About the Simple Rotation Method Used in Ref. [30]

The mathematical evaluations given in Ref. [30] is based on the simple rotation method. I have pointed out the weak point of such a method in my manuscript (see from line 8 on page 4 to the last line on the same page). The usefulness of the simple rotation method is usually restricted to some particular cases.

The system adopted in Ref. [30] is a simple case where the masses of the two oscillatory systems are identical to each other: $m_1 = m_2 = 1$. For such a restricted case, the simple rotation method is applicable. However, for the case of $m_1 \neq m_2$, the simple rotation method cannot be used in the diagonalization of the Hamiltonian of the system. Let us see for this, from a strict evaluation.

If we let $m_1 \neq m_2$ from the system in Ref. [30], the associated Hamiltonian is given by

$$H = \frac{1}{2}\left(\frac{p_1^2}{m_1} + \frac{p_2^2}{m_2}\right) + \frac{1}{2}[m_1\omega_1^2(t)x_1^2 + m_2\omega_2^2(t)x_2^2] - J(t)x_1x_2. \tag{1}$$

We can transformation this Hamiltonian using the simple rotation method based on

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \tag{2}$$

$$\begin{pmatrix} \tilde{p}_1 \\ \tilde{p}_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}, \tag{3}$$

where

$$\alpha = \frac{1}{2}\tan^{-1}\left(\frac{2J}{m_1\omega_1^2 - m_2\omega_2^2}\right). \tag{4}$$

Then, by this transformation, the Hamiltonian becomes

$$H = \frac{1}{2}\left(\frac{\tilde{p}_1^2}{M_1} + \frac{\tilde{p}_2^2}{M_2}\right) + \frac{1}{2}[\Omega_1^2 y_1^2 + \Omega_2^2 y_2^2] + \beta\tilde{p}_1\tilde{p}_2, \tag{5}$$

where

$$\frac{1}{M_1} = \frac{\cos^2\alpha}{m_1} + \frac{\sin^2\alpha}{m_2}, \tag{6}$$

$$\frac{1}{M_2} = \frac{\sin^2\alpha}{m_1} + \frac{\cos^2\alpha}{m_2}, \tag{7}$$

$$\Omega_1^2 = m_1\omega_1^2\cos^2\alpha + m_2\omega_2^2\sin^2\alpha + J\sin(2\alpha), \tag{8}$$

$$\Omega_2^2 = m_1\omega_1^2\sin^2\alpha + m_2\omega_2^2\cos^2\alpha - J\sin(2\alpha), \tag{9}$$

$$\beta = \left(\frac{1}{m_1} - \frac{1}{m_2}\right)\cos\alpha\sin\alpha. \tag{10}$$

From Eq. (5), we see that the cross term $J(t)x_1x_2$ in the original expression of the Hamiltonian has been removed in the transformed Hamiltonian. However, a new cross term $\beta\tilde{p}_1\tilde{p}_2$ is appeared. Hence we cannot go further, because the Hamiltonian of the system cannot be diagonalized in this way. This is the weak point of the simple rotation method adopted in Ref. [30].