Responses to referee report

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We thank the referee for the quick response. I also want to let the referee take some time to have a look at the papers we cite in the replies. We are also glad that some of our replies are accepted by the referee, such as the spin-wave analysis.

1. *R: spin 1/2 is equal to a hard-core boson .(this is the Schwinger boson approach). i agree that soft boson are not equal to spin-1/2 in the absence of strong interaction, but most literature of today on bosonic HOTI is about hardcore boson or spins.*

   We agree that spin 1/2 is equal to a hard-core boson. For spin, the well studied model is Heisenberg model. For hard-core boson, the usual model is Bose-Hubbard model. The two model is totally different. In the spin language, the Bose-Hubbard model with a nearest-neighbor interaction corresponds to a ferromagnetic coupling for the XY component, and an antiferromagnetic coupling for the z component.

   As we have stated: "As far as we know, the bosonic higher-order TI based on Bose-Hubbard model is only numerically studied in "Fractional corner charges in a two dimensional superlattice Bose-hubbard model by Julian Bibo, Izabella Lovas, Yizhi You, Fabian Grusdt, and Frank Pollmann, Phys. Rev. B 102, 041126 (2020)." This study is dealing with a softcore model. We are very confused by the statement of the referee "most literature of today on bosonic HOTI is about hardcore boson or spins." Would the referee list the study on bosonic HOTI of hardcore boson? To be specific, we include the content of Phys. Rev. B 102, 041126 (2020) in Fig.1.

2. *quote: "We hope that the referee can change his/her attitude This statement is totally wrong by reading the following paper: Correlation effects in quadrupole insulators: A quantum Monte Carlo study by Chen Peng, Rong-Qiang He, and Zhong-Yi Lu, Phys. Rev. B 102, 045110 (2020). A noninteracting topological Hamiltonian is constructed from the zero-frequency Greens function, and then the method of nested Wilson loop is used to calculate the higher-order topological invariant for the noninteracting topological Hamiltonian. In the section of introduction, we review the progress of the related studies, and intro- duce the results published in the journal with a good reputation. We hope the referee can change the mind that we have included misleading information."*
   R:I am sorry my attitude is not changed since it is not about attitude, but scientific soundness.
   As I say, the nested Wilson loop is not a topological invariant to measure the strong

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Fractional corner charges in a two-dimensional superlattice Bose-Hubbard model

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Model. We consider the following SL-BHM on a two-dimensional square lattice of size $L \times L$ with L even,

$$\hat{H} = -\left[\sum_{x=1}^{L-1}\sum_{y=1}^{L}(t(x)\hat{a}^{\dagger}_{x,y}\hat{a}_{x+1,y} + \text{H.c.}) + x \leftrightarrow y\right] + \frac{U}{2}\sum_{x,y=1}^{L}\hat{n}_{x,y}(\hat{n}_{x,y} - 1), \quad (1)$$

where $\hat{a}^{\dagger}_{x,y}$ $(\hat{a}_{x,y})$ is the creation (annihilation) operator at site (x, y), and $\hat{n}_{x,y} = \hat{a}^{\dagger}_{x,y}\hat{a}_{x,y}$. The particles can tunnel between neighboring sites with modulated hopping amplitudes

Figure 1: From Phys. Rev. B 102, 041126 (2020)

interacting HOTI. what I mean by topological invariant is it works for all parameter regions in the HOTI phase regardless of what type of interaction you have. Or, an equivalent statement is if I have two Hamiltonians A and B inside the same HOTI phase(with unique GS wave functions) that can be adiabatically connected with finite local unitary steps, if nested Wilson loop works for A, it should work for B.
However, for your model, as long as I add strong ring-exchange interaction or off-diag hopping term between the plaquette, the system is still in the same HOTI phase but the nested Wilson loop disappears.
In addition, one relevant question, as is shown by Hughes et. al., the change of nested Wilson loop does not indicate a bulk gap closing, it could be a mere boundary gap closing. Hence, the nested Wilson loop is more like a fingerprint for boundary obstructed phase. I feel the author is exploring a boundary obstructed phase here. (see 4 for further discussion)

Here we introduce a related study Phys. Rev. B 102, 045110 (2020) in the introduction section, where a noninteracting topological Hamiltonian is constructed from the zero-frequency Greens function, and then the method of nested Wilson loop is used to calculate the higher-order topological invariant for the noninteracting topological Hamiltonian.

We does not want to make further comment, but just put the content of the paper in Fig.2. If we understand correctly, they are using the method of the nested Wilson loop for this specific case.

We are very confused by the referee's comment: "However, for your model, as long as I add strong ring-exchange interaction or off-diag hopping term between the plaquette, the system is still in the same HOTI phase but the nested Wilson loop disappears." We do not say anything about our model using the nested Wilson loop. Please note here it is in the introduction.

We do not want to make more comments, but still thank the referee's interest in our work.

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Correlation effects in quadrupole insulators: A quantum Monte Carlo study

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C. The Green's function formalism for topological invariants

The zero-frequency Green's function $G(\mathbf{k}, i\omega = 0)$ is Hermitian, and we can obtain all the real eigenvalues by diagonalizing it. According to Refs. [44,45], information about the topological properties and gapless surface states is contained in $G(\mathbf{k}, i\omega = 0)$. After defining a so-called topological Hamiltonian $h_t(\mathbf{k}) = -G^{-1}(\mathbf{k}, i\omega = 0)$, we can calculate the topological invariant as if $h_t(\mathbf{k})$ would be a noninteracting Bloch Hamiltonian. The reason why the formalism works well in interacting systems can be simply understood as follows: When we add a small interaction with strength U to a noninteracting system $H_0(\mathbf{k})$, the topological invariant ν should keep unchanged $\partial_U \nu = 0$ [46]. From the relation $G(\mathbf{k}, i\omega) = 1/[i\omega - H(\mathbf{k}) - \Sigma(\mathbf{k}, i\omega)]$, we find that the topological Hamiltonian $h_t(\mathbf{k})$ also adiabatically connects to $H_0(\mathbf{k})$. Hence, it is not surprising that we can simply use $h_t(\mathbf{k})$ as a Bloch Hamiltonian to capture the topological properties in a correlated system.

In conventional topological systems, the Green's function formalism is widely used in calculating topological invariants like winding numbers, Chern numbers, and Z_2 invariants [47]. In our work, we attempt to apply the same idea to the novel topological phase, quadrupole insulators, by calculating the topological invariant $\mathbf{P}^{\nu}$ with the topological Hamiltonian $h_t(\mathbf{k})$, similar to the noninteracting case (shown in Sec. II B).

Figure 2: From Phys. Rev. B 102, 045110 (2020)