Top-quark mass determination in the optimised threshold scan

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Abstract

One of the important goals at the future e^+e^- colliders is to measure the top-quark mass and width in a scan of the pair production threshold. Presented in this work is the most general approach to the top-quark mass determination from the threshold scan at CLIC, with all relevant model parameters and selected systematic uncertainties included in the fit procedure. In the baseline scan scenario the top-quark mass can be extracted with precision of the order of 30 to 40 MeV, already for 100 fb^{-1} of data collected at the threshold. We present the optimisation procedure based on the genetic algorithm with which the statistical uncertainty of the mass measurement can be reduced by about 20%.

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1 Introduction

The threshold scan is expected to be the most precise way to measure the top-quark mass. However, the shape of the threshold cross-section depends not only on the top-quark mass and width but also on other model parameters, such as the top Yukawa coupling and the strong coupling constant. It can also be affected by many systematic effects. The new top-quark threshold scan fit procedures developed for CLIC [1, 2] is more flexible that the one used in the previous study [3]. It allows to take all relevant model parameters and additional parameter constraints coming e.g. from earlier measurements, as well as constraints on data normalisation into account.

At the first energy stage, CLIC running is assumed to include a dedicated scan of the t$\bar{\text{t}}$ threshold with total integrated luminosity of 100 fb^{-1}. Running at 10 equidistant energy points taking 10 fb^{-1} of data for each value of collision energy is assumed as the baseline scan scenario. This is clearly a conservative scenario and the aim of the presented study was to investigate the possibility of reducing the statistical fit uncertainties when using the optimised running scenario. The scan optimisation assumes that the top-quark mass is already known to O(100 MeV).

2 Genetic algorithm

Searching for the best scenario of the top-quark threshold scan at CLIC (or at any other future e^+e^- collider) one needs to take into account many different aspects of the measurement. The top quark mass is just one of the parameters that are to be constrained from the collected data (with the best possible statistical uncertainty). There are other model parameters, measurement of which needs to be optimised at the same time. In a Genetic algorithm, a set of proposed solutions to a problem, called *Individuals*, is evolved to find better solutions. Each Individual has a set of properties, called *genotype*, which can be mutated and altered, and a set of measurable traits, called *phenotype*, that consists the genotype. Phenotype is made of traits that are used for performance evaluation and to choose the most fit Individuals to the next generation. During each iteration of the algorithm, population evolves towards better solutions. Finally, after a given number of iterations, the population should converge to a solution that is close to optimal. We decided to use the Non dominated sorting genetic algorithm II (NSGA-II), proposed by Kalyanmoy Deb in 2002. Thanks to the use of an efficient non dominated sorting algorithm it was possible to lower the time complexity from original $O(MN^2)$ [4] to $O(MNlog^{M-1}N)$ [5], where M is number of objectives and N is the size of the population. For more details refer to [2].

3 NSGA-II set-up

One scan scenario was assumed as an Individual, which genotype is represented by a scan sequence, set of centre-of-mass energy points. Single scan point can be considered a chromosome. Constant total integrated luminosity of $100\,\text{fb}^{-1}$ was assumed and it is equally distributed among all scan points. An initial population was created from the baseline scenario assuming ten scan points equally separated from each other by 1 GeV, starting at 340 GeV. This scenario was studied in detailed in [2,3]. The population size is set to 2000 and number of generations to 30. All results presented here were calculated assuming normalisation uncertainty of $\Delta = 0.1\%$, strong coupling constant uncertainty of $\sigma_{\alpha_s} = 0.001$ and background level uncertainty of $\sigma_{f_{bg}} = 2\%$. When measurement of the top Yukawa coupling was not included in the optimisation objectives, an uncertainty of $\sigma_{y_t} = 0.1$ was assumed. In order to rule out results that could be not representative of Individual's performance, each uncertainty was computed three times and the worst result was chosen.

4 Optimisation

Described in this contribution are the results of the two objective optimisation procedures, optimised for top-quark mass and width or mass and Yukawa coupling measurements. For results of single objective optimisation please refer to [2].

When considering scan optimisation for the mass-width pair, an improvement of around 20-25% could be observed in the final generation for both parameters. For the top-quark mass the uncertainty was reduced from 32 MeV to 26 MeV, while for the width, from 58 MeV to 44 MeV (see Fig 1). If we look at the histogram of measurement points from the last generation (see Fig 1) we can clearly observe three distinct regions on the threshold. One below, one in the middle and one above the threshold. We conclude that cross section measurements in those regions are most sensitive to the two considered model parameters, mass and width.

When optimizing the threshold scan procedure for mass and Yukawa coupling measurements, we can observe a similar behaviour. The mass uncertainty changes from 32 MeV to
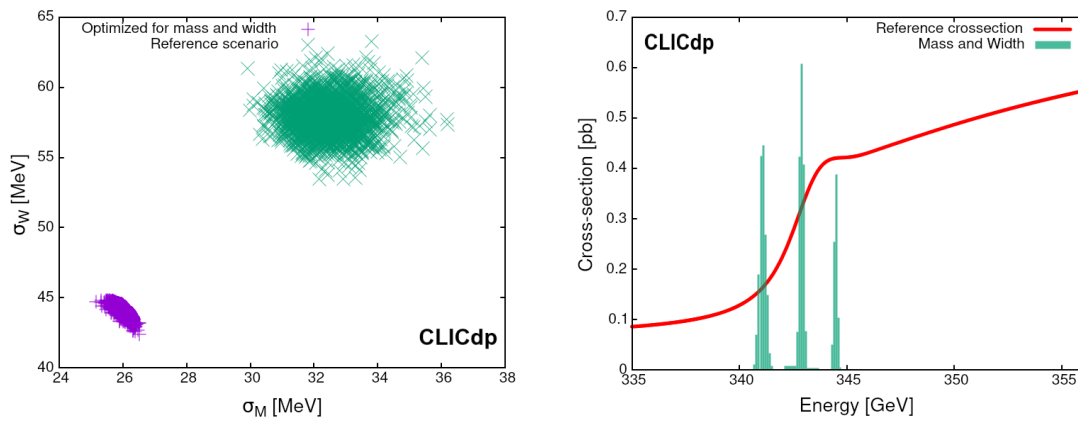
Figure 1: Left: fit uncertainty distribution in the first (green) and the last (blue) generation of the mass and width measurement optimisation procedure. Right: distribution of the scan point energies for the last generation (green; vertical scale arbitrary) compared with the reference cross section template (red). Figure reproduced from [2].
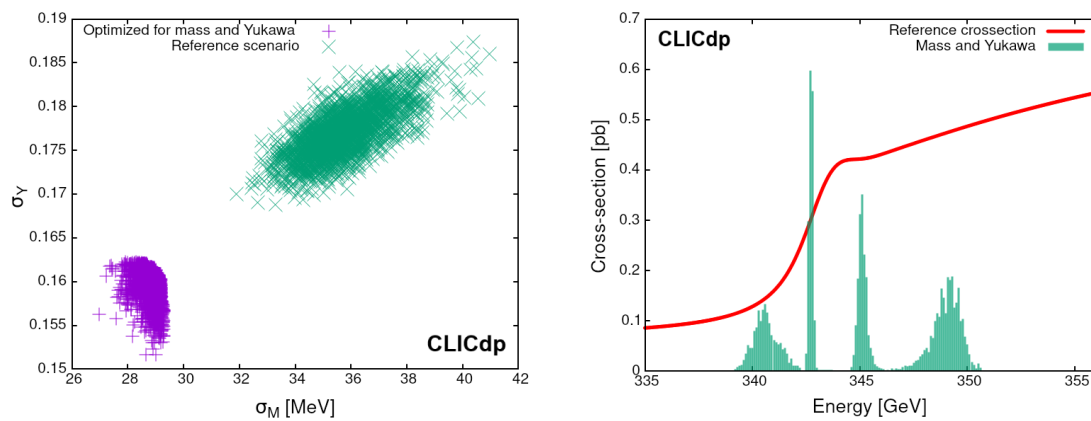

Figure 2: As in Fig. 1 but for mass and Yukawa coupling measurement optimisation. Figure reproduced from [2].

28 MeV, while the Yukawa coupling uncertainty changes from 0.18 to 0.16. The very small improvement for the Yukawa coupling can be explained, when we look at Fig. 2. On the right plot we can clearly see an additional region above the threshold, which is responsible for the determination of the Yukawa coupling. The benchmark scenario from the initial generation has half of its points in this region, not making significant improvement possible.

5 Optimised scan scenarios

To verify the optimisation results we select one test scenario from the final generation for each of the two considered optimisation configurations (See Fig. 3). The algorithm converged well in both cases. For mass and width optimisation 99% of final generation scenarios had 5 scan points, while for mass and Yukawa 97% of them had 9 or 10 scan points. To confirm optimisation results we generated 20 000 pseudo-experiments using selected running scenarios.

Estimates from the optimisation procedure, where each scenario was evaluated based on three pseudo-data sets only, were confirmed with results from a large number of pseudo-
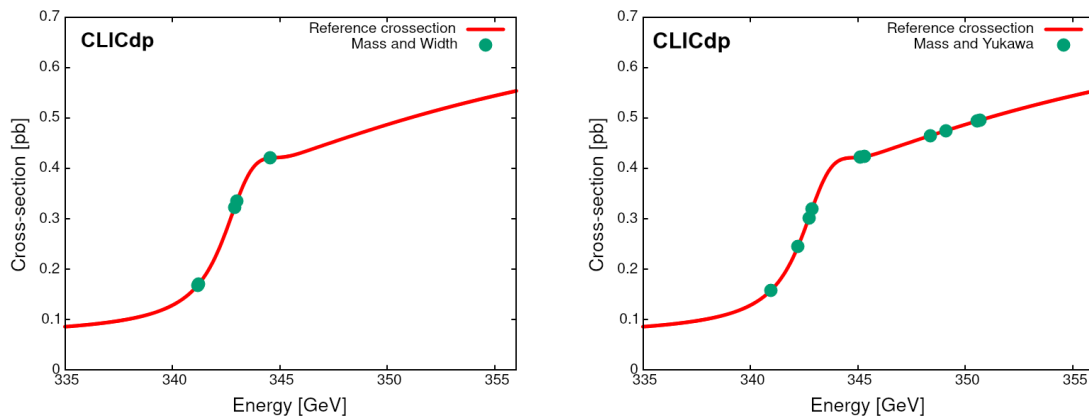
Figure 3: Scan points of the "best scenarios" selected from the last generation compared with the reference cross section template: (left) for mass and width measurement optimisation (5 point scenario: two points below, two in the middle and one above the threshold) and (right) for mass and Yukawa coupling measurement optimisation (10 point scenario). Figure reproduced from [2].
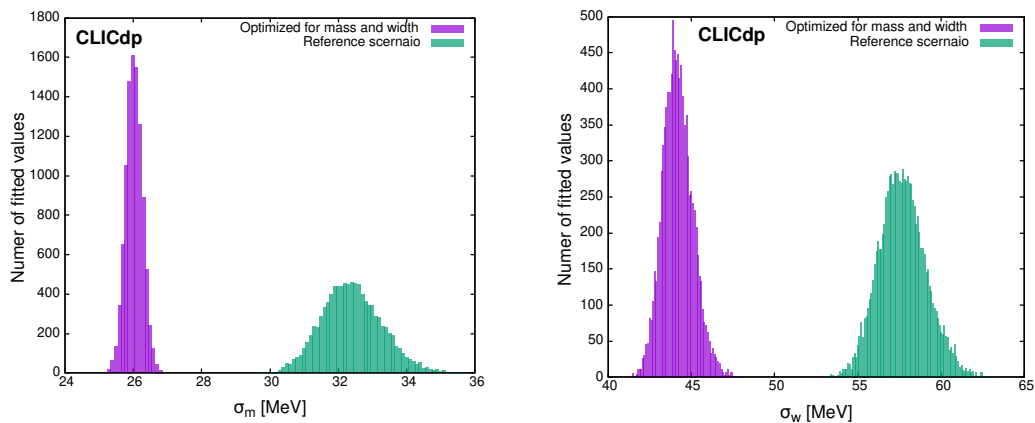

Figure 4: Mass (left) and width (right) uncertainty distributions in scenario optimised for mass and width measurement (see Fig. 3), collated with the the reference scenario's distributions. Figure reproduced from [2].

experiments. For the optimised mass-width scenario the average expected mass uncertainty is around 26 MeV, while for width it is around 44 MeV, see Fig. 4. Furthermore, uncertainty distributions are narrower than those for the reference scenario confirming stability of the fit and shows that it is less sensitive to the statistical fluctuations, which is the result of including three pseudo-experiments for performance evaluation of each Individual.

Also for the scenario when mass and Yukawa coupling measurement is optimised, results obtained from 20 000 of pseudo-experiments, shown in Fig. 5 good agreement with optimisation results is obtained (see Fig. 2). Unfortunately, uncertainty distribution for the mass measurement is significantly wider than the one obtained when mass and width measurement is optimised (Fig. 4). It is also slightly asymmetric, with a larger tail towards high uncertainty values. Nevertheless, the optimised scenario always provides higher mass measurement precision than the reference one.
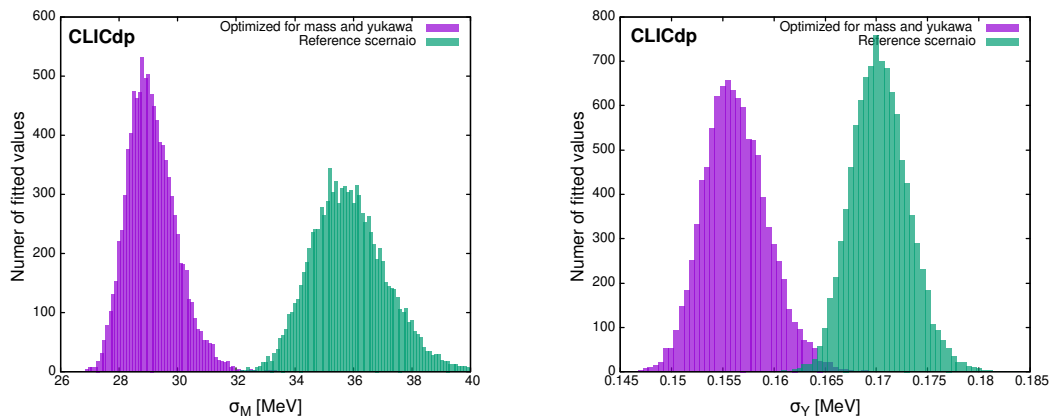
Figure 5: Mass (left) and Yukawa coupling (right) uncertainty distribution in scenario optimised for mass and Yukawa coupling measurement (see Fig. 3), collated with the the reference scenario's distributions. Figure reproduced from [2].

6 Conclusions

An optimisation procedure using a NSGA-II has been applied to the top-quark pair-production threshold scan. Each scan sequence (set of energy points with total equally distributed luminosity of 100 fb^{-1}) is assumed to be a genotype and for a phenotype, parameter fit results are used. Starting from the benchmark scenario (10 scan points, equally separated from each other), stable optimisation results are obtained from the genetic evolution for 30 generations and population size of 2000. By using this optimisation procedure, it was shown that it is possible to reduce the top-quark mass uncertainty by up to 20%, without increase of luminosity or loss of precision in determination of other parameters.

Acknowledgements

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