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T-odd quark-gluon-quark correlation function in the light-front quark-diquark model

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Abstract

We have scrutinized the transverse momentum dependent quark–gluon–quark correlation function. We have utilized the light-front quark-diquark model to study the time-reversal-odd interaction-dependent twist-3 gluon distributions which we have obtained from the disintegration of the transverse momentum dependent quark–gluon–quark correlator. Specifically, we have studied the behavior of $\tilde{e}_L$ and $\tilde{e}_T$ while considering our diquark to be an axial-vector.

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1 Introduction

Theoretically, in the description of hadron containing (semi-)inclusive high energy processes, the cross-sections are generally written in the powers of $1/Q$ with Q being the large momentum transfer of the collision. The convolution of the hard scattering coefficients and leading-twist distribution functions is used to communicate the contribution at the leading power. In the $1/Q$ expansion, the first sub leading power of twist-3 distribution and/or fragmentation functions are added to the cross-section [1–4]. Twist-3 distribution functions are not illuminated in the form of probability which is obvious and contrary to the interpretation of the twist-2 distributions. They are rather used to describe the parton densities in the core the nucleon. Information about the nucleon's parton structure is obtained from twist-3 distribution functions [5], particularly when the parton transverse momenta exist. The appeal on the twist-3 contributions also comes from the fact that they are related to the multi-parton correlation in the interior of nucleon [6,7].

In this paper, we have utilized quark-diquark model to examine the quark distributions of twist-3 level which are ciphered in the quark-gluon-quark correlation. We have emphasized on the time-reversal-odd (T-odd) transverse momentum dependent distributions (TMDs).

In the single-spin asymmetries (SSAs) measured in semi-inclusive deeply inelastic scattering (SIDIS) [8–10] the leading twist T-odd TMDs [11] play important roles in the TMD factorization approach [12–16]. In the TMD factorization approach at the twist-3 level there are eight T-odd distributions that can contribute to various azimuthal asymmetries in the SIDIS [5] and Drell–Yan [17] processes. Even though the twist-3 contributions are suppressed due to $1/Q$, still these experimental observables have potential and may be accessible in the kinematical regime where Q is not so large. The ideal experiments for exploring this kinematical region are at PAX [18] and Jefferson Lab [19, 20].

2 Light-Front Quark-Diquark Model

We contemplate on our problem by considering the light-front quark-diquark model [21], where the proton has a spin-flavor $SU(4)$ structure and is written as a combination of isoscalar-scalar diquark singlet $\left|uS^0\right\rangle$, isoscalarvector diquark $\left|uA^0\right\rangle$ and isovector-vector diquark $\left|dA^1\right\rangle$ states [22–24]. We have

$$|P;\pm\rangle = C_S \left|uS^0\right\rangle^{\pm} + C_V \left|uA^0\right\rangle^{\pm} + C_{VV} \left|dA^1\right\rangle^{\pm}, \tag{1}$$

where S and A denote the scalar and axial-vector diquark and their superscripts are used to depict the isospin of that diquark. Here, we have used the light-cone convention $x^{\pm} = x^0 \pm x^3$ and the frame is chosen such that the proton has no transverse momentum, i.e., $P \equiv \left(P^+, \frac{M^2}{P^+}, \mathbf{0}_\perp\right)$; where the struck quark and diquark have momentum $p \equiv \left(xP^+, \frac{p^2+|\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp\right)$ and $P_X \equiv ((1-x)P^+, P_X^-, -\mathbf{p}_\perp)$, respectively. $x = p^+/P^+$ is used to denote the longitudinal momentum fraction carried by the struck quark. The two particle Fock-state expansion for axial-vector diquark is given as [25]

$$\begin{aligned}
|\nu A\rangle^{\pm} = \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \times \Bigg[&\psi_{++}^{r(\nu)}(x,\mathbf{p}_\perp)\left|+\frac{1}{2}+1; xP^+, \mathbf{p}_\perp\right\rangle \\
&+\psi_{-+}^{r(\nu)}(x,\mathbf{p}_\perp)\left|-\frac{1}{2}+1; xP^+, \mathbf{p}_\perp\right\rangle + \psi_{+0}^{r(\nu)}(x,\mathbf{p}_\perp)\left|+\frac{1}{2}0; xP^+, \mathbf{p}_\perp\right\rangle \\
&+\psi_{-0}^{r(\nu)}(x,\mathbf{p}_\perp)\left|-\frac{1}{2}0; xP^+, \mathbf{p}_\perp\right\rangle + \psi_{+-}^{r(\nu)}(x,\mathbf{p}_\perp)\left|+\frac{1}{2}-1; xP^+, \mathbf{p}_\perp\right\rangle \\
&+\psi_{-}^{r(\nu)}(x,\mathbf{p}_\perp)\left|-\frac{1}{2}-1; xP^+, \mathbf{p}_\perp\right\rangle \Bigg],
\end{aligned} \tag{2}$$

where $\left|\lambda_q \lambda_D; xP^+, \mathbf{p}_\perp\right\rangle$ represents the two-particle state with quark helicity $\lambda_q = \pm\frac{1}{2}$, the helicity vector diquark is $\lambda_D = \pm 1, 0$ (triplet). The superscript of ψ, r denotes the helicity of nucleon and the flavor index for the flavors u and d is denoted by ν.

3 Transverse Momentum Dependent Distributions (TMDs)

3.1 Quark-gluon-quark correlation function

We start our calculations with the transverse momentum dependent quark-gluon-quark correlation function which is defined in [26]

$$\left(\tilde{\Phi}_A^{[\pm]\alpha}\right)_{ij}(x,p_T) \equiv \int \frac{d^2\xi_T d\xi^-}{(2\pi)^3} e^{ip\xi} \times \langle P,S|\bar{\psi}_j(0) g \int_{\pm\infty}^{\xi^-} d\eta^- \mathcal{L}^{[\pm]}\left(0,\eta^-\right) \\ F^{+\alpha}(\eta)\, \mathcal{L}^{\xi_T,\xi^+}\left(\eta^-,\xi^-\right)\psi_i(\xi)|P,S\rangle_c\Big|_{\eta^+=\xi^+=0,\eta_T=\xi_T,p^+=xP^+}, \tag{3}$$

where $F^{\mu\nu}$ is the gluon antisymmetric field strength tensor. By definition, gauge-invariance is certified by the gauge-links $\mathcal{L}^{[\pm]}$ and $\mathcal{L}^{\xi_T,\xi^+}$. The sign " $\pm$ " in the subscript or superscript represents the future/past-pointing [14] nature gauge-link between the quark and gluon in the SIDIS/Drell-Yan processes, respectively.

The correlator can be rewritten further as [27]

$$\left(\tilde{\Phi}_A^{[\pm]\alpha}\right)_{ij}(x,p_T) = ig \int \frac{d^2\xi_T d\xi^- d\eta^-}{(2\pi)^4} \int dx' \frac{e^{ix'p+\eta^-}}{(x'\mp i\epsilon)} \times e^{i\left[(x-x')P^+\cdot\xi^- - p_T\cdot\xi_T\right]} \\ \langle P,S|\bar{\psi}_j(0)\mathcal{L}^{[\pm]}\left(0,\eta^-\right)F^{+\alpha}(\eta) \times \mathcal{L}^{\xi_T,\xi^+}\left(\eta^-,\xi^-\right)\psi_i(\xi)|P,S\rangle\Big|_{\eta^+=\xi^+=0,\eta_T=\xi_T}, \tag{4}$$

where the factor $1/\left(x'\mp i\epsilon\right)$ in Eq. (4) can be written as

$$\frac{1}{(x'\mp i\epsilon)} = \mathrm{P}\left(\frac{1}{x'}\right) \pm i\delta\left(x'\right). \tag{5}$$

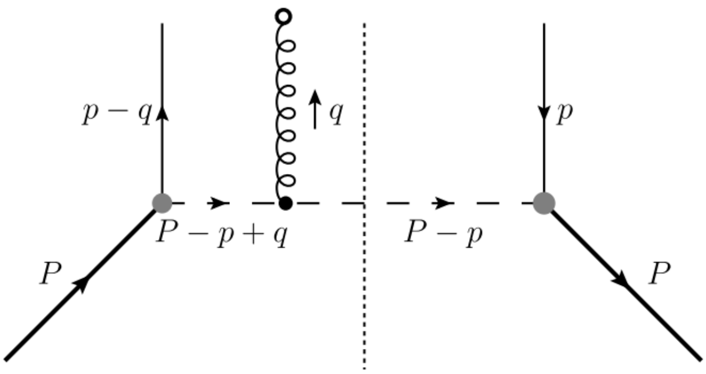

Figure 1: Diagram corresponding to quark-gluon-quark correlator. P, p and q denote the nucleon, quark and diquark momenta respectively [27,28].

3.2 T-Odd Twist-3 Distributions

We can break the quark-gluon-quark correlator as [5]

$$\tilde{\Phi}_A^{\alpha}(x,p_T) = \frac{xM}{2}\left\{\left[\left(\tilde{f}^{\perp} - i\tilde{g}^{\perp}\right)\frac{p_{T\rho}}{M} - \left(\tilde{f}'_T + i\tilde{g}'_T\right)\epsilon_{T\rho\sigma}S_T^{\sigma} - \left(\tilde{f}_s^{\perp} + i\tilde{g}_s^{\perp}\right)\frac{\epsilon_{T\rho\sigma}p_T^{\sigma}}{M}\right] \\ \left(g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\gamma_5\right) - \left(\tilde{h}_s + i\tilde{e}_s\right)\gamma_T^{\alpha}\gamma_5 + [(\tilde{h} + i\tilde{e}) + \left(\tilde{h}_T^{\perp} - i\tilde{e}_T^{\perp}\right)\frac{\epsilon_T^{\rho\sigma}p_{T\rho}S_{T\sigma}}{M}\right] i\gamma_T^{\alpha}\Bigg\}\frac{\not{n}_+}{2}. \tag{6}$$

This equation contains interaction-dependent twist-3 quark distributions, which is in turn dependent on the longitudinal momentum fraction and the transverse momentum denoted by x and p_T respectively. These are denoted by the functions appearing with a tilde. Out of these, $\tilde{g}^{\perp}$, $\tilde{f}_T$ (or $\tilde{f}'_T$), $\tilde{e}_L$, $\tilde{e}_T$, $\tilde{f}_T^{\perp}$, $\tilde{f}_L^{\perp}$, $\tilde{h}$ $\tilde{e}_T^{\perp}$ are T-odd; and $\tilde{f}^{\perp}$, $\tilde{g}_T$ (or $\tilde{g}'_T$), $\tilde{g}_T^{\perp}$, $\tilde{g}_L^{\perp}$, $\tilde{h}_L$, $\tilde{h}_T$, $\tilde{e}$, $\tilde{h}_T^{\perp}$ are T-even.

These TMDs can be projected out by using disparate Dirac matrices. In the right-hand side of Eq. (5), if one takes the real part then they can derive the traces of the T-even TMDs. On the other hand, if one uses the imaginary part, T-odd TMDs are obtained. Here we deal specifically with $\tilde{e}_L$ and $\tilde{e}_T$ [27] by considering the real part of Eq. (5) and using Dirac matrix $i\sigma^{\alpha+}\gamma_5$ as

$$\frac{1}{2Mx}\,\mathrm{Tr}\left[\tilde{\Phi}_{A\alpha} i\sigma^{\alpha+}\gamma_5\right] = S_L\left(\tilde{h}_L + i\tilde{e}_L\right) - \frac{p_T \cdot S_T}{M}\left(\tilde{h}_T + i\tilde{e}_T\right), \tag{7}$$

where S_T and S_L are the transverse and longitudinal polarization vector of the nucleon respectively.

While applying the approximation of the lowest order, we neglect every gauge-link in the correlator (Eq. (4)) and choose the model which has been used extensively in the calculation of TMD distributions [22–24]. Before all else, diquark model shows that the T-odd distributions are non-vanishing. We consider the case in which the diquark is an axial-vector. Here, we calculate the T-odd TMDs emerging in the DIS process (Drell-Yan process comes with a minus sign). Above expression is proportional to the sum of the terms $\psi^{*r}_{-+}(x,p_T)\psi^{r}_{++}(x,p_T)$, $\psi^{*r}_{-0}(x,p_T)\psi^{r}_{+0}(x,p_T)$, $\psi^{*r}_{--}(x,p_T)\psi^{r}_{+-}(x,p_T)$, $\psi^{*r}_{++}(x,p_T)\psi^{r}_{-+}(x,p_T)$, $\psi^{*r}_{+0}(x,p_T)\psi^{r}_{-0}(x,p_T)$ and $\psi^{*r}_{+-}(x,p_T)\psi^{r}_{--}(x,p_T)$. We can solve this further and get the value of desired TMDs by using suitable light-front wave functions [12, 13, 29, 30]. Also in a specific Feynman rule [27, 28, 31], the field strength tensor has been used in the form $F^{+\alpha}\,:\,-i\left(q^+ g^{\alpha\rho} - q^{\alpha} g^{+\rho}\right)$. While using this model some divergences are found which appear to be emerging for a few T-odd TMDs when the integrations are performed over transverse momentum. In the quark-diquark model these kind of divergences are explicitly found very often [27, 32]. So, it can be deduced that T-odd twist-3 distributions has this general feature that when on the nucleon-quark-diquark interaction vertex the point-like coupling is applied the divergences appear. To derive the finite results, one can choose a dipole form factor instead of a point-like coupling constant for the nucleon-quark-diquark coupling [22].

4 Conclusion

We have investigated the prospect to calculate the T-odd interaction-dependent twist-3 quark distributions in the quark diquark model. In the approxmation of lowest order we find that abandoning the gauge-links in the correlator can give results which are not equal to zero for the eight T-odd interaction-dependent quark TMDs of twist-3. Particularly, we find that the projection of T-odd twist-3 correlator with this specific Dirac matrix $i\sigma^{\alpha+}\gamma_5$ in the form of TMDs $\tilde{e}_L$ and $\tilde{e}_T$ while considering the case of a axial-vector diquark, leads to an equation proportional to the the light-front wave functions and the field strength tensor. From this one can get the expression of each TMD individually by specifying the nucleon helicity.

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