
Anonymous Report 2 on 2020-2-25 Invited Report

Requested changes:

1. The authors suggest that the quantum phase transition is the first-order. This statement does not seem to be sufficiently justified by the results presented in Fig. 8. Perhaps add further confirmation of the statement.

Reply: In order to reveal the location and order of the phase transition, we performed new sets of simulations and analyzed our energies more accurately. As discussed in details in response to the first referee, we find that the phase transition from the U(1) QSL to the Nematic phase takes place at $(J_{\triangle}/J_{\triangledown})^{c}=0.05$ at finite bond dimension up to our maximum affordable D=13. The scaling with bond dimension shows that the location of the phase transition does not change with bond dimension for D>10. Our new energy analysis now clearly shows a sharp discontinuity in the first order derivative of the ground state energy at the transition point (as shown with the dashed line in the new inset of Fig. 8-(a)) which is a clear signature of the first order phase transition. Let us further note that the spin-spin correlation on the links of the up and down triangles show an abrupt (discontinuous) change as depicted in Fig. 11 in the main text. This behavior is typical of the first-order phase transitions in which the physical quantities change suddenly (not continuously) at the transition point. Our finding for the first-order nature of the transition is also in agreement with that of the Ref. [44] in which they also observed signatures of first-order transitions on different cylinder geometries.

2. In Fig. 3 it would be useful to show the ground state energy obtained using other methods, similar to Fig. 4. E.g. show the energies obtained using DMRG by F. Pollmann et. al., and Yan et. al.

Reply: We thank the referee for his/her suggestion. We added the DMRG energies obtained by Pollmann et. al., Yan et. al. and Depenbrock et. al. to Fig. 3 to provide a better picture on how our simulations are compared against previous studies.

3. In Fig. 3. there is minus sign missing in the power laws of the fits shown in the inset.

Reply: We thank the referee for pointing this out. We fixed the missing minus sign both in the plot and in the main text.

4. Introduce acronyms SU and FU early (on page 4), and clarify the meaning. One can also remove CDMRG word from the figures for clarity.

Reply: The term SU for (simple-update) and FU for (full-Update) were defined in the second paragraph of Sec. 2 and the corresponding figures were also updated.

5. In most figures the results of Ref. [44] are shown as DMRG, but in Fig. 5 as Nematic-DMRG. Perhaps make this uniform throughout, and make a reference to [44] in the text for the figures.

Reply: The results of Ref. [44] is now shown as **DMRG-Repellin et al.** in all relevant figures.

6. Show results for the isotropic case in Table 1. Perhaps, make in bold the lowest energy value in each row.

Reply: The energies of different ansatzes for the isotropic point were added to the last row of Table .1. The DMRG results of Repellin et al. (last column) for all couplings are lower compared to other methods. However, strictly speaking, these energies are not variational of the infinite system and tend to increase with the diameter of the cylinders, as shown in Fig. 10 in the case of the effective model in the strong breathing limit. Besides, the next lowest energies belong to our D=13 PESS simulations. This was mentioned in the caption of the table but, for these reasons we would prefer to avoid making the DMRG results bold to avoid overemphasizing them.

7. In Ref. [44] the critical value of breathing anisotropy was found to be 0.14. It would be good to cite this value in the beginning of section 4 of the paper, and compare with the value of 0.05 found in this paper.

Reply: We added the critical value 0.14 from the DMRG simulations of Ref. [44] to the beginning of Sec. 4. However, since this value depends on the cylinder geometry used for DMRG calculation, we postponed further discussion to Sec. 5 where we added further discussion. (See also the response to the next question)

8. Perhaps add a more extended discussion of the reason why there is a discrepancy between the critical point found in this paper and Ref. [44]. The current explanation does not seem to be justified.

Reply: In Ref. [44], it has been discussed that the location of the critical points highly depends on the geometry of the cylinder used for DMRG simulations. They report the critical values 0.14, 0.19, 0.07, 0.05 which are obtained on YC8, YC8-2, YC10-2, and YC12-2 geometries, respectively. The main reason that different cylinder geometries lead to different critical points is that the boundary conditions of some of these cylinder geometries such as YC-2 do not allow the nematic pattern and translation symmetry to be present simultaneously. Therefore, the system undergoes a dimer instability to a nematic phase at different critical values for different geometries. However, we perform our tensor network simulations (both iPEPS and PESS) on the infinite plane and our simulations are not biased by any boundary condition. Our simulations for both iPEPS and PESS capture the critical point at $J_{\triangledown}/J_{\vartriangle}\sim0.05$ (see also response to the first referee) and does not tend to change or vanish for D>10 up to D=13 which was the maximum affordable bond dimension within our TN calculations. These extra discussions were added to Sec. 5 for better clarity of the paper.

9. If possible, show scaling of the gap for different values of anisotropy.

Reply: We thank the referee for his/her suggestion. Within the framework of our TN calculations, we are unable to calculate the gap. However as shown in Fig. 7-(b), we were able to calculate the long-range spin-spin correlation for different values of the couplings. The algebraic decay of correlation observed for different coupling signals a gapless ground state.