

# Author response to the second referee report

The referee points out that it is possible to construct states in *electromagnetism* that coincide outside a large ball but do not coincide within the ball. This is *not* a counterexample to the results of our paper. We devote an entire subsection (subsection 4.2) to explaining that nongravitational gauge theories do allow such “split states”. We mention this at several other points in the paper, and even in the Introduction. The referee’s example cannot be generalized to theories of gravity and our paper is about the special properties of gravitational theories.

We first review the referee’s construction and then explain why it cannot be generalized to gravity.

## 1 Electromagnetism

First, let us briefly review the referee’s construction, which is performed in the context of flat space. We canonically quantize a electromagnetic field,  $A_\mu(x)$ , coupled to matter degrees of freedom that we denote as  $\phi(x)$ . Denoting  $\Pi^0(x)$  and  $\Pi^i(x)$  as the canonical conjugate momentum to  $A_0(x)$  and  $A_i(x)$ , with  $i$  running over spatial directions, we immediately find the set of first class constraints.

$$\Pi^0(x) = 0; \quad \partial_i \Pi^i(x) + \rho(x) = 0, \quad (1)$$

where  $\rho(x)$  is the matter charge density. The second constraint is just the familiar Gauss law, since  $\Pi^i(x)$  is proportional to the electric field.

There are two conventional techniques of handling this constraint. One may impose a gauge condition as an additional constraint so that the first class constraints become second class constraints. This allows one to eliminate degrees of freedom. The second technique is to simply impose the constraints (1) on allowed wavefunctionals [1]. The referee appears to do both simultaneously, which we found confusing. Nevertheless, we agree with the physical aspects of the referee’s construction which we refine below.

Choosing the second technique above, we demand that wavefunctionals  $\Psi[A_i, \phi]$  satisfy

$$(\partial_i \Pi^i(x) + \rho(x))\Psi[A_i, \phi] = 0 \quad (2)$$

The first constraint in (1) is trivially satisfied by making the wavefunctional independent of  $A_0$ .

Now, we can choose the wavefunctional to be an arbitrary eigenfunctional of  $\Pi^i(x)$  so that it satisfies

$$\Pi^i(x)\Psi[A_i, \phi] = -E^i(x)\Psi[A_i, \phi], \quad (3)$$

where  $E^i(x)$  is some function in space. Note that, on the left hand side above,  $\Pi^i(x)$  is an operator that acts on the wavefunctional, whereas  $E^i(x)$  is just a function. For any wavefunctional

satisfying (3), we can then look for eigenfunctionals of  $\rho(x)$  that satisfy

$$\rho(x)\Psi[A_i, \phi_i] = \partial_i E^i(x)\Psi[A_i, \phi_i]. \quad (4)$$

The key point is that there is no constraint on the eigenvalues of the charge density. Moreover, the charge density can be simultaneously diagonalized at different points since

$$[\rho(x), \rho(x')] = 0. \quad (5)$$

Note the the data at infinity still imposes a constraint on the bulk data since every wavefunctional must satisfy

$$\left( \int \Pi^i(x) dS^i + \int \rho(x) d^d x \right) \Psi[A_i, \phi_i] = 0. \quad (6)$$

where  $dS^i$  provides a measure for integrating over the sphere at infinity. We need to ensure this constraint is satisfied, but this analysis shows that in electromagnetism it is clearly possible to find wavefunctionals that coincide in the exterior of a ball but not in the interior.

## 2 Impossibility of generalization to gravity

First we note that the constraints in a theory of gravity are more complicated than in electromagnetism. So even at a technical level, the generalization is not as simple as suggested by the referee. In particular, equation (18) in the referee report is incorrect, and we correct it here.

There are two types of constraints, commonly called the momentum constraint and the Hamiltonian constraint. We perform a  $d + 1$  split of the metric and denoting the  $d$  metric as  $\gamma_{ij}$  and the conjugate momentum as  $\Pi^{ij}$ . In addition, we denote the matter degrees of freedom by  $\phi$ . Then these constraints can be written as

$$H(x)\Psi[\gamma_{ij}, \phi] = 0; \quad \chi^i(x)\Psi[\gamma_{ij}, \phi] = 0 \quad (7)$$

where

$$\begin{aligned} \mathcal{H} &= 8\pi G \frac{1}{\sqrt{\gamma}} (\gamma_{ik}\gamma_{jl} + \gamma_{il}\gamma_{jk} - \gamma_{ij}\gamma_{kl}) \Pi^{ij}(x)\Pi^{kl}(x) - \frac{1}{16\pi G} \sqrt{\gamma} R^{(d)}(x) + \frac{\sqrt{\gamma}}{8\pi G} \Lambda + \mathcal{H}_{\text{matter}}(x) \\ \chi^i &= -D_j \Pi^{ij}(x) - \mathcal{P}_{\text{matter}}^i(x) \end{aligned} \quad (8)$$

where  $R^{(d)}$  is the  $d$ -dimensional Riemann tensor,  $\mathcal{H}_{\text{matter}}(x)$  is the Hamiltonian density of the matter sector and  $\mathcal{P}_{\text{matter}}^i(x)$  is its momentum density. We have also included a cosmological constant term,  $\Lambda$ . Note the momentum constraints  $\chi^i(x)$  are similar to the Gauss constraint in electromagnetism, but the Hamiltonian constraint,  $H(x)$  has no analogue in electromagnetism.

It is possible to analyze these constraints in the linearized limit as will be explored in forthcoming work [2]. Nevertheless, the physical point is quite simple. By manipulating the constraints, we again learn that data at infinity places a condition on the bulk energy density. Writing the metric fluctuation away from AdS as  $\gamma_{ij} = \gamma_{ij}^0 + h_{ij}$  we find a condition of the form

$$\frac{1}{16\pi G} \int N(D^i h_{ij} - D_j h_i^i) dS^j \Psi[\gamma_{ij}, \phi] = \int d^d x \sqrt{\gamma^0} H_{\text{bulk}}(x) \Psi[\gamma_{ij}, \phi] \quad (9)$$

On the left we have the ADM expression for the energy, which is defined by integrating the metric fluctuation at the conformal boundary of AdS. On the right we have defined a bulk energy density,  $H_{\text{bulk}}(x)$  that not only includes the matter energy density that appeared in (8) but *also* the energy density of the transverse traceless excitations of gravitons.

Now we see the key physical difference between gravity and electromagnetism. Due to the positivity of energy, specifying the ADM energy through data at infinity provides a powerful constraint. For instance, say that we consider a wavefunctional so that the ADM energy vanishes. This uniquely fixes the wavefunctional to be that of the vacuum of global AdS. In particular, we *cannot* choose wavefunctionals where  $H_{\text{bulk}}(x)$  varies arbitrarily in space. This is entirely different from electromagnetism where setting the charge at infinity (6) allowed an infinite amount of freedom to choose the bulk charge density.

At a technical level, this is because one cannot choose a wavefunctional where  $H_{\text{bulk}}(x)$  takes definite values at different points in space simultaneously. This is different from (4). It is for this reason that the ground state wavefunctional in a theory cannot be obtained by independently minimizing the energy at each spatial point. The ground state wavefunctional in AdS is unique and it is the only state that is consistent with the specification at infinity that the ADM energy vanishes.

Similarly, if we fix the asymptotic energy to be some  $E$ , in AdS units, then there are a *finite* number of wavefunctionals that satisfy (9). This corresponds to the finite degeneracy of an energy eigenstate in global AdS. This degeneracy can be broken by measuring correlators of matter fields as we do in our paper, which allows an identification of the state using correlators at infinity.

### 3 Summary

To summarize, we have provided a technical refinement of the referee’s construction of a split state in electromagnetism. Such a construction does not contradict any result in our paper.

This construction cannot be generalized to gravity for the reasons explained in section 2. So it does not constitute a “counterexample” to our protocol.

## References

- [1] P.A.M. Dirac, *Lectures on quantum mechanics*, Dover, 2001.
- [2] C. Chowdhury, V. Godet, K. Papadodimas, O. Papadoulaki and S. Raju, *in progress* (2021)

# Transcript of Discussion with the Referee

## Description:

This is a copy of the correspondence between the authors and the referee in the context of the Scipost submission: "A physical protocol for observers near the boundary to obtain bulk information in quantum gravity".

The conversation started when the authors first contacted the referee to ask if Prof. Mathur had additional questions beyond the ones he had mentioned in the referee report.

Two pdf notes are referred to in the end of the email-thread, and they are attached at the end of the document.

*For the latest version of the referee's arguments, and for our response, we refer the reader to the second referee report and the note available at the start of this pdf file.*

Very minor edits have been made to the original correspondence for clarity.

**Subject:** Re: discussion

**Date:** Thu, 22 Oct 2020 20:40:17 +0000

**From:** Mathur, Samir <mathur.16@osu.edu>

**To:** Suvrat Raju <suvrat@icts.res.in>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>, Papadoulaki Olga <papadoulaki@ictp.it>

Hi Suvrat,

Let me spend some time understanding what you have written in your reply.

I would like to clarify one point in what you say about Appendix A. Are you saying that by applying operators near the boundary in a small time band I can create a state describing a localized wavefunction of an atom in the center of AdS? That is, are you saying that a set of observers can create an object in a region from which they are spatially separated?

If you are not saying this, then could you clarify what you mean when you say that operators in the small time band near the boundary can create any state?

Best

samir

----- Forwarded Message -----

**Subject:** Re: discussion

**Date:** Fri, 23 Oct 2020 06:16:38 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Mathur, Samir <mathur.16@osu.edu>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>, Papadoulaki Olga <papadoulaki@ictp.it>

Hi Samir,

Thanks.

> I would like to clarify one point in what you say about Appendix A. Are  
> you saying that by applying operators near the boundary in a small time  
> band I can create a state describing a localized wavefunction of an atom  
> in the center of AdS?

Yes, this is correct. The statement is that if  $X$  runs over the set of all operators in the small time band then  $X|0\rangle$  is dense in the full Hilbert space. But this is not a new or a surprising result. It also does not violate locality or require gravity. In fact, this is a trivial example of the so-called "Reeh-Schlieder theorem" that people in axiomatic QFT --- for whom microcausality is a very important axiom --- use all the time.

I should hasten to add that we are not invoking any theorem in Appendix A. We have a separate proof, and we also verified the result numerically. Our code is available on Zenodo (linked in the paper).

> That is, are you saying that a set of observers  
> can create an object in a region from which they are spatially separated?

No, this is not correct. In our paper, as is standard, the observers are allowed to only act with \*unitary\* operators,  $U$ , not with arbitrary operators. This is because unitary operations are the physically allowed transformations of state.

By acting with arbitrary  $U$  on  $|0\rangle$ , we cannot create all states.

There is an unfortunate problem of language here. We use words like "act" and "create" to refer to the application of arbitrary operators on a state. But, physically, only unitary operations are allowed, not arbitrary operations.

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Let me give a qubit example to clarify Appendix A, which you might appreciate.

Take the entangled state of 2 qubits.

$|e\rangle = a|00\rangle + b|11\rangle$

with  $a, b$  nonzero.

Then it is a mathematical fact that by applying operators from qubit 1 to this state, I can generate all four vectors in the Hilbert space.

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 $(1 + \sigma^z) |e\rangle = 2a |00\rangle;$   
 $(1 - \sigma^z) |e\rangle = 2b |11\rangle;$   
 $\sigma^x + i\sigma^y |e\rangle = 2a |10\rangle;$   
 $\sigma^x - i\sigma^y |e\rangle = 2b |00\rangle;$   
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where all sigma matrices are acting only on the first qubit.

I wanted to make two obvious points.

1) This construction, which is just straightforward quantum-mechanics, obviously does not imply any loss of locality. This is because, physically, an observer who has access to only the first qubit can only act with unitary excitations on the first qubit and cannot "act" with arbitrary operators and so cannot change the probability distribution of the second qubit.

2) An observer with access to only the first qubit *cannot* determine the full state of the system. In particular, any measurement on the first qubit --- including expectation values of the operators shown in above --- is insensitive to the action of a unitary operator on the second qubit.

Appendix A is just making the simple and well-known analogous point in a QFT in AdS.

Appendix A is a technical input to our result in the main text. But that result goes beyond Appendix A and shows that by using physical operations, the observers can identify any state in the bulk.

regards,

Suvrat

**Subject:** Discussion  
**Date:** Thu, 29 Oct 2020 18:09:31 -0400  
**From:** Samir Mathur <samirdmathur@gmail.com>  
**To:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in, Suvrat Raju <suvrat@icts.res.in>

Hi Suvrat,

Thanks for your mail; I also read the reply you posted on sci-post. The statements in your mail look fine to me, but I am struggling to understand what is the new result that you are arguing for. So let me write here what I understand, and you can let me know where I do not understand your argument:

(A) I think that to follow what you are saying, I should note three cases: a local field theory

like a scalar field (b) A gauge theory like electromagnetism (c) The theory with gravity.

(B) Even in (a) using the positive frequency modes do not create a localized excitation. This is what leads to the nonvanishing of the Feynman propagator outside the light cone. This nonvanishing however does not allow an observer to send signals/receive outside the light cone.

(C) If there is a spin at the origin, its value can be detected from the frame dragging effect on  $g_{\phi}$  near infinity. This does not however change the fact that this spin can be entangled with another spin near infinity. By doing Stern-Gerlach measurements with the spin at infinity one can check that it is in an entangled state and not an unentangled state; this fact of being entangled does not change just because the spin at the center leads to a nonzero  $g_{\phi}$ .

(D) In classical electrodynamics suppose one tries to find the charge distribution by looking at the potential at infinity. If we have two concentric shells, with charges  $Q$  and  $-Q$ , then the potential at infinity is zero, so we cannot distinguish these shells from the situation with no charges. In quantum mechanics, suppose we take an electron and a positron is spherically symmetric wavefunctions at radii  $r_1, r_2$ . The average potential at infinity will vanish, but the dipole moment will have a distribution: there will be a different dipole for different positions  $(r_1, \theta_1, \phi_1)$  and  $(r_2, \theta_2, \phi_2)$ . Thus one can learn something about the wavefunction in the quantum theory, but this is expected: there is more structure in the quantum theory description (i.e. we have the wavefunction instead of just a classical density) and correspondingly more data that can be recovered from multi-operator correlation functions at infinity.

(E) It seems to me that you are looking for similar effects where the long range force is gravity. With the traditional black hole metric there is a problem: there are infinitely many states close to each other in their moments at infinity. Look for instance at figure 5(b) of arxiv:1402.6378, and let each quantum of the  $r=\text{const}$  part of the slice be an s wave peaked at some location. When these quanta are deep down the throat, they have arbitrarily small differences in moments at infinity. This is what I was pointing out in the context where you say that there are a finite number of primaries below some energy. The black hole puzzle is equivalent to the statement that the traditional geometry gives infinitely many states accumulating at the same energy. (There is no such accumulation of states when you have a fuzzball since there is no horizon, but there is no puzzle to solve anyway if we have fuzzballs in a theory since the fuzzball is just like a piece of coal.)

(F) So I remain confused about what is the new physics point that you are making ... I have just tried to state some general facts above, and I am guessing that you agree with all of these, but maybe these will help to narrow down to what you are saying is the new observation you make.

best

samir

**Subject:** Re: Discussion

**Date:** Fri, 30 Oct 2020 10:29:27 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Samir Mathur <samirdmathur@gmail.com>, papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Dear Samir,

Thank you for your email. Our main conclusion is as follows.

Let  $U$  be a unitary operator made up of fields near  $r = 0$ , at  $t = 0$ . Consider the state  $U|0\rangle$ . We are considering low-energy unitaries, so the energy of  $U|0\rangle$  is well below the Planck scale. For instance,

$$U = e^{i \int \phi(r,t=0) f(r) d r}$$

where  $f(r)$  has support near  $r = 0$ .

The main result is that by making observations near  $r = \infty$ , and for  $t$  in  $[0, \epsilon]$ , one can completely identify the state  $U|0\rangle$ .

There is no analogue of this for local scalar field theories. This is because, if  $X$  is any operator near the boundary we have

$$\langle 0| U^\dagger X U |0\rangle = \langle 0|X|0\rangle$$

as an exact identity by microcausality. So in scalar field theories, one cannot identify  $U|0\rangle$  or even distinguish it from  $|0\rangle$  by making any conceivable observation.

In gauge theories, as you say, if one has charged excitations one gets some information by measuring the probability distribution of the charge. But one could take  $U$  to be a Wilson loop operator near  $r = 0$ . Then again, even in a gauge theory we would find that there is no distinction between  $U|0\rangle$  and  $|0\rangle$  for observables near the boundary. So it is again possible to "hide" information in gauge theories.

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Note that even in gravity, identifying the state is not just a question of measuring the energy. Even at low-energies, there are many exact degeneracies in the energy spectrum.

Moreover, there are also phases, such as



$$e^{i\theta_1} |E_1\rangle + e^{i\theta_2} |E_2\rangle$$

which cannot be resolved by any energy measurement.

We show that when uses the protocol in our paper, which involves measurements of energy \*and\* action by low-energy unitaries near the boundary, one can resolve all these degeneracies and also determine these phases.

I agree with your points A-D below. But in E, you return to the issue of black holes. You are right that our protocol is more difficult to carry out about black holes, but in this paper, our focus is not on black holes at all as we emphasize a number of times 2.3. (We can do this even more in the revised version!)

So could you please let us know if you agree with our result in this paper: if one restricts to low-energy states in AdS (below Planck scale) then observations in a small time band in  $[0, \epsilon]$  near  $r \rightarrow \infty$  are sufficient to completely identify the state in a theory of gravity, but not sufficient in any nongravitational theory.

regards,

Suvrat

P.S: We did discuss black holes as well in a separate paper

<https://arxiv.org/pdf/2002.02448.pdf>

There, a formal proof is given that by measuring sufficiently complicated operators near the boundary one would be able to also extract information about the states that you mention (where there is a pile-up of energies.)

I would be very glad to discuss black holes but if we could agree on the low-energy physics in our current paper, that would be great and we could then move on to black holes.

**Subject:** Re: Discussion

**Date:** Fri, 30 Oct 2020 14:20:48 -0400

**From:** Samir Mathur <samirdmathur@gmail.com>

**To:** Suvrat Raju <suvrat@icts.res.in>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Hi Suvrat,

Thanks for the reply ... I am still struggling to get the physics here, so let us make it more specific.

(a) Can I take the example of a spin at  $r=0$ ? I can detect the value of this spin by looking at

$g_{\{\phi\}}$  at infinity. Is this an example of the kind of measurements that you are thinking about?

(b) Suppose there a a magnetic field  $B$  near  $r=0$ , which flips this spin at a time near  $t=0$ . Are you asking how to measure this change from infinity (at a time near  $r=0$ )?

(c) The angular momentum is conserved, so  $g_{\{\phi\}}$  will not change immediately at infinity. (The angular momentum of the spin changes but that of the apparatus producing  $B$  changes in the opposite way.) What operators will you measure (at large  $r$  near  $t=0$ ) to see the flip of the spin?

(d) I am wondering if you are doing the following. The entire system of the spin and the magnet producing  $B$  has some set of states  $E_n$ , and you seek to find the state  $\sum C_n E_n$  by looking at moments of the stress tensor etc at infinity. Then knowing the  $C_n$  you can compute the state at a later time, thus predicting that the spin will flip. Is this what you wish to do? This would be equivalent to saying that you find the state using its energy (and other moments of the stress tensor) at infinity.

(e) Can you expand the above example to help me understand what you are trying to say?

best

samir

**Subject:** Re: Discussion

**Date:** Sat, 31 Oct 2020 09:16:06 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Samir Mathur <samirdmathur@gmail.com>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Dear Samir,

Thanks very much for your email.

Let me start with (d).

> (d) I am wondering if you are doing the following. The entire system of  
> the spin and the magnet producing  $B$  has some set of states  $E_n$ , and you  
> seek to find the state  $\sum C_n E_n$

This is exactly the right idea. The only caveat is that our protocol does not involve looking at moments of the energy. Looking at moments of the energy is insufficient to get  $c_n$  because of two problems

1) Degeneracies: There are states with approximately, or even exactly the same energy. eg. for a massless scalar field in  $AdS_5$ , the state, corresponding to the operator  $O$  and a 4th-level-descendant of  $O$ , both have energy 8. Obviously energy measurements cannot distinguish these two.

2) Phases: if we consider  $\langle H^q \rangle$  in the state  $\sum c_n |E_n\rangle$  we get  $\sum_n |c_n|^2 (E_n)^q$ . This is insensitive to the phase of  $c_n$ , which are important (see below).

So we need to surmount these problems. The main point of our protocol is that by the simple process of the action of a low-energy unitary near the boundary followed by an energy measurement, we can get both the magnitude and phase of the coefficients  $c_n$ . This is the main result of section 3.

Let me now turn to your specific example. If I understand correctly, there is a spin interacting with a magnetic field so that after some time, the spin flips. In the example you mention, neither  $\int g_{\phi}$  nor  $\int g_t$  will change near the boundary since both energy and angular momentum are conserved.

But, indeed, the time evolution will take  $\sum c_n |E_n\rangle \rightarrow \sum c_n e^{-i E_n t} |E_n\rangle$

Note that only the phases of the  $c_n$  have changed. With one choice of phase for  $c_n$ , the spin is up and the B-field has less angular momentum. With another choice, the spin is down and the B-field has picked up some angular momentum.

Since our protocol can read off the phases, we can determine what the state is, and which of the two possibilities above is the right one.

regards,

Suvrat

**Subject:** Re: Discussion

**Date:** Sat, 31 Oct 2020 23:10:19 -0400

**From:** Samir Mathur <samirdmathur@gmail.com>

**To:** Suvrat Raju <suvrat@icts.res.in>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Hi Suvrat,

Thanks for the mail. Let me restate what I followed from your last mail, and then continue with a question:

(A) I understand that you wish to identify the state  $|\psi\rangle = \sum c_n |E_n\rangle$ , and thus predict its evolution at  $r=0$  using the known Hamiltonian of the theory. In this way you do not violate causality to learn what will happen at  $r=0$ , since you do not directly look for a signal to emerge from  $r=0$  to the large radius  $R$  where the observers are.

(B) Let me understand what you were saying about global modes. Let there be two real scalar free fields  $\phi_1, \phi_2$  coupled to gravity. Let me make the state in a following way:

(i) A quantum of  $\phi_1$  is sent in at the speed of light from infinity, so that it reaches  $r=0$  at  $t=0$  (i.e., the pulse moves along  $r+t=0$ ). The waveform is spherically symmetric ( $l=0$ ) and well peaked in  $r$ .

(ii) A set of observers make measurements near  $r=R$  (where  $R$  is large but not infinity), in the time interval  $(0, \epsilon)$  and wish to know if the quantum was of the field  $\phi_1$  or  $\phi_2$ .

Can you list the measurements they will need to make? Do you wish to look at the small tail of the wavefunction that will be left at  $r=R$  even when the peak of the wavefunction has reached  $r=0$ ? This can be done, but does not need gravity; this would be just local field theory. The gravitational field of both species is the same, so I wanted to know exactly what observations of the gravitational field you will make to decide between  $\phi_1$  and  $\phi_2$ .

(This example may help me understand how gravity is important in your analysis)

best

samir

**Subject:** Re: Discussion

**Date:** Sun, 1 Nov 2020 11:09:33 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Samir Mathur <samirdmathur@gmail.com>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Hi Samir,

Lets assume that the two states we want to distinguish are:

$$1) |\Psi_1\rangle = U_1 |0\rangle$$

$$2) |\Psi_2\rangle = U_2 |0\rangle$$

$U_1$  is a unitary made out of fields  $\phi_1$  near  $r = 0$ , at  $t = 0$ . and  $U_2$  is the same unitary made out of fields  $\phi_2$ . I'll assume for simplicity that  $\langle 0|U_1|0\rangle = \langle 0|U_2|0\rangle = 0$ .

Situation in LQFT:

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Say that excitation has compact support. Therefore there is \*no tail\* near the boundary. In particular, let  $O$  be any operator at  $t = 0$ , near  $r = \infty$  in a LQFT. For all such measurements we therefore have

$$\text{"LQFT locality": } \langle \Psi_1 | O | \Psi_1 \rangle = \langle \Psi_2 | O | \Psi_2 \rangle = \langle 0 | O | 0 \rangle$$

i.e. near the boundary both states look exactly like the vacuum.

Nevertheless, even in LQFT there exist distinct operators,  $X_1$  and  $X_2$  in the time band  $[0, \epsilon]$  so that

$$|\Psi_1\rangle = X_1 |0\rangle$$

$$|\Psi_2\rangle = X_2 |0\rangle$$

These operators  $X_1$  and  $X_2$  can be found explicitly using our Appendix A.  $X_1$  involves fields  $\phi_1$ , whereas  $X_2$  involves fields  $\phi_2$ .

This does not violate the relation "LQFT locality" above. In particular, note that

$$U_1^\dagger X_1 U_1 = X_1 \text{ and}$$

$$U_2^\dagger X_2 U_2 = X_2.$$

so the observers cannot just use  $X_1$  or  $X_2$  to distinguish the states.

I hope we are on the same page till this point. Everything till here is just in LQFT, so if there is some disagreement till this point, please let me know.

### Situation in Gravity

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Now, in gravity, our protocol works as follows.

1) The observers act with the unitary

$$e^{iJX_1}$$

measure the energy, and determine the probability of getting 0. Obviously, the second step of measuring the energy is possible only in gravity. Here in state  $|\Psi_1\rangle$  they would find that the  $O(J^2)$  term in this probability is "1". In the state  $|\Psi_2\rangle$  they would find that the  $O(J^2)$  term in this probability is 0.

2) Alternately, the observers could act with the unitary

$$e^{iJX_2}$$

and measure the energy and determine the probability of getting 0. Here in the state  $|\Psi_2\rangle$  they would find that the  $O(J^2)$  term in this probability is "1", whereas in  $|\Psi_1\rangle$  they would find that it is 0.

The algebra for these results is given in the introduction of our paper, so I will not repeat it here.

Therefore they are able to successfully distinguish the two states, which is clearly impossible in LQFT.

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If you want to think in even simpler terms, the point is that the correlators of

$$H \phi_1$$

and

$H \phi_2$

are different in the two states. If you would like to see the explicit expressions for these two-point functions, could you please take a look at equations 3.25 and 3.26 in our paper of <https://arxiv.org/pdf/2002.02448.pdf>

The setting is flat space but otherwise it is exactly what you describe. We have a pulse of light being sent in, which is near  $r = 0$ , at  $t = 0$ , and we need to determine its shape from near infinity at  $t = 0$ .

For the state with the pulse of the field  $\phi_1$ , the correlator of  $\langle H \phi_1 \rangle$  will be non zero. For the state with the pulse of the field  $\phi_2$ , the correlator of  $\langle H \phi_2 \rangle$  will be non zero. (Note that in both cases, there is no observable "tail" without the insertion of  $H$ .)

When people say the "gravitational field" is the same, usually they mean that  $\langle H^n \rangle$  is the same in the two states. This is correct but, as I mentioned, correlators of  $H$  together with other dynamical fields do succeed in distinguishing these states.

best,

Suvrat

**Subject:** Re: Discussion

**Date:** Sun, 1 Nov 2020 08:20:18 -0500

**From:** Samir Mathur <samirdmathur@gmail.com>

**To:** Suvrat Raju <suvrat@icts.res.in>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Hi Suvrat,

Thanks for the detailed mail ... let me pursue this example in full detail so that I may follow what you are saying. I will discretize everything and write the wavefunctional explicitly at each stage; let me know at what step you are saying something different:

(A) Let the spacetime be a set of lattice points  $x_1 \dots x_n$  with  $x_1$  being at the origin and  $x_n$  at the boundary radius  $R$ . (I really mean the 3-d case but am writing just 1-d for simplicity; you should just extend the notation to 3-d at every stage)

(B) The scalar field  $\phi_1$  is modelled as a ball at each site joined by a spring to its left and right neighbors. Waves along this set of balls and springs travel at the speed of light, and this gives an explicit model of my scalar field which I can put on a computer if I wish. Similarly, a second set of balls and springs models  $\phi_2$ .

(C) The vacuum wavefunctional has the form  $\Psi[\phi_i(x), \phi_2(x), g(x)]$ . More explicitly,

this means the following.

- (i) The scalar field  $\phi_1$  is given by a set of different configurations  $\phi_1(x)$ .
- (ii) Each of these configurations is a set of  $n$  real numbers  $\{\phi_1(x_1), \dots, \phi_1(x_n)\}$  which describe the displacement of the balls at their respective positions  $x_n$ .
- (iii) A wavefunctional  $\Psi[\phi(x)]$  assigns a complex number to each such set of numbers  $\phi_1(x) = \{\phi_1(x_1), \dots, \phi_1(x_n)\}$
- (iv) Since we have three fields involved,  $\phi_1(x)$ ,  $\phi_2(x)$ ,  $g(x)$ , the wavefunctional  $\Psi$  will assign a complex number to each set of numbers  $\{\{\phi_1(x_1), \dots, \phi_1(x_n)\}, \{\phi_2(x_1), \dots, \phi_2(x_n)\}, \{g(x_1), \dots, g(x_n)\}\}$

(D) Start with the vacuum wavefunctional. There are certainly correlations in the above wavefunctional that stretch from  $x_1$  to  $x_n$ . But causality is still maintained. Let us see how this should happen.

(E) At  $t=0$  we make a unitary operation at  $x_1$ ; for convenience I will call this location the Left side of my space. Thus each field configuration  $\phi_1(x)$  changes as

$$\{\phi_1(x_1), \dots, \phi_1(x_n)\} \rightarrow \{\phi_{1L}(x_1), \dots, \phi_1(x_n)\}$$

where the subscript  $L$  on the first entry on the RHS means that this entry has been altered. None of the other entries have been altered in this field configuration. We call the modified field configuration  $\phi_{1L}(x)$ . We do not change  $\phi_2(x)$  at all. The metric deforms everywhere; we call its configuration  $g'(x) = \{g'(x_1) \dots g'(x_n)\}$ . The full deformed wavefunction is called  $\Psi_L$ ; note again that in this wavefunctional the configurations  $\phi_1$  are altered only on the left and not elsewhere. (This follows from the locality of the applied operator).

(F) Now suppose an observer at the Right endpoint  $x_n$  wants to find out if  $\phi_1$  or  $\phi_2$  was altered in the above way. His local operations affect the waveforms  $\{\phi_{1L}(x_1), \dots, \phi_1(x_n)\}$  and  $\{\phi_2(x_1), \dots, \phi_2(x_n)\}$  only at the endpoint  $x_n$ . Of course each change in  $\phi_1$ ,  $\phi_2$  deforms  $g$  everywhere.

(G) Now perhaps you can tell me explicitly how the observer can know if the Left operation has deformed  $\phi_1$  and not  $\phi_2$ . I am trying to understand which numbers in the above set of variables are changed by your operations.

The difficulty I am having is that the initial action changes the waveform of  $\phi_1$  only at  $x_1$ , and I do not follow how change this can be 'undone' by the local operation at  $x_n$  to bring the  $\phi_1$  wavefunctional back to the vacuum state. I think you will simply end up with a deformation of each configuration  $\{\phi_1(x_1), \dots, \phi_1(x_n)\}$  to configuration which is deformed both at the Left and the Right

$$\{\phi_1(x_1), \dots, \phi_1(x_n)\} \rightarrow \{\phi_{1L}(x_1), \dots, \phi_1(x_n)\} \rightarrow \{\phi_{1L}(x_1), \dots, \phi_{1R}(x_n)\}$$

The metric will respond to these deformations, but the final  $g''$  will be the same regardless

of which Left and Right operations were performed; i.e., which of the fields I applied at  $x_1$  and which I applied at  $x_n$ .

(H) So I have just restated my core confusion above ... clarifying it in the above language should help me ...

samir

**Subject:** Re: Discussion

**Date:** Sun, 1 Nov 2020 20:36:58 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Samir Mathur <samirdmathur@gmail.com>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Dear Samir,

Thanks for your email. We are now moving somewhat away from the formalism of our paper to a discussion of how this procedure is implemented in the WdW formalism. We have not worked this issue out in detail, but we have thought about it a little and here is how we think the answer works in your example. I will be a little schematic below.

Let us throw away all the metric degrees of freedom except for  $\int g_{00}$ , which is the energy. I'll call this H. Then, the ground state wavefunction should look like

$$|\Psi_{00}\rangle[\phi_1(x_1) \phi_2(x_1) \dots \phi_1(x_n) \phi_2(x_n)] |H=0\rangle$$

I've called the wavefunctional for the scalar fields,  $|\Psi_{00}\rangle$ , because it corresponds to the ground state functional for both  $\phi_1$ , and  $\phi_2$ .

Now, you would like to change the amplitude for  $\phi_1$  at the point  $x_1$ . However, the action of this unitary necessarily creates a \*superposition\* of energy eigenstates (since localized excitations can never correspond to an exact energy eigenstate). So the new wavefunction necessarily looks like:

$$\sum_n c_n |\Psi_{n0}\rangle[\phi_1(x_1) \phi_2(x_1) \dots \phi_1(x_n) \phi_2(x_n)] |H=n\rangle$$

where  $|\Psi_{n0}\rangle$  is the wavefunctional corresponding to the  $n^{\text{th}}$  eigenstate for field  $\phi_1$  and the ground state for  $\phi_2$  and  $c_n$  are some coefficients depending on your unitary.

Please note the correlation between the matter part of the wavefunctional and the energy part.

Second, please also note that the difference between  $|\Psi_{n0}\rangle$  and  $|\Psi_{00}\rangle$  stretches all the way to infinity because energy eigenstates are always delocalized.

On the other hand if I act with a unitary on  $\phi_2$ , I will find

$$\sum_n c_n |\Psi_{0n}\rangle[\phi_1(x_1) \phi_2(x_1) \dots \phi_1(x_n) \phi_2(x_n)] |H=n\rangle$$



where the wavefunctionals that appear in the sum now correspond to  $n^{\text{th}}$  eigenstate for  $\psi_2$  and the ground state for  $\psi_1$ .

It is obvious that the correlators  $\langle H \psi_1(x_n) \rangle$  and  $\langle H \psi_2(x_n) \rangle$  differ between the two states and that we can use these correlators to determine the eigenstate.

This also displays the key difference between gravity and a LQFT. If we had not had the additional variable,  $E$ , we would simply have had the wavefunctionals

$$\sum_n c_n \Psi_{\{n\}} [\psi_1(x_1) \psi_2(x_1) \dots \psi_1(x_n) \psi_2(x_n)]$$

in one case and

$$\sum_n c_n \Psi_{\{0\}} [\psi_1(x_1) \psi_2(x_1) \dots \psi_1(x_n) \psi_2(x_n)]$$

and indeed these two LQFT wavefunctionals would have been identical at all points away from  $x_1$ . So no observable near infinity would have changed at all.

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I have been a little imprecise above since this WdW formalism has not been worked out in detail anywhere. In particular, the structure of the wavefunctionals is more complicated in the interacting theory than I have indicated. But I think the explanation above captures the essential physical aspects.

regards,

Suvrat

**Subject:** Re: Discussion

**Date:** Sun, 1 Nov 2020 20:39:28 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Samir Mathur <samirdmathur@gmail.com>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

sorry, I realized a notational issue in my email. I am using the variable  $n$  for the sum over eigenstates, but  $n$  also denoted the right-most point on the lattice. These are, of course, separate and I hope this will not cause confusion!

**Subject:** Re: Discussion

**Date:** Sun, 1 Nov 2020 12:45:04 -0500

**From:** Samir Mathur <samirdmathur@gmail.com>

**To:** Suvrat Raju <suvrat@icts.res.in>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Hi Suvrat,

Good, we may have narrowed things down to the point which is worrying me. I think that you are arriving at your conclusion because you are being schematic here, and if you actually computed the  $c_n$  for this toy model you will find that you cannot actually make the detection that you mention. This fact is very nonobvious in the energy basis, since as you say, the eigenfunctions of energy are very nonlocal. Causality is also very nonobvious in the operator language. That is why I asked about the wavefunctional language. What I actually think will happen is the following:

- (a) Suppose the excitation at the origin modifies  $\phi_1$  and not  $\phi_2$
- (b) The constant mode of  $g$  becomes nonzero at the same time, as you write, but would be the same whether I had modified  $\phi_1$  or  $\phi_2$
- (c) The new value of this constant  $g$  mode changes the wavefunctional at the outer boundary, but the change at  $x_n$  is the same in both the  $\phi_1$  and  $\phi_2$  wavefunctionals (since the wavefunctional at  $x_n$  is responding to the modification of the gravitational potential away from flat space, and this modification affects both fields the same way)
- (d) The local efforts at  $x=x_n$  by an observer can only detect the local values of  $\phi_1$ ,  $\phi_2$  at  $x_n$ , and these are affected by  $g$  the same way for both fields. Thus the detection you make here cannot tell you if  $\phi_1$  or  $\phi_2$  was modified at the origin.

Let us focus for a while on this picture of locality in the wavefunctional formulation and see if we are both using the same language and asking the same question. To reiterate, I would worry that by using the energy basis where things look nonlocal (but are really not) one can get confusing answers. The only way that locality is easily manifest is in terms of the local values for  $\phi_1$ ,  $\phi_2$  and  $g$  at points of a lattice. That is, write the full wavefunctional as numbers at a set of lattice points; then ask which numbers are changing and why.

best

samir

**Subject:** Re: Discussion

**Date:** Mon, 2 Nov 2020 08:34:12 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Samir Mathur <samirdmathur@gmail.com>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Dear Samir,

Thank you for your email.

I think you did not notice the entanglement between the matter and the energy part of the wavefunctional in my previous email. Let me explain again. To avoid confusion with previous notation, I will call the end of the spin-chain "M" rather than "n".

Situation in LQFT.

-----

The ground state wavefunctional for field 1, looks like

$$Q_{\{0\}}(\phi_1) = e^{\{-\sum_{\{i \neq j\}} \phi_1(x_i) \phi_1(x_j) G_{\{i j\}}\}}$$

where  $G_{\{i j\}}$  is a Greens function. Similarly there are the excited state wavefunctionals.

$$Q_1(\phi_1), Q_2(\phi_1) \dots Q_n(\phi_1).$$

There is a similar set of wavefunctions for field 2. I will denote them by  $R_0(\phi_2) \dots R_n(\phi_2(x_i))$

In LQFT if one makes an excitation in field 1, one gets  $F_{\{1\}} = \sum_n c_n Q_n(\phi_1) R_0(\phi_2)$

Even though this wavefunction is a sum over many energy eigenstates, it is a completely local excitation because it has the property that the coefficients  $c_n$  conspire so that all expectation values of  $\phi_1(x_i)$  and  $\phi_2(x_i)$  are unchanged except for  $\phi_1(x_1)$ . For instance, if we measure  $\phi(x_M)$  in the excited state above we find that the answer is  $\sum_{\{n,m\}} c_n c_m^* \langle Q_n | \phi(x_M) | Q_m \rangle$

which vanishes because the coefficients  $c_n$  ensure perfect cancellation away from point 1. This cancellation continues to happen until a time  $t = M$  when the excitation reaches  $M$ .

Similarly, one can make an excitation in field 2, and one would get  $F_{\{2\}} = \sum_n c_n Q_0(\phi_1) R_n(\phi_2)$

This has the property that all expectation values of  $\phi_1(x_i)$  and  $\phi_2(x_i)$  are unchanged at  $t = 0$  except for  $\phi_2(x_1)$ .

I hope we are on the same page so far. Please let me know if you disagree with anything above.

Situation in gravity.

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Now, let us keep track of the energy variable. The correct generalization of the wavefunctional  $F_1$  is

$$F_{\{1\}} = \sum_n c_n Q_n(\phi_1) R_0(\phi_2) |H=n\rangle$$

Please notice that there are \*correlations\* between the energy and the matter part. This wavefunctional has the property that if I measure  $H$  and find it to be  $n$ , the corresponding matter part is in the sector

$$Q_n(\phi_1) R_0(\phi_2)$$

where field 1 is contributing energy  $n$ , and field 2 is contributing energy  $0$ .

The correct generalization of the wavefunctional  $F_2$  is

$$F_2 = \sum_n c_n Q_0(\phi_1) R_n(\phi_2) |H=n\rangle$$

Please notice that the correlations between the energy and the matter part are now different. This wavefunctional has the property that if I measure H and find it to be n, the corresponding matter part is in the sector

$$Q_0(\phi_1) R_n(\phi_2)$$

In this case field 2 is contributing energy n and field 1 is contributing energy 0.

We see very clearly from here that the correlators of H with  $\phi_1$  and of H with  $\phi_2$  are different in the two cases. For the wavefunctional  $F_1$ , I would get

$$\langle H \phi_1(x_M) \rangle_{F_1} = \sum_{n,m} n c_n c_m^* \langle Q_n | \phi_1(x_M) | Q_m \rangle$$

This is nonzero because of the additional factor of n that appears in the sum. Without that factor, this would have vanished by the property of the  $c_n$  above.

Also

$$\langle H \phi_2(x_M) \rangle_{F_1} = 0$$

because  $\phi_2$  is always in its ground state in this wavefunction.

On the other hand for the wavefunction  $F_2$ , I would get

$$\langle H \phi_1(x_M) \rangle_{F_2} = 0$$

but

$$\langle H \phi_2(x_M) \rangle_{F_2}$$

nonzero as above.

I emphasize that in both cases  $\langle H \rangle = \sum |c_n|^2 n$  is the same and it is the correlator of H with the other matter fields that distinguishes the state.

Contrast with the picture in your email

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I think I also understand where we are disagreeing with you. Please correct me if I have misunderstood. If I understand correctly, the picture you have in mind is that we make some local excitation near point 1, and then adjust the metric everywhere according to the constraints. This would suggest that we write down some state

$$(Q_{exc}[\phi_1]) \otimes R_0[\phi_2] \otimes f_{grav}(H)$$

where the first part indicates a local excitation near "1" and where the last part of the wavefunctional,  $f_{grav}$ , has somehow been adjusted to respond to the excitation. If this was indeed a valid wavefunctional we would indeed not be able to identify states from near infinity.

But this wavefunctional does not satisfy the constraints. Since the energy can be measured through the metric, it must be \*correlated\* with the matter sector. By the Born rule, when we make an energy measurement, we can get different possible values. If we find that the value is some "n", then the matter sector must have precisely energy value, n. This is what forces us to the entangled wavefunctional

$$\sum (c_n Q_n[\phi_1] \otimes R_0[\phi_2] \otimes |H=n\rangle)$$

where the coefficients  $c_n$  are related to  $Q_{\text{exc}}$  by  $Q_{\text{exc}} = \sum c_n Q_n$

To say this another way, the constraints are not that the metric far away measures the "average energy", which would suggest  $\langle H_{\text{grav}} \rangle = \langle H_{\text{matter}} \rangle$

but rather the constraint is that

$$(H_{\text{grav}} - H_{\text{matter}})$$

annihilates the wavefunctional. This forces the entangled structure that I have written down, and allows our protocol to work.

Correlator computation

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I said we were being schematic only because we are not writing down the precise wavefunctional.

But the computation of  $\langle H_{\phi_1} \rangle$  and  $\langle H_{\phi_2} \rangle$  near the boundary, can be done precisely and explicitly in perturbation theory, and it clearly distinguishes between the two states that you mention. In the attached pdf, we are attaching this computation in AdS, which shows how a local excitation in the bulk can be distinguished by these correlators. Please take a look.

The wavefunctional picture above is completely consistent with this result.

Please also note that one doesn't even have to go to gravity to see the effect we are discussing here. This simple effect is visible even in QED. If one takes a state which has an electron at the center and another state with a muon at the center, then the electric field far away is the same. But correlators of  $\langle Q_e \rangle$  and  $\langle Q_\mu \rangle$  at infinity will distinguish between the two states.

In fact, this is the scenario that we discuss in section 4.2.1 of our paper.

I hope this helps to clarify the issue.

Thanks very much.

regards,

Suvrat

**Subject:** Re: Discussion

**Date:** Mon, 2 Nov 2020 09:29:44 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Samir Mathur <samirdmathur@gmail.com>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

I should clarify another small thing in my previous email to avoid confusion. When I discussed "electron" and "muon", I did not mean that all the effects we discuss are present in QED. I emphasize that I was only referring to the special case, where one is given a priori that the state is of the form 4.8 in section 4.2.1 of our paper.

Of course, in general, one can hide information in QED as we have already discussed in previous emails

**Subject:** Re: Discussion

**Date:** Mon, 2 Nov 2020 10:04:18 -0500

**From:** Samir Mathur <samirdmathur@gmail.com>

**To:** Suvrat Raju <suvrat@icts.res.in>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Hi Suvrat,

Thanks for the detailed reply. I will work through it in more detail, but it will help me if you can also send me a pdf file with the following computation: this is just so I can understand what you are allowing observers to do.

(i) Take 1+1 dimensional free scalar field  $\phi(t,x)$  with mass  $m$ . (There is no gravity.)

(ii) The state at  $t \rightarrow -\infty$  is  $|0\rangle$ .

(iii) The operator  $e^{i \int J_1 \phi}$  is applied at  $(t,x)=(0,0)$

(iii) An observer is allowed to make local measurements in a small neighbourhood of  $(t,x)=(T,X)$  with  $T>0$ . Write down the operator he will use and compute explicitly what he will get. (For example if you want to place  $e^{i \int J_2 \phi}$  at  $(T,X)$  and compute its expectation value, then I would like to see the explicit calculation using the expansion of  $\phi$  in creation and annihilation modes, rather than an indirect argument for what one should get. I am asking for an operator language rather than a path integral, since you are using operators in most of your arguments.)

(iii) This computation should show that if  $|X|>T$ , there is no effect of placing the excitation at  $(0,0)$ ; i.e., we get the same result of the measurement whether or not  $e^{i \int J_1 \phi}$  was placed at  $(0,0)$ .

I did this computation explicitly at some point just to be clear about the difference between

Wightman correlators and what can be detected by causality, and have notes somewhere that I can find and send to you. But I would also like to see how you do this in your own language, so that I see how you are separating correlations (that always exist in the vacuum even without gravity) from what can be measured causally.

best

samir

**Subject:** Re: Discussion

**Date:** Mon, 2 Nov 2020 20:55:19 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Samir Mathur <samirdmathur@gmail.com>

**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Hi Samir,

Could you please look at the pdf file that we sent you earlier today? It does precisely this computation in  $d$  dimensions and also explains the difference between LQFT and gravity. There is no indirect argument. It is an explicit computation. The equations in the paper also do not use any indirect argument. The computations in the paper are very explicit, including some numerical computations in the Appendix.

I feel it would be really great if you could tell us where precisely where you disagree with us. If either in the paper or in the emails that we have sent, you find any equation that is incorrect, please let us know.

Could you please also let us know if you agree with us on the wavefunctional argument that we were discussing in previous emails? We also explained where precisely there was a gap in reasoning in the argument that you sent us suggesting that  $\langle H \phi \rangle$  could not be used to detect the excitations.

I understand that you may be uncomfortable with the conclusion that we are reaching. But I wanted to assure you that we are not making some trivial error like confusing retarded correlators with Wightman correlators or forgetting to expand something to the appropriate order, or not recognizing that energy eigenstates are delocalized but that an appropriate linear combination can be localized!

At this point, I am sure that you will agree that we have made a very significant effort to answer all the questions you have raised, and we also did some additional computations like the ones we sent you earlier.

So before we jump to another topic, I would like to request you to work through these emails and point out a sharp disagreement. If, in any equation, you feel that we have made an indirect argument and feel that the result should be different, please mark it up and send it to us. Please also let us know what you feel the correct result should be.

regards,

Suvrat

---

from: **Samir Mathur** <samirdmathur@gmail.com>  
to: Suvrat Raju <suvrat@icts.res.in>  
cc: papadoulaki@ictp.it,  
chandramouli.chowdhury@icts.res.in  
date: Nov 2, 2020, 9:40 PM

Hi Suvrat,

I did browse through your file, but was actually asking for something more explicit. No operators like  $H$ ; and no use of commutators etc; no momentum operator  $\hat{P}_i$  (I am calling these 'arguments'). I would like to see the state at all stages. Thus one should write

$$|\phi\rangle = \int d^3x a e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)} + \text{cc}$$

and write the state that one gets by action of  $e^{iJ_i|\phi\rangle}$  (0,0) in terms of particle occupation numbers. You can work to quadratic order in all quantities like  $J_1$ , but let us not use arguments to throw away terms like  $J_1 J_2$  which will automatically arise (here  $e^{iJ_2|\phi\rangle}$  is the kind of operator that you may want to place at  $(T, X)$ ). You will find Bessel functions arising in this computation. At the end of course, the quantity you are computing should vanish for  $X > T$  but not for  $X < T$ . Please stay with 1+1 d, since I want to go through the computation in full detail and it will be easier for me.

It is a little nontrivial to see causality, so I want to be sure that this is being done in the way that I can understand explicitly and see the state at all times, ... we can then see what is happening if we have gravity,

best

samir

---

**Subject:** Re: Discussion  
**Date:** Wed, 4 Nov 2020 10:40:09 +0530  
**From:** Suvrat Raju <suvrat@icts.res.in>  
**To:** Samir Mathur <samirdmathur@gmail.com>  
**CC:** papadoulaki@ictp.it, chandramouli.chowdhury@icts.res.in

Dear Samir,

Thanks for your email.

The question in your latest email seems to be some elementary QFT exercise. I am not sure I understand the relevance to our paper or the link with our previous discussion. So before we embark on that exercise, I would like to summarize our discussion so far, including the discussion at Scipost and the discussion in these emails. I have only provided summaries below, and details are given in the original emails and the reply on Scipost.



-----  
1) You asked us about second order terms in  $J$ , and whether our result was perhaps just a consequence of not keeping these terms.

Our reply was that we had been very careful with the second order terms. The entire point of the unitary discussed in Appendix B is to ensure that these terms do not interfere with the result in our paper. We have not displayed them in some equations, not because we haven't kept track of them, but because their contribution vanishes.

2) You asked us how the result in Appendix A is consistent with the elementary mathematical fact that two infinitely differentiable functions can agree in an interval and disagree outside.

Our reply was that the result in Appendix A simply requires that given a function with support on  $[0, \pi]$  it is possible to find another function with support on  $[0, \epsilon]$  so that the positive frequency parts of the two functions coincide to arbitrary accuracy.

We provided the formal proof of this statement, and also gave an example using qubits. Appendix A also contains a numerical check. We pointed out that this statement holds in LQFT even without gravity.

3) You asked us whether Appendix A meant that an observer near a boundary could create spatially separated excitations.

We explained that Appendix A is a statement about the mathematical action of arbitrary boundary operators on the vacuum. Such an action does indeed produce all states, even in a local quantum field theory i.e. for any state  $|X\rangle$ , there exists local operators  $X$ , near the boundary and in the time-band  $[0, \epsilon]$  so that  $X|0\rangle = |X\rangle$ .

But, physically allowed transformations of state are those where the observer acts with a unitary operator. In general the  $X$  above is not unitary. So, in a LQFT, no action of unitaries near the boundary creates a spatially separated excitation. So, by using simple operations of the kind used in our paper, an observer near the boundary cannot physically create a spatially separated excitation.

4) You asked us about the difference between gauge and gravity theories, and why conserved charges in gauge theories do not allow a similar method of reading off information from the boundary.

We explained that gauge theories contain exactly local gauge invariant operators. A unitary made up of such operators from the bulk of AdS, such as a Wilson loop near the middle of AdS, commutes with all observations near the boundary. So observers near the boundary cannot distinguish one state from another state where such a unitary has acted. In contrast, there are no local gauge invariant operators in gravity.

For our protocol, the important difference is that in a gauge theory, the projector onto states of zero charge projects onto an infinite dimensional manifold. In contrast, in gravity, the projector onto zero energy states in global AdS projects onto a unique state, which is a

key input in our protocol.

5) You asked us how our protocol would work for states where a magnetic field was interacting with a spin in the middle of AdS and caused it to flip. How would one determine if the spin was up or down?

We explained that in this case, the energy and angular momentum would not change since both are conserved. But the interaction of the B-field with the spin would cause the state to evolve as

$$\sum c_n |E_n\rangle \rightarrow \sum c_n e^{-i E_n t} |E_n\rangle$$

Since our protocol can determine the coefficients  $c_n$  at any point of time, we can determine if the spin is up or down at a given point of time.

6) You asked us to clarify the main point of our result and how gravity was different from a LQFT.

We clarified that the main point was that in a theory of gravity, one can distinguish the state  $|0\rangle$  from  $U|0\rangle$  where  $U$  is a unitary excitation localized in the bulk. This is impossible in a LQFT.

Even in a gauge theory, as already mentioned above, if  $U$  is a Wilson loop in the bulk, no measurement near the boundary can distinguish  $|0\rangle$  from  $U|0\rangle$

7) Then you asked us about the following question. If one has two identical scalar fields, 1 and 2, how does one distinguish a state with an excitation of 1 from a state with an excitation of 2?

We explained the details of how our protocol works in this case. In the state with an excitation of 1, a unitary near the boundary comprising scalar field 1, followed by a measurement of the energy will detect the excitation. Whereas, in the state with an excitation of 2, a unitary comprising scalar field 2 followed by an energy measurement will detect the excitation.

8) Then you asked us how this is consistent with the wavefunctional picture, which you felt was useful in keeping track of causality. A naive analysis in this picture would suggest that excitations of identical fields,  $\phi_1$  and  $\phi_2$ , would produce the same far-gravitational field and so it would be impossible to distinguish between them using gravitational effects.

So we adopted that framework and pointed out that the wavefunctional would have entanglement between the far-gravitational field and the excitations of the scalar field. We explained the structure of this entanglement in detail: an eigenvalue of the zero-mode of the metric far away would be partnered with an energy eigenstate of the matter field, which is delocalized. This entanglement is what our protocol exploits to determine the state.

Therefore, even in the wavefunctional picture, it is clear that correlators of the energy with

dynamical fields can be used to distinguish excitations of the two fields.

9) Separately from all these questions we also performed an additional computation of two point functions  $\langle H \phi \rangle$  to show how such correlators could be used to distinguish between excitations of identical scalar fields.

-----

I feel you will agree that we have taken significant efforts to answer all your questions so far and explain the paper to you. Also, so far, we have clearly justified all the steps in the paper that you had questions about.

So, at this point, before we start anything new, we would like to understand your precise remaining disagreements. We would also like to move away from generalities to specifics.

\*\*\*\*\*

In particular, we have the following request. Could you please work through our paper and the emails we sent and let us know if there is any equation --- either in the emails or the paper --- that has an error.

If you feel that we used some formal arguments, and reached an incorrect result, please point out the specific equation and please also let us know what you feel the correct result should be.

For instance, if in some equation you feel that keeping second order terms would have made a difference to the result, please point out that specific equation, and please indicate the correct answer.

In our emails, and in the paper, we have been mathematically precise and have performed a number of calculations. So if there is any error anywhere in the argument, it should be possible to locate it.

\*\*\*\*\*

I feel that once we have a clear sense of what your remaining objections are, we can move forward with the discussion.

regards,

Suvrat

**Subject:** moving forward

**Date:** Wed, 4 Nov 2020 10:42:45 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Samir Mathur <samirdmathur@gmail.com>, Papadoulaki Olga <papadoulaki@ictp.it>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>

Dear Samir,

This is in continuation of my previous email. Of course, we should continue the scientific discussion but it would also be good to move forward with the publication of our paper.

So, I hope it will be possible for you to respond to our reply at Scipost. If you have any additional suggestions regarding the paper specifically, please do post them there. Once again, as mentioned in my previous email, if you have objections it would be very useful for us if you could localize them to specific equations in the paper.

Of course, if you have general concerns about the program we are advocating, including implications for black holes we can separate that from this paper and discuss it separately over email.

I feel that the discussion we have had in the emails has also been useful, and will benefit other readers. This is a unique advantage of Scipost's "peer witnessed" review model where they can see the interaction between the authors and the referee.

So if you agree, we can post a complete copy of our correspondence publicly. You can either include it in your referee report, or we can include it in our author reply.

regards,

Suvrat

**Subject:** Re: moving forward

**Date:** Wed, 4 Nov 2020 15:05:43 -0500

**From:** Samir Mathur <samirdmathur@gmail.com>

**To:** Suvrat Raju <suvrat@icts.res.in>

**CC:** Papadoulaki Olga <papadoulaki@ictp.it>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>

Hi Suvrat,

Actually in my referee report to scipost, in 'comments to the editor' I wrote that the paper should be published even though I felt that the result there was wrong. This is because I think readers will benefit either way; if an argument is published and its counters are also visible, then everyone benefits, even if the initial argument was incorrect I assumed that your paper was published once it was on scipost (sort of like the arxiv), but I am not very familiar with the procedures of this forum.

I can post a summary of my difficulty to scipost; let me write here what I would post (with my current understanding of things):

\*\*\*

I thank the authors for email correspondence that helped narrow down where I think the

problem lies in their computation. As an example we discussed, one can consider a theory with two free scalar fields,  $\phi_1$ ,  $\phi_2$ , coupled to the gravitational field  $g$ . A localized excitation near  $r=0$  is created for  $\phi_1$ , at time  $t=0$ . The claim is that an observer near infinity can detect which of the two fields were excited, by making observations only around  $t=0$ ; i.e., in a region that is well outside the light cone of the region  $r=0$ ,  $t=0$  where the field was deformed. No details of the quantum gravity theory are assumed; the argument proceeds with just the low energy properties of scalar fields and gravity.

This result appears to be in conflict with causality, and I do not believe the result is correct. At an intuitive level, one has deformed a field only near  $r=0$ ,  $t=0$ . The wavefunctionals of the fields have not been deformed near infinity. So it should not be possible to look near infinity and detect which of the two fields was deformed at the origin.

At a technical level let me write down the argument they gave, and state what I believe is wrong with it. The authors note that a localized excitation for  $\phi_1$  must be a sum over many energy eigenstates  $E_n$ . They argue that each such eigenstate will create a different value for the gravitational field at infinity by the Gauss law, and this gravitational effect can be detected. One can therefore project onto a particular eigenstate of the energy, and this projection in turn forces a projection onto the corresponding eigenstate  $E_n$  for the field  $\phi_1$ . But these energy eigenstates of  $\phi_1$  are not localized; they stretch across the full space as plane waves. Thus if one measures the value of  $\phi_1$  near infinity, one can observe the deformation of  $\phi_1$ , and note that the deformation is not in the field  $\phi_2$ . In symbols, the authors ask for the expectation value of  $H \phi_1$  at infinity, which implements the above projections.

I believe that this argument contains the following error. One must start not with the vacuum state in the interior, but with a state where we have  $\phi_1$ ,  $\phi_2$  in the vacuum state, together with an apparatus described schematically by some field  $\phi_3$ . It is this apparatus that will create the deformation of  $\phi_1$ . The total energy as seen from infinity is some value  $E_T$ , arising at this stage from  $\phi_3$ . Now the apparatus creates the localized excitation of  $\phi_1$ , and this excitation can indeed be written as a sum over different energy states  $E_n$ . But for each value of  $E_n$ , the apparatus will be left in a state with a corresponding energy  $E_T - E_n$ . Thus the gravitational field at infinity will not change; the total energy of the apparatus and  $\phi_1$  is fixed at  $E_T$  and only this value can be seen from infinity. Thus there will not be a way to project onto different values of  $E_n$  using the gravitational field at infinity. As a result, it will not be possible to tell from infinity which of the two fields  $\phi_1$  or  $\phi_2$  was excited at the origin.

One may further refine the discussion by taking superpositions over different values of  $E_T$  in the starting state (to get the evolution of the system), but this should not change the essence of the point made above. The main point is that any operations that happen near the origin conserve energy locally, so they do not generate an 'entanglement with gravity' which can be seen from infinity. If one looks only at the scalar field  $\phi_1$  and not at the apparatus creating it, then one is violating energy conservation, and this can lead to immediate inconsistencies.

I have encouraged the authors to write out more explicitly all the fields that are involved in the creation and measurement processes that they have in mind, using toy models where needed but with the full quantum state described explicitly at all times. Hopefully this will lead to a clarification of their issue.

\*\*\*\*

I did not know that scipost was expecting a response from me, but I can post this as a summary of my current understanding. If you send me additional arguments I will take them into account and modify accordingly ...

best

samir

---

from: **Suvrat Raju** <suvrat@icts.res.in> via messagingengine.com  
to: Samir Mathur <samirdmathur@gmail.com>  
cc: Papadoulaki Olga <papadoulaki@ictp.it>,  
Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>  
date: Nov 5, 2020, 12:51 PM

Dear Samir,

Thanks for your email. Scipost functions like a usual journal, except that the refereeing process is open. I did not realize that they had not asked you for a response to our comment. Maybe the editor has invited another referee because we have not yet received a decision on our paper.

But, in any case, we are free to post additional comments. So if you could give us a few days, we will prepare a readable summary of our correspondence that can be posted there after your permission (together with the comment that you have below, and a responses to that comment.)

-----  
In terms of the specific situation that you mention, I think the presence of an apparatus will not change the fact that we can detect excitations from infinity. Recall that the apparatus itself must be made of some dynamical field. Lets call the energy in this field  $F$ . Let us call the energy in the field we are probing,  $E$ , and  $H$  is the energy as measured near infinity using the metric.

Then the initial state is a localized state of the apparatus with the physical field in the vacuum. So it reads  
$$\sum_n c_n |E=0\rangle |F=n\rangle |H=n\rangle$$

for some choice of  $c_n$ . After a while, the apparatus and the other field interact. So the state becomes

$$\sum_{n,m} c_n d_{n,m} |E=m\rangle |F=n-m\rangle |H=n\rangle$$

where  $d$  are some coefficients controlled by the interaction. Notice that the energy is conserved in each branch of the wavefunction. But also notice the form of the entanglement between the metric and the other fields.

If  $H = n$  and  $F = n-m$  then  $E = m$ .

To determine the coefficients  $c$  and  $d$ , we now need \*three point functions\* of the metric, the apparatus-field, and the field we are probing. Clearly such three-point functions will determine all the coefficients above.

----

The point you raise is an interesting one, and we discuss it briefly in the paper. By coming up with configurations of the kind that you mention, one can make the information harder to obtain --- and move it from 2-point functions into 3-point functions. But provided the "complexity" of the state we are preparing and the "ability" of the observers to make measurements is scaled up in the same way, the observers can always detect the state.

regards,

Suvrat

-----

**Subject:** Re: moving forward

**Date:** Thu, 5 Nov 2020 23:50:02 -0500

**From:** Samir Mathur <samirdmathur@gmail.com>

**To:** Suvrat Raju <suvrat@icts.res.in>

**CC:** Papadoulaki Olga <papadoulaki@ictp.it>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>

Hi Suvrat,

Your mail confused me, for two reasons:

(A) You say "By coming up with configurations of the kind that you mention, one can make the information harder to obtain --- and move it from 2-point functions into 3-point functions."....

But I was not trying to make a more intricate example. I was just saying that you do not have a choice: you have to include the apparatus which creates the excitation, and the total energy of the apparatus + field excitation does not change. This is what makes the state of the field not detectable from

infinity. (Could you let me know the page where you discuss this in the paper; I should read that before posting my comments)

(B) I was looking at what you said about using 3-point functions. You take the state of a field E, a field F, and the gravitational field H to be of the form

$$\sum_{n,m} c_n d_{n-m} |E=m\rangle |F=n-m\rangle |H=n\rangle$$

Suppose I now drop gravity from this setup; i.e., I just consider two scalar fields E,F in a state

$$\sum_n c_n |E=m\rangle |F=n-m\rangle$$

This looks just like what you had written in your earlier mail for the case of one scalar field E and gravity H. In that case you had argued that you could project onto different components of this sum, to detect the nature of the state entangled between E and H. But your treatments of all fields E, F, H looks the same in terms of the operators you allow yourself to measure. So in the above case with only E, F, by your logic, will I not be able to detect the entanglement of E with F from infinity? In other words, if I write the E,F state in terms of eigenfunctions (which each extend to infinity in their support), then it appears that I can get information about the E,F state from infinity. But of course in this case we do not expect to see anything from infinity, since we have just localized field excitations near  $r=0$ , with no gravity.

So I imagine that you are ignoring the effect of cancellations that take place in the sum that you write; it is these cancellations that shield information from being seen at infinity, and the formal arguments may be too abstract to show you that causality is in fact maintained.

By the way, you are welcome to post all the correspondence, but in that case you should not summarize it, since that may distort what was being said (we can fix typos etc of course for readability). I can post my own summary opinion of the situation, but will wait to clarify any remaining points where we disagree.

best

samir

**Subject:** Re: moving forward

**Date:** Fri, 6 Nov 2020 15:44:33 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Samir Mathur <samirdmathur@gmail.com>

**CC:** Papadoulaki Olga <papadoulaki@ictp.it>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>

Dear Samir,



Thanks for your email. I only said "more intricate" because the apparatus was not there in the earlier setup. But it is perfectly fine to include it, provided we also include the "apparatus field" in our set of observables.

The answer to your puzzle in (B) is quite simple.  $E$  below stands the energy of the dynamical field,  $F$  for the energy of the apparatus and  $H$  for the energy as measured by the metric near infinity.

Situation in a LQFT

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Let us start with the case where there were two identical fields and we had to distinguish between excitations of these fields.

Recall that in a LQFT, the energy of the excitation is not an observable near the boundary.

So, by using observations near the boundary, the state  $|E=m\rangle$

cannot be distinguished from the state  $U |E=m\rangle = \sum_n u_{\{n m\}} |E=n\rangle$

where  $U$  is a unitary acting in the middle of AdS.

On the other hand, if I give the observers a \*prior guarantee\* that the system is in an energy eigenstate of either 1 or 2, and I only ask them to determine if the energy in the state is coming from field 1 or field 2, they can easily answer that by measuring expectation values of fields 1 and fields 2.

Please note the importance of the prior information that the observers have in this case.

Now let us turn to the case with the apparatus coupled to a field. As you note, with observations near the boundary, the state

$|\text{apparatus-field}\rangle = \sum_{\{n\}} c_n |E=m\rangle |F=n-m\rangle$

cannot be distinguished from another state where a unitary from the middle of AdS has acted and changed the state of both the apparatus and the field:

$U |\text{apparatus-field}\rangle = \sum_{\{n,q,q'\}} u_{\{n m q q'\}} c_n |E=q\rangle |F=q'\rangle$

But notice that if the observers are given a \*prior guarantee\* that the sum of the total energy in the two fields is  $m$ , and they are just asked to determine the coefficients  $c_n$  in the state  $|\text{apparatus-field}\rangle$ , then they can do this by measuring two-point functions of the field and the apparatus.

Again, please note the importance of the prior guarantee that the total energy has a given value.

Everything is obviously consistent with locality.

Situation in gravity

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In gravity, the entanglement between the matter part of the wavefunctional and the zero-mode of the metric near infinity is precisely of the kind that we need. The wavefunctional is a sum of terms of the form:

$$|\text{matter-energy}=n\rangle|H=n\rangle$$

Therefore once one measures the zero-mode of the metric, one gets precise the prior information that one needed above for the matter part.

In the first case above (distinguishing between fields), we need one additional observation to successfully distinguish the excitations. So, including the observation of the metric, we need a two point function  $\langle H \ \phi \rangle$

In the second case above (field + apparatus), we need two point functions even after measuring the energy to determine the coefficients. So, in total, we need three point functions  $\langle H \ \phi_{\text{field}} \ \phi_{\text{apparatus}} \rangle$

In either case, these correlators completely distinguish the state. If you look through our concrete computations of the two-point function in the pdf we sent, you will see this, and also see the obvious generalization to 3-point functions when the apparatus is also involved.

-----  
I wanted to request you once again to work through the paper and see if you can find any error. So far you have not pointed out an error in any equation to us, but have instead mentioned a number of separate puzzles that our results appear to raise for you.

We have been able to immediately provide clear answers to all these puzzles because our protocol is fundamentally robust. I hope also that our answers have persuaded you that we have thought quite carefully about the possible cancellations that occur with unitary local excitations in LQFTs, about the effect of second order terms, the difference between gravity and LQFT, the difference between the classical and the quantum case etc.

The last issue is one that I would like to emphasize since it appears to me that the puzzles you mention appear to come from thinking about gravity from a classical perspective. In the classical theory, it is indeed the case that gravitational effects cannot be used to identify the excitation. But there is significantly more information available in the quantum wavefunctional as our protocol and our discussion shows

I feel that if you were to work through our arguments in detail, you will see that there is no error and will also immediately see the answer to the puzzles above, and also to other puzzles of this kind. And, as I said, if there is an error in our arguments, it should be possible to locate it directly rather than through any indirect arguments.

regards,

Suvrat

P.S: On the summary, we were, of course, planning to take your approval before posting anything. I felt that we could prepare some document summarizing the questions and answers. You could edit the document to clarify anything that you felt required clarification. The end-result would be a joint document involving the authors and referee. Something like this might more helpful for readers.

We can, of course, just post the entire email correspondence but we will have to check with Scipost and the editor-in-charge about whether that is okay. If they are happy with it, that would also be fine.

**Subject:** Re: moving forward

**Date:** Mon, 16 Nov 2020 08:32:02 -0500

**From:** Samir Mathur <samirdmathur@gmail.com>

**To:** Suvrat Raju <suvrat@icts.res.in>

**CC:** Papadoulaki Olga <papadoulaki@ictp.it>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>

Hi Suvrat,

I came back from my travels, and looked over your arguments some more ... I am attaching a note where I write out my understanding of the issue of locality, and where it might differ from what you are saying ...

best

samir

**Subject:** Re: moving forward

**Date:** Thu, 19 Nov 2020 13:43:55 +0530

**From:** Suvrat Raju <suvrat@icts.res.in>

**To:** Samir Mathur <samirdmathur@gmail.com>

**CC:** Papadoulaki Olga <papadoulaki@ictp.it>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>

Dear Samir,

Thank you for your email, and for the note. We looked through it carefully, and we think we understand where the issue might be.

We agree with you till equation 8, and even equation 9 is okay if H and G are functionals.

But we think there is an error in equations (10),(11), (12). This issue carries over to the gravity case and through the rest of the note.

Recall that  $\Psi[\phi_1, \phi_2, A_0]$  is a functional so that it assigns a number for every field configuration. In particular, note that  $j_0(x)$  defined as in equation (4) is an operator on such functionals and  $\Psi[\phi_1, \phi_2, A_0]$  is, in general, *not* an eigenfunctional of  $j_0(x)$ .

For instance, consider the factorized ansatz that is written in Equation (12).

$$\Psi[\phi_1, \phi_2, A_0] = Q[\phi_1, \phi_2] R(A_0)$$

( $Q = H e^{iG}$  in your notation.  $R = \delta(\dots)$ )

But now you will find that

$$j_0(x) Q[\phi_1, \phi_2]$$

is a *different functional* of  $\phi_1, \phi_2$  from  $Q$ . (You say below equation (11) that the coefficient is a "c number", but this "c number" depends on the configuration  $\phi(x)$ . So this "c number" times the original functional is a new functional.)

Said another way, if views the functionals as vectors then  $j_0 Q[\phi_1, \phi_2]$  is *not* parallel to  $Q[\phi_1, \phi_2]$ . As a result the factorized ansatz of Equation (12) does not work and does not satisfy Equation (6).

-----

Perhaps the simplest case to see this is to just consider the vacuum. The vacuum, in the absence of any electromagnetic constraints, for a single scalar field is just given by

$$\Psi[\phi] = \text{Exp}[\int d x d y K(x,y) \phi(x) \phi(y)]$$

And you can check that the factorized ansatz does not correctly satisfy the constraints of Equation (6) even in this case.

-----

This issue is quite important. As we mentioned in previous emails, the constraints force there to be entanglement between the charge-sector and the matter-sector. In the gravity case, there must be entanglement between the energy-sector and the matter-sector. This entanglement is the key physical effect that allows our protocol to work.

We can explain this in more detail, but first we just wanted to check if you agree with us on the issue above.

regards,

Chandramouli, Olga, Suvrat

**Subject:** Re: moving forward  
**Date:** Thu, 19 Nov 2020 16:51:12 -0500  
**From:** Samir Mathur <samirdmathur@gmail.com>  
**To:** Suvrat Raju <suvrat@icts.res.in>  
**CC:** Papadoulaki Olga <papadoulaki@ictp.it>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>

Hi Suvrat,

I have added a section at the end (before the summary section) to clarify the nature of the wavefunctional I write down (it is not in a factorized form) ... I attach the updated file,

samir

**Subject:** Re: moving forward  
**Date:** Sun, 22 Nov 2020 09:25:25 +0530  
**From:** Suvrat Raju <suvrat@icts.res.in>  
**To:** Samir Mathur <samirdmathur@gmail.com>  
**CC:** Papadoulaki Olga <papadoulaki@ictp.it>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>

Dear Samir,

Thanks for your email. I should not have used the word "factorized". What I meant was that your ansatz has the form

$$\sum \langle \text{definite-value-of-}\phi(x) | \text{definite-value-of-}A^0(x) \rangle$$

whereas what you would need to satisfy the constraint is

$$\sum \langle \text{definite-value-of-}j^0(x) | \text{definite-value-of-}A^0(x) \rangle$$

where  $j^0$  is the charge density. These two are not the same since the charge density involves the conjugate momentum and so its eigenstates are not the same as the eigenstates of  $\phi$ . The distinction is not unimportant, and is the key difference between the classical and the quantum case.

----

Could you please see the attached note. See sections 2,3,4, which we have added to the original 2-pt function calculation.

In section 2, we have shown that the ansatz in your note does not satisfy the original constraint in your note. The problem is precisely the one above: eigenstates of  $\phi$  are not eigenstates of  $j$ .

Please also see the quantum mechanical example in section 3. where one can exactly solve the constraint, and compared it with the ansatz that you mentioned. We feel this

quantum mechanical example is quite instructive so it may be useful to even focus on only this example.

We have also indicated how we think the procedure should work in QFT in a simplified setting in section 4.

Please let us know if you agree and if our point is now clear.

best,

Chandramouli, Olga, Suvrat

**Subject:** Re: moving forward

**Date:** Mon, 23 Nov 2020 04:07:52 -0500

**From:** Samir Mathur <samirdmathur@gmail.com>

**To:** Suvrat Raju <suvrat@icts.res.in>

**CC:** Papadoulaki Olga <papadoulaki@ictp.it>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>

Hi Suvrat,

Thanks very much for the note, since this clarified many things for me. The extra term you note when you apply the constraint (and see it to not vanish) is second order in  $q$ . I have written the wavefunction to first order in  $q$  only. I think you are saying that the solution for general  $q$  will be an arbitrary functional of  $[\phi_i, A]$  and not a wavefunctional confined to a curve in  $[\phi_i, A]$  space. This looks correct, and I am happy to have learned something about such wavefunctionals.

Let me now get back to the physics question of what can be measured and what the consequences of the measurement would be. First, about the electromagnetic case which I find simpler. In the kind of measurements that you are allowed to make at infinity, can we only measure the total charge or can we also measure things like electric dipole moments and magnetic dipole moments? That is, suppose I have a dipole  $\vec{p}$  at the origin. The measurements at large  $r$  in the time interval  $[0, \epsilon]$  (that is causally disconnected from the origin) -- will these allow me to measure the dipole moment from looking at the electromagnetic field?

best

samir

-

Subject: thanks

Date: Sat, 10 Apr 2021 12:32:55 +0530

From: Suvrat Raju <suvrat@icts.res.in>

To: Samir Mathur <samirdmathur@gmail.com>, Papadoulaki Olga <papadoulaki@ictp.it>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>

Dear Samir,

Thanks very much for your second report on our paper.

As we had discussed previously, with our response, we felt it would be very nice if we could also make our correspondence available to readers since we discussed a large number of issues. Here is the pdf that we have currently. It contains the new version of your note and the first page contains a line directing readers to the Scipost website for the latest version of the arguments.

If you would like to make any edits for clarity to the emails, you can do so in this Google Doc:

We have already shared edit privileges with you.

We have included the latest note you sent us. But if you have made further changes, please send us the latest version and we will include that instead.

We have not yet updated our part of the note but we will update that in response when we post on Scipost.

regards,

Suvrat

----- Forwarded Message -----

Subject: Re: thanks

Date: Sun, 11 Apr 2021 23:04:16 -0400

From: Samir Mathur <samirdmathur@gmail.com>

To: Suvrat Raju <suvrat@icts.res.in>

CC: Papadoulaki Olga <papadoulaki@ictp.it>, Chandramouli Chowdhury <chandramouli.chowdhury@icts.res.in>

Hi Suvrat,

I summarize my current questions here; you can add this to the other emails when you post.

I believe you are arguing that the effect of unitary operators at one location can be detected by distant observations without allowing light to have time to travel to the latter locations. In these arguments I have been concerned that the measurements you want to use may not correspond to gauge-invariant objects that actually be measured locally. Of course you say that these measurements are near flat infinity where gravity is weak, but the effect you are measuring is weak as well, since the gravitational field is small there.

To look for a flaw in your argument, I have tried to construct two wavefunctionals that (i) satisfy the gauge constraints) (ii) differ in some region  $r < R$  (iii) are identical at  $r > R$ . If this can be done, then I believe this would be a counterexample to your claim. Is this correct?

I had earlier constructed such wavefunctionals (for the electromagnetic case) in the WKB approximation to first order in the coupling  $q$ . You had pointed out that this construction would not work at order  $q^2$ . So I then tried to make a nonperturbative construction that would hold for all  $q$ ; this is the last section in the notes.

Anyway, I dont know if I am understanding your issue correctly ... I hope your paper is published and that we can continue to discuss the physics separately,

best

samir



# 1 The issue

It has been observed before that measuring the gravitational field at infinity gives the energy of the state in the interior. Thus if we assume that all states of a system have different energies, then we can identify the state in the interior by measuring the gravitational deflection of particles at infinity.

In this paper the authors seek to extend this notion in the following way. Suppose we have two scalar fields  $\phi_1$  and  $\phi_2$  with the same mass; in fact we assume that these fields are identical in every way except that they carry a different flavor index  $i = 1, 2$ . We create a localized excitation near  $r = 0$  with field 1, leaving field 2 unexcited. In a theory without gravity, the authors note that there is no way to tell from infinity which of the two fields was excited. With perturbative quantized gravity, however, they argue that measurements at infinity will, in fact, be able to reveal which of the two fields was excited near  $r = 0$ . These measurements will be performed near infinity in a small time band  $t \in (0, \epsilon)$ , so that there is no time for signals to be sent from this measurement region to  $r = 0$  and back.

The argument for this claim appears to be as follows:

(i) Measuring the gravitational field will project onto a definite energy eigenstate of the field energy. This is argued to be the case from the following form of the overall wavefunction

$$\Psi = \sum_n \Phi_n h_n \tag{1}$$

where  $\Phi_n$  is an energy eigenfunction for the matter part with energy  $E_n$ , and  $h_n$  is the gravitational field wavefunction corresponding to the energy  $E_n$ .

(ii) Next the authors argue that energy eigenstates of matter are not localized states; they are more like plane waves that extend to infinity. Thus the energy eigenstate  $\Phi_n$  will have a nonvanishing tail at infinity, which can be identified by measurements of the field variables at infinity. Thus measuring both gravity and field variables at infinity will identify the state near  $r = 0$ , though the actual field excitation was localized with compact support near  $r = 0$  and so did not have any tail that reached infinity.

(iii) The authors note that the case for gauge fields is different from the case for gravity for the following reason. Charge can also be measured from infinity, so one can project onto states of a definite charge. But a state of definite charge can have compact support. On the other hand states with definite energy typically have noncompact support, so they must have the above mentioned tail at infinity, and this tail, in turn, allows an identification of the flavor of the field excitation in the above example.

This argument suggests that turning on weak perturbative gravity leads to a change in the locality properties of the quantum theory. This appears strange, so we examine the situation in more detail.

## 2 The electromagnetic case

Let us first write the wavefunctional for the electromagnetic case. Before the gauge coupling is introduced, we can take the matter wavefunctional as

$$\Phi_0[\phi_1(x), \phi_2(x)] \quad (2)$$

where  $\phi_1, \phi_2$  are real fields.

The Lagrangian is

$$L = -\frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\partial_a\phi_1\partial^a\phi_1 - \frac{1}{2}\partial_a\phi_2\partial^a\phi_2 + qA_a j^a \quad (3)$$

where

$$j_a = \phi_1\partial_a\phi_2 - \phi_2\partial_a\phi_1 \quad (4)$$

In the classical theory, the constraint equation for the electromagnetic potential  $A_0$  has the form

$$\Delta A_0(x) = qj_0(x) = q(\phi_1\partial_0\phi_2 - \phi_2\partial_0\phi_1) \quad (5)$$

There are many ways to pass to the quantum theory. One possibility is to impose this as an operator relation

$$\left(\Delta\hat{A}_0(x)\right)\Psi[\phi_1(x), \phi_2(x), A_0(x)] = q(\hat{\phi}_1\hat{\Pi}_2 - \hat{\phi}_2\hat{\Pi}_1)\Psi[\phi_1(x), \phi_2(x), A_0(x)] \quad (6)$$

In this basis, the operator  $\hat{A}_0(x)$  is just multiplication by  $A_0(x)$ . The operators  $\hat{\phi}_1, \hat{\phi}_2$  are also multiplications by  $\phi_1(x)$  and  $\phi_2(x)$  respectively. We also have

$$\hat{\Pi}_1(x) \rightarrow -i\frac{\partial}{\partial\phi_1(x)}, \quad \hat{\Pi}_2(x) \rightarrow -i\frac{\partial}{\partial\phi_2(x)} \quad (7)$$

Let us solve the constraint equation to leading order in the coupling. We start with any state of the matter fields of our choice, for example (2). Then we make it a solution of the full theory by solving (7) as follows. For any choice of the point  $\{\phi_1(x), \phi_2(x)\}$  in the matter configuration space, for any point  $x$ , we must have

$$\Delta A_0(x)\Psi[\phi_1(x), \phi_2(x), A_0(x)] = q\left(\phi_1(y)\left(-i\frac{\partial}{\partial\phi_2(y)}\right) - \phi_2(y)\left(-i\frac{\partial}{\partial\phi_1(y)}\right)\right)\Psi[\phi_1(x), \phi_2(x), A_0(x)] \quad (8)$$

This equation may look complicated, but it looks simpler if we work to leading order in the coupling  $q$  and use the WKB form of the wavefunction

$$\Psi[\phi_1(x), \phi_2(x)] = H[\phi_1, \phi_2]e^{iG[\phi_1, \phi_2]} \quad (9)$$

where  $H$  is slowly varying and  $G$  is quickly varying. Then we get

$$A_0(x) = \Delta^{-1}(x, y)q\left(\phi_1(y)\left(-\frac{\partial}{\partial\phi_2(y)}\right) - \phi_2(y)\left(-\frac{\partial}{\partial\phi_1(y)}\right)\right)G[\phi_1, \phi_2] \quad (10)$$

Note that for any point  $\{\phi_1(y), \phi_2(y)\}$  in configuration space, the quantity

$$q \left( \phi_1(y) \left( -i \frac{\partial}{\partial \phi_2(y)} \right) - \phi_2(y) \left( -i \frac{\partial}{\partial \phi_1(y)} \right) \right) G[\phi_1, \phi_2] \quad (11)$$

is just a c-number function of  $y$ . The kernel  $\Delta^{-1}(x, y)$  is just the usual propagator, equal to  $1/|x - y|$  in 3-d. Thus  $A_0(x)$  can be explicitly computed as a function of  $x$ , for the given point in configuration space  $\{\phi_1(y), \phi_2(y)\}$ . Let us call this function  $A_0^{\phi_1, \phi_2}$ . The full wavefunction is then

$$\Psi[\phi_1, \phi_2, A_0] = H[\phi_1, \phi_2] e^{iG[\phi_1, \phi_2]} \delta \left( A_0 - A_0^{\phi_1, \phi_2} \right) \quad (12)$$

Thus there is no big conceptual change to the free theory wavefunctional when we include the electromagnetic field; the wavefunctional just moves from being at  $A_0 = 0$  to a new curve  $A_0 = A_0^{\phi_1, \phi_2}$  in the  $\{\phi_i, A_0\}$  space.

### 3 Gravity

At the formal level at which one is working, the gravity set up looks similar to this electromagnetic set up. The gauge potential  $A_0(x)$  is replaced by  $h_{00}(x)$ , and the current is replaced by the stress tensor  $j_0 \rightarrow T_{00}$  where

$$T_{00} = (\nabla \phi_1)^2 + (\nabla \phi_2)^2 + \Pi_1^2 + \Pi_2^2 \quad (13)$$

At the quantum level we can write

$$\hat{T}_{00} = (\nabla \hat{\phi}_1)^2 + (\nabla \hat{\phi}_2)^2 + \hat{\Pi}_1^2 + \hat{\Pi}_2^2 \quad (14)$$

which becomes, in the representation  $\Psi[\phi_1(x), \phi_2(x), h_{00}(x)]$

$$\hat{T}_{00} = (\nabla \phi_1)^2 + (\nabla \phi_2)^2 - \frac{\partial^2}{\partial \phi_1^2} - \frac{\partial^2}{\partial \phi_2^2} \quad (15)$$

The analogue of (10) becomes

$$h_{00}(x) = \Delta^{-1}(x, y) \left( (\nabla \phi_1(y))^2 + (\nabla \phi_2(y))^2 - \frac{\partial^2}{\partial \phi_1(y)^2} - \frac{\partial^2}{\partial \phi_2(y)^2} \right) G[\phi_1, \phi_2] \quad (16)$$

and the analogue of (12) becomes

$$\Psi[\phi_1, \phi_2, h_{00}] = H[\phi_1, \phi_2] e^{iG[\phi_1, \phi_2]} \delta \left( h_{00} - h_{00}^{\phi_1, \phi_2} \right) \quad (17)$$

## 4 Localized excitations

So far we have not taken any particular form for  $\Psi_0$ , so we have not come across the questions of locality etc. Let us make the wavefunctional more explicit now.

Let the vacuum wavefunctional be

$$\Phi_{vac}[\phi_1(x), \phi_2(x)] = \Phi_{1,vac}[\phi_1(x)]\Phi_{2,vac}[\phi_2(x)] \quad (18)$$

We wish to make a deformation near  $r = 0$  to  $\Phi_{1,vac}[\phi_1(x)]$  while leaving the field variables unchanged at other positions. One way to do this is to take the excited state

$$\Psi_{ex,1} = e^{i \int dx f(x) \hat{\Pi}_1(x)} \Phi_{vac}[\phi_1(x), \phi_2(x)] \quad (19)$$

where  $f(x)$  is a function with compact support near  $r = 0$ . The operation  $e^{if\Pi(x)}$  changes the value of  $\phi_1$  at the position  $x$  as  $\phi_1(x) \rightarrow \phi_1(x) + f(x)$ . Since  $f$  vanishes outside a compact region, the field wavefunctional is left unaffected outside this compact region. In particular, the wavefunctionals are not altered in the region near infinity where measurements are being made. Thus the wavefunctional of  $\phi_1$  is just like the wavefunctional of  $\phi_2$  near infinity.

Let us now add the gravity field. Let us take a wavefunctional  $\Psi_{vac}$  of the form (9)

$$\Psi_{vac} = H[\phi_1, \phi_2] e^{iG[\phi_1, \phi_2]} \quad (20)$$

The action (19) converts this to

$$\Psi_{ex,1} = H[\phi_1 - f, \phi_2] e^{iG[\phi_1 - f, \phi_2]} \quad (21)$$

where the symbol  $\phi_1 - f$  means that  $\phi_1(x)$  is replaced by  $\phi_1(x) - f(x)$ . Note that this change happens only for the points  $x$  that in the compact region around the origin where  $f$  is nonvanishing.

Looking at (17), we see that we expect the full wavefunctional for the excited state to be

$$\Psi_{ex,1}[\phi_1, \phi_2, h_{00}] = H[\phi_1 - f, \phi_2] e^{iG[\phi_1 - f, \phi_2]} \delta \left( h_{00} - h_{00}^{\phi_1 - f, \phi_2} \right) \quad (22)$$

where  $h_{00}^{\phi_1 - f, \phi_2}$  is now computed using the wavefunctional  $\Psi_{ex,1}$ . (Recall that in (16) to compute  $h_{00}(x)$  we need not only the values of the wavefunctional at that point but also upto two derivatives of this wavefunctional. Thus  $h_{00}^{\phi_1, \phi_2}$  depends not only on the point in configuration space  $\{\phi_1, \phi_2\}$  but also on the wavefunctional that we have taken.)

## 5 The wavefunctionals at infinity

Suppose we had excited field  $\phi_2$  instead of  $\phi_1$ . Then we would have, in place of (22)

$$\Psi_{ex,2}[\phi_1, \phi_2, h_{00}] = H[\phi_1, \phi_2 - f] e^{iG[\phi_1, \phi_2 - f]} \delta \left( h_{00} - h_{00}^{\phi_1, \phi_2 - f} \right) \quad (23)$$

Let us compare this to (22). We have

$$h_{00}^{\phi_1-f, \phi_2} = h_{00}^{\phi_1, \phi_2-f} \quad (24)$$

since the two fields have the same energy when they are in the same configuration. Further, at points outside the compact region where  $f$  is supported, there is no difference between the matter wavefunctionals in the two states; in fact they are both locally the same as the wavefunctional in the vacuum state. In detail, this means the following. Suppose we look at the probability to find some configuration  $\phi_1(x)|_{x \in R}$  where  $R$  is a spatial region far from the region where  $f$  is supported. This shape of  $\phi_1$  in the region  $R$  can arise from different configurations  $\phi_\alpha(x)$  which all agree with the given  $\phi_1(x)$  in the region  $R$ . Consider the vacuum wavefunctional  $\Psi_{vac}$ . Let each such configuration  $\phi_\alpha$  have an amplitude  $\psi_\alpha$ . Then the probability to find the configuration  $\phi_1(x)|_{x \in R}$  is

$$P_{vac}[\phi_1(x)|_{x \in R}] = \sum_{\alpha} |\psi_{\alpha}|^2 \quad (25)$$

Now consider the state (19). Each configuration  $\phi_\alpha$  in the above discussion is now modified to a configuration  $\phi'_\alpha$ , where the modification is only in the region of support of  $f$ . Thus for the excited state  $\Psi_{ex,1}$  we get the same amplitude  $\psi_\alpha$  for the configuration  $\phi'_\alpha$  as we had in  $\Psi_{vac}$  for the configuration  $\phi_\alpha$ . Thus we find

$$P_{ex,1}[\phi_1(x)|_{x \in R}] = \sum_{\alpha} |\psi_{\alpha}|^2 = P_{vac}[\phi_1(x)|_{x \in R}] \quad (26)$$

Similarly,

$$P_{ex,2}[\phi_1(x)|_{x \in R}] = \sum_{\alpha} |\psi_{\alpha}|^2 = P_{vac}[\phi_1(x)|_{x \in R}] \quad (27)$$

Since both the gravity and matter wavefunctionals at infinity are the same for  $\Psi_{ex,1}$  and  $\Psi_{ex,2}$ , it is unclear how looking near infinity can distinguish them.

## 6 Comparing to the argument of the paper

Let us now look at the arguments of the paper:

(a) Let us project onto a given value of the energy at infinity. At infinity we have the form

$$h_{00}^{\phi_1, \phi_2} = \frac{k}{r} \quad (28)$$

Here  $k$  depends on the wavefunctional and the point in field space; it is given by

$$k = \int dy \left( (\nabla \phi_1(y))^2 + (\nabla \phi_2(y))^2 - \frac{\partial^2}{\partial \phi_1(y)^2} - \frac{\partial^2}{\partial \phi_2(y)^2} \right) G[\phi_1, \phi_2] \quad (29)$$

Thus by projecting onto a given value of  $k$ , we select a given subset of configurations  $\{\phi_1, \phi_2\}$  in configuration space. But this does not look like projecting onto an eigenstate of the field wavefunctions as indicated in (1). (We cannot for example say that we have projected onto a matter wavefunctional that is a delta function on the configurations satisfying (29), since we use the gradients of the wavefunctional in getting (29) and the delta function will not have these gradients.)

(b) In the paper I am puzzled by the argument that

$$Q \equiv \langle 0|X|g\rangle \tag{30}$$

is nonzero, and possibly close to  $\langle 0|X|X\rangle \approx 1$ . Here  $|g\rangle$  arises from a unitary operator applied near  $r = 0$ , and  $X$  is part of a local unitary operator applied near the boundary. One would have thought that  $Q \approx 0$ .

Let us first check this in an approximation. Let us ignore vacuum fluctuations, and take the vacuum state to have the field configuration

$$\phi_1 = 0, \quad \phi_2 = 0 \tag{31}$$

In the above discussion we had considered the operator  $e^{if\hat{P}_i}$  which translates  $\phi \rightarrow \phi + f$ . Let us use this operator at  $r = 0$  to get

$$|g\rangle = e^{if\hat{\Pi}_1}|0\rangle \tag{32}$$

with  $f = \delta(\vec{r})$ . In the state  $|g\rangle$  we then have

$$\phi_1 = f\delta(\vec{r}), \quad \phi_2 = 0 \tag{33}$$

Let us now apply the opposite unitary at a point  $\vec{R}$  near the boundary

$$e^{i\hat{X}} = e^{i\int dx \tilde{f}(x)\hat{\Pi}_1} \tag{34}$$

where

$$\tilde{f} = -\delta(\vec{r} - \vec{R}) \tag{35}$$

In our approximation, if we apply (34) to (32) then we will get

$$\phi_1 = \delta(\vec{r}) - \delta(\vec{r} - \vec{R}), \quad \phi_2 = 0 \tag{36}$$

We see that (36) is orthogonal to (31). Thus we get

$$\langle 0|e^{i\hat{X}}|g\rangle = 0 \tag{37}$$

and its expansion (30) should not be taken as a nonvanishing contribution.

Now let us look at the actual quantum computation including vacuum fluctuations. Let us look at the corresponding computation for a 1+1 dimensional free field  $\phi$ :

$$Q_1 = \langle 0 | e^{-iJ\hat{\Pi}(0)} e^{iJ\hat{\Pi}(R)} | 0 \rangle \quad (38)$$

We can write  $\Pi = \partial_+ \phi - \partial_- \phi$ , and find

$$Q_1 = \frac{1}{R^{J^2}} \quad (39)$$

This falls off with  $R$ , so in the limit  $R \rightarrow \infty$ , it vanishes. But why does it not vanish exactly? This is because the vacuum state has quantum fluctuations, so that in the vacuum the value of  $\phi$  is not given by (31) but instead has a gaussian spread. The translation  $\phi \rightarrow \phi + J$  then gives a new state that is not exactly orthogonal to the vacuum. The overlap does fall off with larger translations  $J$ , and that is why (39) falls off faster for larger  $J$ . Overall, we see that the nonvanishing of  $Q_1$  is just due to the correlations present in the vacuum state; these are the same correlations that give the nonvanishing of the Wightman function outside the light cone.

## 7 A comment on the nature of the wavefunctional

Consider the nature of the wavefunctional that is obtained for a constrained system. In the electromagnetic case we had written this wavefunctional as a solution of the relation (8). As a simpler example we can just look at quantum mechanics rather than quantum field theory. We consider a wavefunction of two variables  $\psi(z_1, z_2)$  with the constraint

$$(\hat{z}_2 - q\hat{z}_1^2)\psi = 0 \quad (40)$$

The mapping from the quantum field theory case to the toy quantum mechanical case is

$$\Psi[\phi_i(x), A_0(x)] \rightarrow \psi(z_1, z_2), \quad \phi_i(x) \rightarrow z_1, \quad A_0(x) \rightarrow z_2 \quad (41)$$

Let us see how we would impose the constraint in the quantum mechanics case. Suppose we work perturbatively in  $q$ , just as we did in the analysis following (8). For  $q = 0$ , the constraint is

$$\hat{z}_2\psi = 0 \quad (42)$$

We can write

$$\psi(z_1, z_2) = \tilde{\psi}(z_1)\delta(z_2) \quad (43)$$

and we see that it satisfies the constraint. At the next order in  $q$ , we can write

$$\psi(z_1, z_2) = \tilde{\psi}(z_1)\delta(z_2 - qz_1^2) \quad (44)$$

and we see that it satisfies the constraint again. (In this case this works to all order in  $q$ , but in more complicated cases one will get corrections at higher order; in particular there is in general a Jacobian multiplying the delta function which is determined by gauge symmetry in the electromagnetic case and handled using ghosts.)

Now let us return to the electromagnetic case. We had the expression in (12)

$$\Psi[\phi_1, \phi_2, A_0] = H[\phi_1, \phi_2] e^{iG[\phi_1, \phi_2]} \delta(A_0 - A_0^{\phi_1, \phi_2}) \quad (45)$$

This is similar to the quantum mechanical case above. Note that  $A_0^{\phi_1, \phi_2}$  depends on the matter configuration  $\{\phi_1(x), \phi_2(x)\}$ , so the wavefunctional is not in a factorized form between the matter and the gauge field. The relevant point is that since the two fields are similar except in their flavour quantum number, we have

$$A_0^{\phi_1, \phi_2}(x) = A_0^{\phi_2, \phi_1}(x) \quad (46)$$

Thus the gauge field configuration attached to any matter configuration does not change if we flip the two matter fields with each other. This is important in ensuring that we cannot distinguish which of the fields was excited in the interior by looking at the gauge field (and the matter fields) at infinity.

## 8 Higher orders

In the above we have solved the constraint equation to order  $q$ . Let us see how we would carry out this solution to higher orders. Thus consider a theory with a scalar field  $\phi(x)$  and a gauge field  $A(x)$  which satisfies the constraint

$$A = q\phi\Pi \quad (47)$$

at the classical level. In the quantum theory we write

$$\left(\hat{A}(x) - q\hat{\phi}(x)\hat{\Pi}(x)\right) \Psi[\phi, A] = 0 \quad (48)$$

At order  $q^0$  we have

$$\hat{A}\Psi[\phi, A] = 0 \quad (49)$$

which is satisfied by

$$\Psi[\phi, A] = \tilde{\Psi}[\phi]\delta(A) \quad (50)$$

More generally we write

$$\Psi[\phi, A] = \tilde{\Psi}[x](1 + J^{(1)} + J^{(2)} + \dots)\delta(A - A_\phi^{(1)} - A_\phi^{(2)} - \dots) \quad (51)$$

where

$$J \equiv 1 + J^{(1)} + J^{(2)} + \dots \quad (52)$$



is a Jacobian which tells us the correct width of the delta function when it is written in the form that is chosen, and

$$A_\phi \equiv A_\phi^{(1)} + A_\phi^{(2)} + \dots \quad (53)$$

is the value of  $A$  that the point  $\phi(x)$  in field space is lifted to when the perturbation is turned on. Thus the constraint relation is

$$\left( A(x) + iq\hat{\phi}(x)\frac{\delta}{\delta\phi(x)} \right) \left( \tilde{\Psi}[x](1 + J^{(1)} + J^{(2)} + \dots)\delta(A - A_\phi^{(1)} - A_\phi^{(2)} - \dots) \right) = 0 \quad (54)$$

To go to higher orders we will find it convenient to write

$$\delta(A - A_\phi^{(1)} - A_\phi^{(2)} - \dots) = \delta(A - A_\phi^{(1)}) - A_\phi^{(2)}\delta'(A - A_\phi^{(1)}) + \dots \quad (55)$$

Thus the equation is

$$\left( A(x) + iq\phi(x)\frac{\delta}{\delta\phi(x)} \right) \left( \tilde{\Psi}[x](1 + J^{(1)} + J^{(2)} + \dots) \left( \delta(A - A_\phi^{(1)}) - A_\phi^{(2)}\delta'(A - A_\phi^{(1)}) + \dots \right) \right) = 0 \quad (56)$$

The equation to order  $q^0$  is satisfied since

$$A\delta(A) = 0 \quad (57)$$

At order  $q$  we have

$$A(x)\tilde{\Psi}[x]\delta(A - A_\phi^{(1)}) + iq\phi(x)\frac{\delta\tilde{\Psi}[\phi]}{\delta\phi(x)}\delta(A - A_\phi^{(1)}) = 0 \quad (58)$$

This gives

$$A_\phi^{(1)}(x)\tilde{\Psi}[x]\delta(A - A_\phi^{(1)}) + iq\phi(x)\frac{\delta\tilde{\Psi}[\phi]}{\delta\phi(x)}\delta(A - A_\phi^{(1)}) = 0 \quad (59)$$

This is satisfied by

$$A_\phi^{(1)}(x) = -iq\phi(x)\frac{\delta\tilde{\Psi}[\phi]}{\delta\phi(x)} \quad (60)$$

## 9 Summary

We have tried to look at the explicit wavefunction for the situation with weak quantum gravity, and did not find that the wavefunctional at infinity has any signature of the local excitations near the origin. Projecting to a definite energy projects onto a subset of configurations, but these do not appear to give the energy eigenstate of the kind the authors have in mind (one which will have a support that will extend to infinity). The authors use amplitudes like  $\langle 0|X|g \rangle$  in their argument, but these seem to be only picking up vacuum fluctuations, and so may not have the kind of information the authors seek to capture.

## 10 Looking at the electromagnetic equation again

Let us return to the electromagnetic case again. Our goal is to see if we can get a solution to the quantum wavefunctional that (1) satisfies the constraints (2) changes can be made to this wavefunctional at  $r < R$  while making no change outside  $r = R$ ; we will argue this means that the state inside  $r = R$  cannot be detected from  $r > R$  by any means.

We start by recollecting the problem and making some observations:

(a) We will start with a gauge where

$$\vec{\nabla} \cdot \vec{A} = 0 \tag{61}$$

The actual  $t$  component of the gauge field equation gives the constraint

$$F_{0i,i} = j_0 \tag{62}$$

where

$$F_{0i} = A_{i,0} - A_{0,i}, \quad F_{0i,i} = A_{i,0i} - A_{0,ii} = (\vec{\nabla} \cdot \vec{A}) + \Delta\Phi \tag{63}$$

In the gauge (61) this becomes

$$\Delta\Phi = j_0 \tag{64}$$

but we note that this is not the constraint equation in a more general gauge. This will be relevant because sometimes apparently nonlocal effects can arise because of a gauge choice, but there is really no nonlocality because the observables must also be defined carefully in each gauge.

(b) The equation (64) gave rise to (8)

$$\Delta\Phi(x)\Psi[\phi_1(x), \phi_2(x), \Phi(x)] = q \left( \phi_1(y) \left( -i \frac{\partial}{\partial \phi_2(y)} \right) - \phi_2(y) \left( -i \frac{\partial}{\partial \phi_1(y)} \right) \right) \Psi[\phi_1(x), \phi_2(x), \Phi(x)] \tag{65}$$

where we have written

$$\Phi = -A_0 \tag{66}$$

Recall that where we had assumed two free scalar fields with the same mass; these make a complex scalar and we are looking at the charge produced by this scalar and the potential  $\Phi$  created by this charge.

(c) In section 2 we had tried to solve this equation for the wavefunctional  $\Psi$  by a WKB approximation, assuming small  $q$ . This answer could be used only to linear order in  $q$ , but now to higher orders in  $q$ . We will now look for a solution without using the small  $q$  approximation or a WKB type approximation.

The wavefunctional  $\Psi$  depends on  $\phi_1(x), \phi_2(x), \Phi(x)$ . To construct a wavefunctional satisfying the constraints, We proceed in the following steps:

(i) First choose any one function  $\Phi(x) = \Phi_1(x)$ . Compute

$$U(x) \equiv \Delta\Phi_1(x) \quad (67)$$

which is just a function over 3-dimensional space.

(ii) Now look at any point  $x = x_1$ . At this point we have a number

$$U_1 \equiv \Delta\Phi_1(x) \Big|_{x=x_1} \quad (68)$$

At this point in function space  $\Phi = \Phi_1(x)$  and further at the spatial point  $x = x_1$  the functional  $\Psi$  has to satisfy the constraint

$$\left( \phi_1(y) \left(-i \frac{\partial}{\partial \phi_2(y)}\right) - \phi_2(y) \left(-i \frac{\partial}{\partial \phi_1(y)}\right) \right) \Psi[\phi_1(x_1), \phi_2(x_1)] = \frac{U_1}{q} \Psi[\phi_1(x_1), \phi_2(x_1)] \quad (69)$$

Now this is an equation for a function  $\Psi$  of just two real number arguments  $\phi_1(x_1), \phi_2(x_2)$ . There are many solutions of this equation. Let us call them

$$[\phi_i(x_1)]_1, [\phi_i(x_1)]_2, [\phi_i(x_1)]_1 \dots \quad (70)$$

for later use. (Here  $i = 1, 2$  ranges over the two flavors of scalar fields that we have taken.) Note that this equation does not involve the functional  $\Psi$  at any other space point  $x \neq x_1$  right now. For any other point  $x_2$  we will get similar solutions

$$[\phi_i(x_2)]_1, [\phi_i(x_2)]_2, [\phi_i(x_2)]_1 \dots \quad (71)$$

and so on, where in (69) we replace  $U_1$  by its value  $U_2$  at the point  $x_2$ . Thus we can choose a solution to this equation at each point  $x$  separately. Suppose we choose the first solution  $[\phi_i(x)]_1$  for each  $x$ . This gives  $\Psi$  that satisfies the constraint everywhere for the point in function space  $\Phi = \Phi_1(x)$ .

(iii) Now choose some other point  $\Phi = \Phi_2(x)$  in the space of functions  $\Phi(x)$ . Proceed as above, getting a solution that satisfies the constraint at this point  $\Phi_2(x)$  in function space. Doing this for all functions  $\Phi(x)$  gives us a complete functional  $\Psi[\phi_1(x), \phi_2(x), \Phi_0(x)]$  that satisfies the constraint.

(iv) Now we observe that since for each  $\Phi(x)$  the construction above proceeded separately for each point  $x$ , we can choose two different functionals  $\Psi[\phi_1(x), \phi_2(x), \Phi(x)]$  that are the same for all points outside a sphere  $r = R$  and differ inside. Then we will not be able to do any observations outside  $r = R$  to find what is the state inside.

(v) Note that the above state was a factored product of states at each  $x$ . This is not a generic low energy state. But we can take a state that has some entanglement

between nearby points, by taking superpositions of the above constructed  $\Psi$ ; thus we can for example add the  $\Psi$  that we get from choosing  $[\phi_i(x)]_1$  at each  $x$  to the wavefunctional that we get from choosing  $[\phi_i(x)]_2$  at each  $x$ . This is a more general class of states, but again, we can arrange that the wavefunctional outside  $r = R$  remain unchanged while the wavefunctional inside  $r = R$  is altered.

## 11 Questions

Let us see if this construction above has any bearing on the issue:

- (a) Is the above construction incorrect for the electromagnetic case itself?
- (b) Is the above construction correct, but the gravitational case is different in some essential way? (It seemed to me that I can extend the above construction to the gravitational case as well).
- (c) Is the construction valid for both electromagnetic and gravitational cases, but somehow the issue of interest is not addressed by these constructions?

# Response from authors to the note from the referee

## 1 Two point functions

We have two propagating fields,  $\phi_1$  and  $\phi_2$  of the same mass. We denote the corresponding operator dimension by  $\Delta$  and consider the states

$$\begin{aligned} |1\rangle &= e^{i \int dr f(r) \phi_1(r, t=0, \Omega) d^{d-1} \Omega} |0\rangle \\ |2\rangle &= e^{i \int dr f(r) \phi_2(r, t=0, \Omega) d^{d-1} \Omega} |0\rangle \end{aligned} \quad (1)$$

Here  $f(r)$  is some function with compact support near  $r = 0$ . Consider the correlation function

$$\langle 1 | HO | 1 \rangle \quad (2)$$

where  $O$  is some operator. Expanding the unitary above and using the fact that  $H|0\rangle = 0$ , we find that

$$\langle 1 | HO | 1 \rangle = i \int dr d^{d-1} \Omega f(r) \langle 0 | \phi_1(r, t=0, \Omega) HO | 0 \rangle + O(f^2) \quad (3)$$

Now we note that

$$\phi_1(r, t=0, \Omega) H = H \phi_1(r, t=0, \Omega) + [\phi_1(r, t=0, \Omega), H] = H \phi_1(r, t=0, \Omega) - i \Pi_1(r, t=0, \Omega) \quad (4)$$

where  $\Pi_1$  is the canonically conjugate momentum. Again using  $\langle 0 | H = 0$ , the correlator above just becomes

$$\langle 1 | HO | 1 \rangle = \int dr d^{d-1} \Omega f(r) \langle 0 | \Pi_1(r, t=0, \Omega) O | 0 \rangle \quad (5)$$

Now, let us see what happens if we take  $O$  to be  $\phi_1$  or  $\phi_2$  near the boundary. In one case we find

$$\lim_{r \rightarrow \infty} r^\Delta \langle 1 | H \phi_1(r, t', \Omega) | 1 \rangle = N_\Delta \int f(r, \Omega) \frac{r^{d-1}}{\sqrt{1+r^2}} dr d^{d-1} \Omega \sin(t') \left[ \frac{1}{\sqrt{1+r^2} \cos(t') - r\Omega \cdot \Omega'} \right]^{\Delta+1}, \quad (6)$$

up to  $O(f^2)$  terms, which does not vanish provided  $t' \neq 0$  even if  $t'$  is very small. In section 4.2.1 of the paper it is shown that by means of such correlators in the range  $t' = [0, \epsilon]$  one can completely reconstruct the function  $f$ . For our purposes we only note that

$$\lim_{r \rightarrow \infty} r^\Delta \langle 1 | H \phi_2(r, t', \Omega) | 1 \rangle = 0 \quad (7)$$

Conversely in the state 2 we will find that  $\langle 2 | H \phi_2 | 2 \rangle$  does not vanish whereas  $\langle 2 | H \phi_1 | 2 \rangle$  does.

Notice that without the insertion of  $H$ , all correlators of  $\phi_1$  and  $\phi_2$  near the boundary in the time band  $t' = [0, \epsilon]$  are precisely those of the vacuum since the operator in (1) is a unitary operator. Also notice that insertions of only  $H$  are insufficient to distinguish the state.

## 2 Checking the constrained ansatz in QFT

### The ansatz.

We are looking to solve the constraint

$$\nabla^2 A_0(y) \Psi[\phi_1(x), \phi_2(x), A_0(x)] = q \hat{j}_0(y) \Psi[\phi_1(x), \phi_2(x), A_0(x)] \quad (8)$$

where the operator  $\hat{j}_0$  is given by

$$\hat{j}_0(y) = -i \left( \phi_1(y) \frac{\partial}{\partial \phi_2(y)} - \phi_2(y) \frac{\partial}{\partial \phi_1(y)} \right) \quad (9)$$

The conjectured ansatz is that starting with some choice of the wavefunctional for the matter fields, which we denote by  $\Psi^{\text{ng}}$ , we can lift it to a solution of the constraints in the following manner.

$$\Psi[\phi_1(x), \phi_2(x), A_0(x)] = \Psi^{\text{ng}}[\phi_1(x), \phi_2(x)] \delta(A_0(x) - A_0^{\phi_1, \phi_2}(x)) \quad (10)$$

where

$$A_0^{\phi_1, \phi_2}(x) = -q \int \mathcal{G}(x, x') \hat{j}_0(x') G[\phi_1(x'), \phi_2(x')] \quad (11)$$

and  $G[\phi_1(x'), \phi_2(x')] = -\log[\Psi^{\text{ng}}[\phi_1(x'), \phi_2(x')]]$ . Here  $\mathcal{G}$  is the Green's function (inverse of  $\nabla^2$ ). The functional delta function that appears in (10) can be interpreted as follows. Let us perform a lattice discretization so that the fields are defined at points  $x_1, x_2 \dots x_N$ . Then the delta function means

$$\delta(A_0(x) - A_0^{\phi_1, \phi_2}(x)) \equiv \prod_{i=1}^N \delta(A_0(x_i) - A_0^{\phi_1, \phi_2}(x_i)) \quad (12)$$

### Failure of the ansatz.

But this ansatz does not satisfy the constraint (8) since

$$\left( \nabla^2 A_0(y) - q \hat{j}_0(y) \right) \Psi^{\text{ng}}[\phi_1(x), \phi_2(x)] \delta(A_0(x) - A_0^{\phi_1, \phi_2}(x)) = R_1 - R_2 \quad (13)$$

where one term vanishes because

$$R_1 = \left[ \nabla^2 A_0^{\phi_1, \phi_2}(y) \Psi^{\text{ng}}[\phi_1(x), \phi_2(x)] - q \hat{j}_0(y) \Psi^{\text{ng}}[\phi_1(x), \phi_2(x)] \right] \delta(A_0(x) - A_0^{\phi_1, \phi_2}(x)) = 0 \quad (14)$$

but the other term does not vanish.

$$R_2 = \Psi^{\text{ng}}[\phi_1(x), \phi_2(x)] \delta'^{(y)}(A_0(x) - A_0^{\phi_1, \phi_2}(x)) q \hat{j}_0(y) A_0^{\phi_1, \phi_2} \neq 0. \quad (15)$$

Here the notation  $\delta'^{(y)}$  means the following. In the definition of the functional delta function in equation (12) we replace the delta function at the point  $y$  with a  $\delta'$  and leave all other delta functions unchanged. Note that the term  $R_2$  does not vanish even when we take  $\Psi^{\text{ng}}$  to correspond to the *vacuum* of the scalar fields in the absence of any gauge constraint. We

remind the reader that the vacuum of the scalar fields, in the absence of gauge fields, is given by<sup>1</sup>

$$\Psi^{\text{ng}}[\phi_1, \phi_2] = e^{-\int \phi_1(x)K(x-y)\phi_1(y) + \phi_2(x)K(x-y)\phi_2(y)d^3x d^3y} \quad (16)$$

where the kernel  $K$  can be found explicitly and is given by

$$K(x-y) = \int \frac{d^3k}{2(2\pi)^3} \sqrt{k^2 + m^2} e^{ik \cdot (x-y)} \quad (17)$$

In the notation of the note, this state corresponds to  $H = 1$  and  $G$  is the exponent in the expression (16). We see that the remainder (15) does not vanish for the choice of  $\Psi^{\text{ng}}$  in (16).

One might hope that perhaps since the remainder (15) looks like it is quadratic in  $q$ , one can fix the expression (11) by adding a quadratic term. But if one does this, then the term in equation (14) does not vanish. And moreover, it does not appear possible to cancel any quadratic remainder in (14) with (15) since equation (14) has a delta function whereas (15) has a derivative of a delta function. We show this in greater detail in section 3.

### 3 Constraints in Quantum Mechanics

To see the problem with the ansatz above in a simplified setting, let us consider a quantum mechanical model of the constraint. Say that we have two operators  $z_1$  and  $H$ . The operator  $\Pi_1$  is the canonical conjugate to  $z_1$ . We will denote eigenkets of  $z_1$  with the subscript 1 and eigenkets of  $H$  with subscript 2.

Say that we impose the constraint

$$\left[ H - \frac{q}{2}(\Pi_1^2 + z_1^2) \right] |\Psi\rangle = 0. \quad (18)$$

It is important that the constraint involves both  $\Pi_1$  and  $z_1$ . If one imposes a constraint that involves only  $z_1$  and not  $\Pi_1$  as is done in Prof. Mathur's note, then one does not see this issue.

**Correct solution.** The solution to this constraint can be found by inspection and the solutions are

$$|\Psi\rangle = \sum_{n=0}^{\infty} a_n H_n(z_1) e^{-\frac{z_1^2}{2}} |z_1\rangle_1 \otimes |q(n + \frac{1}{2})\rangle_2, \quad (19)$$

where  $H_n(z_1)$  are the Hermite polynomials and  $a_n$  are arbitrary coefficients. Note that the spectrum of the operator  $H$  is discrete. The eigenkets of that operator are normalized so that  ${}_2\langle q(m + \frac{1}{2}) | q(n + \frac{1}{2}) \rangle_2 = \delta_{mn}$ .

**Comparison with procedure in the note.** Now, let us compare this with the procedure in the note. The procedure of the note, if we understand correctly, is as follows. We start with an arbitrary wavefunction for  $z_1$  of the form

$$e^{g(z_1)}.$$

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<sup>1</sup>In particular, note that the vacuum is not the state  $\delta(\phi(x))$ , which would seem to correspond to " $\phi(x) = 0$ ". The vacuum has Gaussian fluctuations of the field and its conjugate momentum.

Then the procedure of the note seems to suggest that the solution of the joint wavefunction should be

$$\Psi_{\text{inc}}(z_1, H) = e^{g(z_1)} \delta\left(H - \frac{q}{2} (z_1^2 - g''(z_1) - g'(z_1)^2)\right), \quad (20)$$

so that the state can be written as

$$|\Psi\rangle = \int dz_1 dH \Psi_{\text{inc}}(z_1, H) |z_1\rangle_1 \otimes |H\rangle_2 \quad (21)$$

We see that equation (21) is not the same as equation (19) and is, therefore, incorrect. In particular, we find the following problems with this procedure.

1. **Spectrum:** The result (21) suggests a continuous spectrum for the observable  $H$  which is incorrect. As we see from equation (19), the correct spectrum for  $H$  should be discrete.
2. **Failure of linearity:** The constraint (18) is a linear constraint. So a sum of solutions should be a solution. But the ansatz (21) is not linear. If we take the starting wavefunction to be  $e^{g(z)} = e^{g_1(z_1)} + e^{g_2(z_1)}$  then the conjectured joint wavefunction does not superpose correctly.
3. **Failure to satisfy original constraint:** For a general choice of  $g(z_1)$  the ansatz (21) fails to satisfy the original constraint. For instance, let us take

$$g(z_1) = -2z_1^2 \quad (22)$$

Then the ansatz (20) leads to

$$\Psi_{\text{inc}}(z_1, H) = e^{-2z_1^2} \delta\left(H - \frac{q}{2} (4 - 15z_1^2)\right) \quad (23)$$

This wavefunction does not satisfy the constraint (18). We find that

$$\begin{aligned} \left[H - \frac{q}{2}(\Pi_1^2 + z_1^2)\right] \Psi_{\text{inc}}(z_1, H) &= \frac{15}{2} q^2 e^{-2z_1^2} \left(15qz_1^2 \delta''\left(\frac{15qz_1^2}{2} - 2q + H\right)\right. \\ &\quad \left.+ (1 - 8z_1^2) \delta'\left(\frac{15qz_1^2}{2} - 2q + H\right)\right) \end{aligned} \quad (24)$$

4. **Perturbation theory?** One might imagine that the remainder on the RHS of (24) is “higher order” in  $q$ . However, given that we have started with an ansatz that is not close to the correct answer in (19), it is not clear how adding higher order terms in  $q$  will help in finding a solution. From the note, it appears that adding higher order terms will result in a wavefunction

$$e^{-2z_1^2} \delta(H - f(q, z_1)), \quad (25)$$

where  $f(q, z_1)$  has some series expansion in  $q$ . But this leads to the equation

$$\begin{aligned} &\left(f(q, z_1) - \frac{q}{2}(4 - 15z_1^2)\right) \delta(H - f(q, z_1)) \\ &- q \frac{\partial^2 f(q, z_1)}{\partial z_1^2} \delta'(H - f(q, z_1)) - q \left(\frac{\partial f(q, z_1)}{\partial z_1}\right)^2 \delta''(H - f(q, z_1)) = 0 \end{aligned} \quad (26)$$



which does not appear to have any solution for  $f(q, z)$ . One might have tried an ansatz of the form

$$e^{-2z_1^2} (\delta(H - f(q, z_1)) + q\delta'(H - g(q, z_1)) + q^2\delta''(H - h(q, z_1)) + \dots), \quad (27)$$

but we are unable to find any procedure of choosing the functions  $f, g, h, \dots$  to make this expansion work.

This is not surprising since we know that, in the end, the ansatz (20) works *only* for a discrete set of very special choices of  $g(z_1)$  where  $e^{g(z_1)}$  is an eigenfunction of the operator  $-\frac{\partial}{\partial z_1^2} + z_1^2$ . So it only works if we restrict the starting point to be

$$g(z_1) = -\frac{z_1^2}{2} + \log(H_n(z_1)) \quad (28)$$

where  $H_n$  is a Hermite polynomial.<sup>2</sup> If we start with some arbitrary initial guess for  $g(z_1)$  that is not close to (28) (as, for instance, the initial guess (22)), we do not see how the perturbative procedure will converge to the correct answer.

**Conclusion.** If one was trying to solve the classical constraint

$$\langle H \rangle = \frac{q}{2} \langle \Pi_1^2 + z_1^2 \rangle \quad (29)$$

then it appears that a modification of the procedure above could be adopted to yield solutions. However, the proposed procedure does not correctly solve the quantum mechanical constraint (18).

## 4 Simplified constraint in QFT

We do not know how to write down an explicit solution to the full constraint equation 18 in quantum field theory. But let us consider a simplified constraint equation in analogy with section (3). In this simplified context, we can indicate how the analogy with section 3 should work.

Say that we have two dynamical fields  $\phi_1, \phi_2$  in AdS (so that we have a discrete spectrum) and we introduce one more degree of freedom  $H$  and impose the constraint

$$(H - qH_1 - qH_2)\Psi[\phi_1(x), \phi_2(x), H] = 0, \quad (30)$$

where  $H_1$  is the Hamiltonian of  $\phi_1$  and  $H_2$  is the Hamiltonian of  $\phi_2$ . In particular

$$H_1 = \int : [\Pi_1^2(x) + \sqrt{g}g^{ij}\partial_i\phi_1(x)\partial_j\phi_1(x) + m^2\phi_1^2] : d^3x \quad (31)$$

and similarly for  $H_2$ . We have inserted a normal ordering to ensure that the eigenvalues of  $H_1$  and  $H_2$  start from 0. The difference between this simplified constraint and the full Hamiltonian constraint is, of course, that we are not imposing a pointwise constraint but only a constraint between one degree of freedom “ $H$ ” and the sum of the energies of the two dynamical fields.

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<sup>2</sup>Even then the wavefunction (20) is not quite correct since the delta function does not lead to a normalizable wavefunction, and we do not see how to normalize it correctly.

Then we see that the solutions to this constraint, in analogy with the quantum mechanical case are

$$\sum a_{n,m} \Psi_{n,m}[\phi_1(x), \phi_2(x)] |H = q(n + m)\rangle \quad (32)$$

where  $\Psi_{n,m}[\phi_1(x), \phi_2(x)]$  is the wavefunctional corresponding to the  $n^{\text{th}}$  energy eigenstate of  $\phi_1$  and  $m^{\text{th}}$  energy eigenstate of  $\phi_2$  and  $a_{nm}$  are arbitrary coefficients. We now see that this has the right structure to give us the correlators in section 1.

On the other hand, the procedure in the note suggests that we should start with some wavefunction  $\Psi^{\text{ng}}[\phi_1(x), \phi_2(x)]$  and then simply lift the wavefunction, including  $H$  to

$$\Psi^{\text{ng}}[\phi_1(x), \phi_2(x)] \delta(H - qH^{\phi_1, \phi_2}) \quad (33)$$

where

$$H^{\phi_1, \phi_2} = \langle H_1 + H_2 \rangle \quad (34)$$

and the expectation value is taken in the state  $\Psi^{\text{ng}}$ . This again does not satisfy the original constraint (30). It leads to an incorrect spectrum for  $H$ , which is continuous rather than discrete. It also does not give the right correlators of section 1 since it does not have the right entanglement structure.