gamma + X$  total cross section at the different center-of-mass energies,  $\sqrt{s}$ . Numerical error of calculation is equal 0.1%.

$\sqrt{s}$ [TeV]	$\sigma_{\text{LO}}$	$\sigma_{\text{NLO}}$	$K_{\text{NLO}}$	$\sigma_{\text{NNLO}}^{\text{CPM}}$ [5]
8	$32.50^{+9.80}_{-7.46}$	$71.00^{+4.93}_{-2.65}$	2.18	$67.42^{+7.41}_{-5.73}$
13	$53.91^{+18.14}_{-14.11}$	$126.79^{+10.43}_{-7.30}$	2.35	$114^{+13.64}_{-10.54}$
27	$115.25^{+45.09}_{-34.45}$	$298.54^{+30.71}_{-25.55}$	2.59	$245.91^{+32.46}_{-24.34}$

in which use the simple collinear picture of initial state radiation may be a bad approximation. In the present paper, we calculate different multi-scale variables in three-photon production from a point of view of high-energy factorization (HEF) [6, 7]. We use the parton Reggeization approach (PRA) which is a version of HEF formalism, based on the modified multi-Regge kinematics (mMRK) approximation for QCD scattering amplitudes [8, 9]. This approximation is accurate both in the collinear limit, which drives the transverse-momentum-dependent (TMD) factorization and in the high-energy (multi-Regge) limit,  $\hat{s} \gg (-\hat{t}) \sim \mathbf{p}_T^2 \sim \mu^2$ .

In same manner of PRA, we studied previously one-photon production [10], two-photon production [11] and photon plus jet production [12] in proton-(anti)proton collisions at the Tevatron and LHC.

## 2 Parton Reggeization Approach

### 2.1 High-energy factorization

The cornerstones of PRA are  $k_T$ -dependent factorization formula, unintegrated parton distribution functions (uPDF's) and gauge-invariant amplitudes with off-shell initial-state partons. The second one is constructed in the same manner as it was suggested by Kimber, Martin, Ryskin and Watt [13, 14], but with sufficient revision, see Ref. [15]. The off-shell amplitudes are derived using the Lipatov Effective Field Theory (EFT) of Reggeized gluons [16] and Reggeized quarks [17]. More details of PRA can be found in Ref. [8], the inclusion of real NLO corrections is studied in Ref. [9], the development of PRA in the full one-loop NLO approximation is further discussed in [18–20].

Factorization formula of PRA for the process  $p + p \rightarrow \gamma\gamma\gamma + X$ , can be presented in a  $k_T$ -factorized form:

$$d\sigma = \sum_{i,\bar{j}} \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \Phi_i(x_1, t_1, \mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \Phi_{\bar{j}}(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{\text{PRA}}, \quad (1)$$

where  $t_{1,2} = -\mathbf{q}_{T1,2}^2$ , the off-shell partonic cross-section  $\hat{\sigma}_{\text{PRA}}$  is determined by squared Reggeized amplitude,  $|\mathcal{A}_{\text{PRA}}|^2$ . Despite the fact that four-momenta of partons in the initial state are off-shell ( $q_{1,2}^2 = -t_{1,2} < 0$ ), the PRA hard-scattering amplitude is gauge-invariant.

## 2.2 New unintegrated PDFs

To resolve collinear divergence problem, we require that uPDF  $\Phi_i(x, t, \mu)$  in (1) should be satisfied exact normalization condition:

$$\int_0^{\mu^2} dt \Phi_i(x, t, \mu^2) = F_i(x, \mu^2) \text{ or } \Phi_i(x, t, \mu^2) = \frac{d}{dt} [T_i(t, \mu^2, x) F_i(x, t)], \quad (2)$$

where  $T_i(t, \mu^2, x)$  is referred as Sudakov form-factor, satisfying the boundary conditions

$$T_i(t = 0, \mu^2, x) = 0 \text{ and } T_i(t = \mu^2, \mu^2, x) = 1.$$

UPDF can be written as follows from KMR model:

$$\Phi_i(x, t, \mu) = \frac{\alpha_s(\mu)}{2\pi} \frac{T_i(t, \mu^2, x)}{t} \sum_{j=q,\bar{q},g} \int_x^1 dz P_{ij}(z) F_j\left(\frac{x}{z}, t\right) \theta(\Delta(t, \mu) - z). \quad (3)$$

Here, we resolved also infra-red divergence taking into account the cutoff:  $z < 1 - \Delta_{\text{KMR}}(t_{1,2}, \mu^2)$ , where  $\Delta_{\text{KMR}}(t, \mu^2) = \sqrt{t}/(\sqrt{\mu^2} + \sqrt{t})$  is the KMR-cutoff function [13].

The solution for Sudakov form-factor in Eq. (2) has been obtained in Ref. [15] (see equations (26)-(28)). There are important differences between the Sudakov form-factor obtained in the PRA (3) and in the KMR approach [13]. At first, the Sudakov form-factor in PRA contains the  $x$ -dependent  $\Delta\tau_i$ -term in the exponent which is needed to preserve exact normalization condition for arbitrary  $x$  and  $\mu$ . The second one is that in PRA the rapidity-ordering condition is imposed both on quarks and gluons, while in KMR approach it is imposed only on gluons.

## 2.3 LO and NLO subprocesses

In presented study, we take into consideration the LO subprocess of three-photon production in quark-antiquark annihilation

$$Q\bar{Q} \rightarrow \gamma\gamma\gamma \quad (4)$$

and NLO contributions of quark(antiquark)-gluon scattering subprocesses

$$QR \rightarrow q\gamma\gamma\gamma. \quad (5)$$

We don't consider NLO contributions from subprocesses  $Q\bar{Q} \rightarrow g\gamma\gamma\gamma$  which is negligibly small ( $\leq 5\%$ ) at high energy.

In the Lipatov EFT, the LO (4) and NLO (5) subprocesses are described by gauge-invariant sets of 13 and 40 Feynman diagrams, respectively. The direct integration of squared amplitudes in the LO approximation of Lipatov EFT can be done in the numerically-stable form. To calculate contributions from  $2 \rightarrow 4$  NLO subprocesses with initial Reggeized partons such method is not efficient and we use parton-level event generator KaTie [21–23]. The LO contribution of subprocess (4) were calculated for crosscheck both with event generator KaTie [21] and semi-analytically with the help of Feynman rules of Lipatov EFT. The approach used KaTie

for numerical generation of off-shell amplitudes is equivalent to the Lipatov EFT at the tree-level [12, 24].

Next important usage is matching LO and NLO calculation in PRA [9, 11]. To extract specific double counting between LO (4) and NLO (5) subprocesses with emission of additional quark which is separated in rapidity from three-photon cluster. Such additional quark should be considered as emitted parton during perturbative QCD evolution and should be absorbed in uPDF. Accordingly to KMR-PRA model of uPDFs, it has strong angular (rapidity) ordering for emitted partons, such a way we should extract events with rapidity configuration of final photons and quark when rapidity of final quark smaller or large of photon's rapidities, depending of sign of initial quark rapidity. Such procedure decreases NLO contribution about 40-50 %.

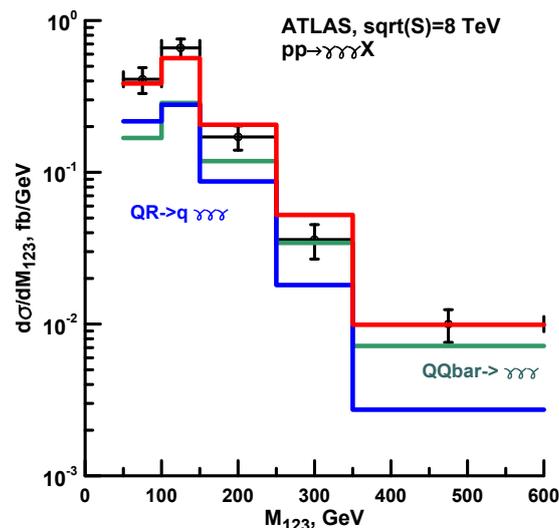


Figure 1: The differential cross sections as function of three-photon invariant mass  $M_{123}$ . The hard scale in PRA calculation is taken as  $\mu = M_{123}$ . The green histogram corresponds LO contribution (4), the blue histogram corresponds NLO contribution (5) and the red histogram is their sum.

### 3 Results

We calculate cross section at different choice of factorization ( $\mu_F$ ) and renormalization ( $\mu_R$ ) scales, which we take equal to each other,  $\mu_F = \mu_R = \mu$ . In the Table 1 we compare predictions obtained in PRA with  $\mu = M_{3\gamma}$  - invariant mass of three-photon system;  $\mu = k_{T,3\gamma}$  - sum of transverse momentum moduli; and  $\mu = E_{T,3\gamma}$  - transverse energy of three-photon system. Table 2 collects total cross sections at three energies  $\sqrt{s} = 8, 13, \text{ and } 27$  TeV. We compare PRA predictions with result of calculation in NNLO CPM [4, 5]. PRA results in LO with real NLO corrections are roughly coincide with full NNLO predictions of CPM for  $\sqrt{s} = 8$  TeV. At higher energies (13 and 27 TeV) PRA predicts larger cross sections, up to  $\sim 10\%$  and  $\sim 20\%$ , respectively.

The differential cross sections as function of three-photon invariant mass  $M_{123}$  is shown in Fig. 1. The hard scale is taken as  $\mu = M_{123}$ . We find good agreement also for invariant mass spectra of different photon-pairs ( $M_{ij}$ ), rapidity ( $|\Delta y_{ij}|$ ) and azimuthal angle ( $|\Delta\phi_{ij}|$ ) differences and transverse momenta of leading  $p_{T1}$  and subleading ( $p_{T,2,3}$ ) photons.

## 4 Conclusion

We describe cross section and spectra for three-photon production in LO PRA with real NLO corrections. We demonstrate applicability of new KMR-type uPDFs for using in HEF calculations.

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