Reply to Report 2

We thank the referee for their positive assessment of our work and helpful comments. Our responses to the specific question the referee posed are as follows:

(1) The reason why we stopped our simulation at 80-120ms is two-fold. First, we aimed to consider broadly the same range as the experiments of Refs [10,14] (35ms) and [15] (60ms). Second, our simulations reveal a very strong finite-size effect that occurs around 45ms in Fig. 11(a) occurring at the same time as the breathing motion of the trap shown in Fig. 13: the amplitude of the Josephson oscillations begin to increase again. The damping time was extracted from the initial decay of the oscillations, which occurs before this reemergence begins. The latter has not been observed in the existing experiments, and our result therefore constitutes a new and interesting prediction in the parameter regime accessible to us (which as we stress is not the same as the one in the experiments). The time scale of the simulations in section 6 is such that the reemergence effects are clearly shown. The referee's question alerted us to the fact that we had not sufficiently stressed the fact in our parameter regime the Josephson oscillations revive, and that the relevant time scale appears to be related to the breathing motion of the trap. We have therefore extended our discussion of this point.

(2) As the referee points out, the experiments observe a damping of the fluctuations of the relative phase. However, it is important to keep in mind that this observation is based on a particular definition of the relative phase and its fluctuation for each single-shot experiment. As we explain in some detail in section 3, our method does not provide us with access to single-shot experiments, and hence we extract the relative phase from an average over many absorption images. This precludes us from making any statements about fluctuations of the phase on the basis of the results presented in our manuscript unless we make additional assumptions. The HF approximation should of course allow one to develop a theory for single-shot experiments as it provides, in principle, an expression for the full quantum state. We have considered this problem and found that it is not a simple matter. In particular, unlike in the low-energy limit (cf [38]), the eigenvalue problem of the operators entering the expression of the time-of-flight density (31) is non-trivial. Given these facts we decided against making any statements on the fluctuations of the relative phase.

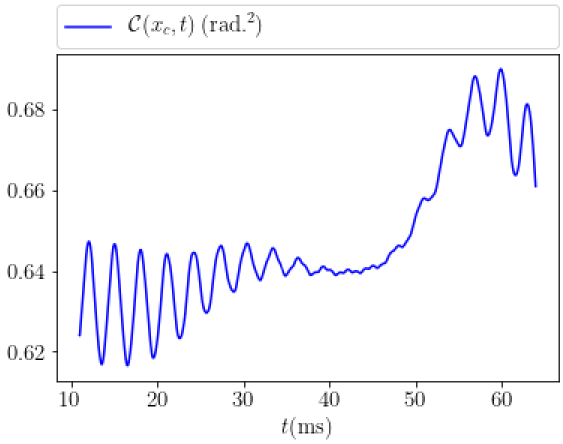
The referee's argument assumes that density fluctuations are negligible. Given the relatively small particle numbers we consider in order to remain within the regime of validity of the HF approximation we see no reason why this should be the case. However, if it were true then one could use a number-phase representation to obtain

$$\frac{C_{LR}(x,x,t)}{\sqrt{C_{LL}(x,x,t)C_{RR}(x,x,t)}} \approx \langle e^{i\Phi(x,t)} \rangle = e^{i\langle \Phi(x,t)\rangle - \frac{1}{2}\langle \Phi^2(x,t)\rangle_c + \dots} \ . \tag{1}$$

In order to proceed we then need to make a further assumption that the third and higher cumulants of the phase defined in this way are small. If this holds true we obtain

$$\begin{aligned} \langle \Phi(x,t)\rangle &\approx \arg C_{LR}(x,x,t) \ , \\ \langle \Phi^2(x,t)\rangle_c &\approx -2\log\left[\frac{|C_{LR}(x,x,t)|}{\sqrt{C_{LL}(x,x,t)C_{RR}(x,x,t)}}\right] \end{aligned} \tag{2}$$

We have computed the right hand side of (2) in the relevant parameter regime and timescales and find damped, oscillating behaviour around a fairly large, finite value $\langle \Phi^2(x,t)\rangle_c \approx 0.62$ as shown in the figure below. These results suggest that if all the aforementioned assumptions were to hold,



the variance of phase fluctuations remains large throughout the time evolution.

(3) We agree with the referee's comment. Our remark was meant to refer to the regime where Δ_1 is small and we have changed it in order to make this clear. It now reads *In particular, in the regime where* Δ_1 *is small the value of* Δ_2 *provides an indication of the strength of connected* 4*-point correlations, which vanish in HF.*

(4) We are grateful to the referee for spotting this typo, which has been corrected.