Ms. Ref.: scipost_202105_00029v1 (*Gravity waves in a rainbow universe*)

Dear editor and reviewer,

Thank you for giving us the opportunity to submit a revised draft of our manuscript for publication process in the *SciPost Physics* journal. We appreciate the time and effort that you and the reviewers dedicated to provide feedback on our manuscript and are grateful for the insightful comments on and valuable improvements to our submission. We have carefully considered the suggestions to improve our manuscript. We hope that the revised version of the submission is now suitable for publication in the *SciPost Physics* journal.

Responses to Reviewer:

The paper entitled "Gravity waves in a rainbow universe" deals with the polar modes of GWs employing the formalism of gravity's rainbow, which is based largely on the product of the disparity between quantum mechanics (QM) and general relativity (GR). They have adopted the perturbation of the spatially flat conformal Friedmann-Lemaitre-Robertson-Walker (FLRW) metric using the Regge-Wheeler formalism.

There are some serious questions:

Comment #1: Is the Planck length fundamental?

Response #1:

As is well known, Planck realized that a length, mass and time interval can be defined by considering fundamental constants ($\hbar, c, G$) as follows:

$$l_P = \sqrt{\frac{\hbar\, G}{c^3}}, \quad m_P = \sqrt{\frac{\hbar\, c}{G}}, \quad t_P = \sqrt{\frac{\hbar\, G}{c^5}}, \quad E_P = \sqrt{\frac{\hbar\, c^5}{G}}.$$

The Planck scale sets a limit for known physics. This means we need a new physics under the Planck scale. Moreover, In Ref. [5], Calmet showed that a unification of quantum mechanics and general relativity implies that there is a fundamental length in Nature in the sense that no operational procedure would be able to measure distances shorter than the Planck length. Thus, one can naturally take into account that they are in some sense fundamental constants for the time being. Otherwise, whether they are a fundamental constant is still a current problem (please see

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5255898/).

Comment #2: In rainbow gravity [Joao Magueijo and Lee Smolin, Class. Quantum Grav., 21, 1725, (2004).], the geometry of spacetime is energy (E) dependent and then quanta of different energies may support different classical geometries. In this context, is the fundamental length energy dependent?

Response #2:

We have used a set of units in which $\hbar = c = 1$, so that $l_P = \sqrt{G} = E_P^{-1}$. Newton's constant did not depend on energy, and all energy dependence was gathered in rainbow functions in the original formalism of rainbow gravity introduced in Ref. [29]. In other words, l_P is energy independent in our study.

Comment #3: Justify the choices of the rainbow functions (eq. 3), since this type of rainbow functions cannot be seen in Ref. 29, as stated in your paper.

Response #3:

The referee is right. We have rewritten the relevant part of the manuscript as follows:

"By using the varying light speed idea, Feng and Yang [30] introduced a choice for the rainbow functions:

$$f(\varepsilon) = \frac{1}{1 - \gamma\varepsilon}, \quad g(\varepsilon) = 1.$$

This model indicates that spacetime has an energy-dependent velocity c=$1 - \gamma\varepsilon$, where γ is the rainbow parameter, and the varying velocity of light takes smaller values while the energy of photons increases [30]."

Besides, we have replaced reference [30] with the following correct reference:

"Z. W. Feng and S. Z. Yang, Phys. Lett. B 772 (2017) 737."

Comment #4: Justify the relation between the last paragraph of page 24 and the Fig-1 of page 25 i.e., with experimental (Planck data) result.

Response #4:

The referee is right. We have redrawn the FIG. 1 to include the constraints given in Eq. (25). We have also checked whether the information given under Eq. (24) is consistent FIG. 1. We have not seen any inconsistency.

Comment #5: Does the value of the H0 change in the presence of Rainbow functions?

Response #5:

H_0 represents the Hubble constant, which is the present value of the Hubble parameter, and it is based on experimental observations. Thus, rainbow functions do not affect the value of the Hubble constant. It is significant to emphasize that the Hubble parameter, $H_c = \frac{\dot{A}}{A}$ is also energy independent since a factor of rainbow functions in both numerator and denominator.

Comment #6: Briefly explain how the equation 85 obeys Huygens' principle.

Response #6:

It is known that Huygens principle is not satisfied by gravitational waves. However, based on the fact that electromagnetic waves satisfy the Huygens principle in flat space-time, Malec and Wylezec [45] showed that in the Friedmann space, the propagation equations of gravitational waves (axial modes) provide the Huygens principle under the Regge-Wheeler condition in the radiation era. This means form of the equation of motion in Friedman universe (Eq. 84) is exactly that of the electromagnetic fields. Malec and Wylezec also proved that $\Phi_l(\tau, r)$ is a general solution of the Eq. (84) for the radiation area (w=1/3). So, we can say that Eq. (85) holds Huygens' principle.

Comment #7: In equation 89, the authors have used the new parameters /tao and /xi. Then the equation 89 becomes free from rainbow functions explicitly, how is it physically justified?

Response #7:

The main purpose here is to show that gravitational waves satisfy the Huygens' principle. The way to do this is to reach an equation similar to the differential equation provided by electromagnetic waves, which hold the Huygens' principle, as has been done before in the literature [45]. These transformations are purely mathematical tools, and thus, we cannot say that Eq. (89) is free from rainbow functions. When the explicit solution of Eq. (89) is made, which is the subject of another study, can be expressed in terms of rainbow functions by reusing $\eta = f\tau$ and $r = g\varsigma$ transformations.

Gravity waves in a rainbow universe

M. Salti*[1] and O. Aydogdu†[1]

[1] *Department of Physics, Faculty of Arts and Science,*
Mersin University, Mersin, TR 33343, Turkey

Abstract

The dawn of the epoch of gravitational wave (GW) astronomy, which initiated with the detection of a fundamental noise, was the period when the search for a theory in which gravity could be quantized began to increase significantly. In this paper, we have mainly intended to focus on the polar modes of GWs in the formalism of gravity's rainbow, which is based largely on the product of the disparity between quantum mechanics (QM) and general relativity (GR). For this purpose, we have perturbed the spatially flat conformal Friedmann-Lemaitre-Robertson-Walker (FLRW) metric, material distribution and the components of four-velocity by making use of the polar Regge-Wheeler gauge and formulated the corresponding field equations for both the zeroth-order (unperturbed) and the first-order (perturbed) cases of the metric. Subsequently, these field equations have been taken into account simultaneously to get exact expressions of the gauge functions. From a graphical perspective, we have studied the impact of rainbow parameters on the amplitude of GWs. At the final step, we have discussed the Huygens Principle and concluded that the GWs obey the principle only in the radiation-dominated era and the principle is broken otherwise.

PACS numbers: 04.30.-w, 04.50.+h, 12.38.Bx

Keywords: Gravitational waves, gravity's rainbow, cosmology, perturbation.

* msalti@mersin.edu.tr
† oktaydogdu@mersin.edu.tr

I. INTRODUCTION

It is generally believed that the Planck length ($l_p \sim 10^{-33}$cm)[1] is the minimum length scale that can be observed or measured in a laboratory. On the other hand, in a cosmological framework, there is an idea that the unification of GR and QM indicates the notion of a fundamental length[2]. It has been long supposed that the Planck length can set a limit, which is eliminating the big bang singularity problem, on spacetime curvature in the quantum gravity framework[3, 4]. Recently, it has been proven that there is no any operational procedure that can exclude the discreteness of spacetime on distances shorter than the Planck length[3]. Therefore, we should take the Planck length as a fundamental length, or in other words, any quantum gravity theory must imply that measuring a distance shorter than the Planck length is prohibited.[5]. One of the generally accepted properties of quantum gravity is the existence of a minimum measurable length, however, the difficulty of constructing a quantum theory of gravity is notoriously known[6–8].

Although recent investigations performed in loop quantum gravity (LQG), which aims to merge QM and the age-old theory of GR, have greatly strengthened the belief in the existence of a minimal measurable length, these developments have also inspired numerous investigations and argumentations on the Lorentz symmetry[9–12]. Nowadays, it is commonly believed that doubly special relativity (DSR) can be used to solve the paradox, which comes from the apparent confliction between the Lorentz symmetry and the existence of minimal length[13–16]. The DSR is a deformed version of the special theory of relativity (SR) and it was proposed to keep inertial frames relative while making the Planck energy an invariant scale[13, 17, 18]. Actually, this goal can be achieved with the help of a non-linear Lorentz transformation in momentum space, thus the usual formulations of energy-momentum (or dispersion relations) in the SR may be altered by making use of adjustments in the order of Planck length[19, 20]. Modification of energy dispersion relation can also be performed via the semi-classical limit of LQG[21, 22]. According to the observational perspective[23–26], such modifications cause not only threshold anomalies of TeV photons but also ultra-high-energy cosmic rays, which means the DSR also faces with problematic issues. Also, in Refs.[27] and [28], it was shown that a modified dispersion relation (MDR) may lead to alternatives to inflationary cosmology and this can be tested via the future measurement of cosmic microwave background spectrum. In a recent paper, Magueijo and Smolin[29] have extended the MDR idea to the curved spacetime by proposing a deformed equivalence principle (DEP) of the GR. In this formalism, it is mainly stated that[29] (i) the free-falling observers will encounter the same laws as in the DSR, and (ii) there is no single classical spacetime geometry explored by a particle traveling in it when the impacts of the probe itself are considered. According to this idea, the spacetime geometry is altered by energy of the test particle. Therefore, particles having diverse amounts of energy feel different space-time tissue and the energy-dependent definitions construct a rainbow of metrics. As pointed out in Ref.[29], in a DSR formalism[18], the MDR may be written as

$$f^2(\varepsilon)E^2 - g^2(\varepsilon)P^2 = m^2 \tag{1}$$

with $\varepsilon = \frac{E}{E_{pl}}$, where E_{pl} denotes the Planck energy. Here, $f(\varepsilon)$ and $g(\varepsilon)$ are called rainbow functions, which are necessary to approach to unit when $\frac{E}{M_{pl}} \ll 1$ due to the correspondence principle and to fulfill the requirements

$$\lim_{\varepsilon\to 0} f(\varepsilon) = \lim_{\varepsilon\to 0} g(\varepsilon) = 1. \tag{2}$$

By using the varying light speed idea,Feng and Yang[30] introduced a choice for the rainbow functions:

$$f(\varepsilon) = \frac{1}{1-\gamma\varepsilon}, \quad g(\varepsilon) = 1. \tag{3}$$

This model indicates that spacetime has an energy-dependent velocity $c = 1 - \gamma\varepsilon$, where γ is the rainbow parameter, and the varying velocity of light takes smaller values while the energy of photons increases[30]. In literature, there are other significant scenarios for the rainbow functions[15, 31, 32]:

$$f(\varepsilon) = \sqrt{1-\varepsilon^2}, \quad g(\varepsilon) = 1, \tag{4}$$

$$f(\varepsilon) = g(\varepsilon) = \frac{1}{1-\varepsilon}, \tag{5}$$

$$f(\varepsilon) = 1, \quad g(\varepsilon) = 1 + \frac{\varepsilon}{2}. \tag{6}$$

It is generally accepted that predicting the existence of GWs is one of the most significant achievements of the GR[33]. Important events such as the big bang and the mergence of two black holes may cause the creation of such waves. Nearly 100 years later after Einstein's prediction of the existence of GWs, the LIGOVIRGO cooperation observed them for the first time by detecting a signal corresponding to the event GW150914 (two black holes crashed into each another)[34]. Later, this conclusion has been endorsed by other events such as the GW151226[35], GW170104[36], GW170608[37], GW170814[38] and GW170817[39] (please visit the Gravitational Wave Open Science Center website[40] for detailed information and more events). Detection of such fundamental noise may justify the quantization of gravity and reveal significant features of its sources. On this purpose, the cosmological perturbations have become a noteworthy achievement not only in identifying the formation of structure in the Universe but also modeling dynamics of the GWs theoretically. Regge and Wheeler[41] introduced gauge independent perturbation formalisms for the axial and polar modes of GWs in the Schwarzschild spacetime and found stable solutions. Thereafter, focusing on the gravitational radiation propagated by a black hole, Zerilli[42] improved the polar mode calculations by correcting the minor mistakes made by Regge and Wheeler. Later on, Lindblom and Detweiler[43] investigated the f-modes for various equations-of-states numerically by making use of the ReggeWheeler gauge, Chandrasekhar and Ferrari[44] discussed polar perturbations in a diagonal gauge, Malec and Wylężek[45] used the ReggeWheeler formalism in the FLRW cosmology to investigate the propagation of GWs and check the validity of Huygens' principle, Passamonti et al.[46] studied the nature of GWs by focusing on nonlinear couplings of radial and polar non-radial modes in relativistic stars, and Bradley et al.[47] worked on the StewartWalker gauge[48] to investigate GWs locally rotationally symmetric class-II spacetimes.

Studying the behavior of GWs by making use of perturbation formalisms such as the ReggeWheeler[41] and the StewartWalker[48] schemes has not been discussed widely in alternatives to the GR theory. Recently, Sharif and Siddiqa investigated axial[49] and polar[50] modes of GWs in the $f(R,T)$-gravity, Salti[51] studied the propagation of axial GWs in Rastall gravity, and Salti et al.[52] investigated polar modes of GWs in Rastall cosmology. In this study, we intend to discuss the impact of MDR on GW physics. The article has the following format. In the second section, we present briefly the background metric, material content and the perturbation scheme. Subsequently, we focus on the zeroth-order (or unperturbed) case and its cosmological indications in the third section. The fourth section includes the perturbed background equations and effects of polar perturbations. The fifth section consists of discussions on Huygens principle. The summary of our investigation is presented in the final section. We accomplish some parts of the investigation and graphical analyses by means of the high-level programming environment Mathematica. Note that, throughout this study, the natural Planck unit $c = \hbar = 8\pi G = 1$ is adopted.

II. THE PERTURBATION SCHEME

It is known that the ordinary time t is connected to the conformal one η by the expression $Ad\eta = dt$, where the factor A represents cosmic scale. Making use of such transformation is beneficial especially while discussing dynamics of photons traveling radially from an observer. Thus, such photons obey the relation

$$d - d_0 = \eta - \eta_0 = \int \frac{dt}{A}, \tag{7}$$

which purposes time and distance can be replaced each other. Therefore, the conformally flat FLRW metric tensor reads

$$g_{\mu\nu} = A^2(\eta)\left[\delta^1_\mu\delta^1_\nu + r^2\delta^2_\mu\delta^2_\nu + (r\sin\theta)^2\delta^3_\mu\delta^3_\nu - \delta^0_\mu\delta^0_\nu\right]. \tag{8}$$

Here, we can define a conformal Hubble parameter:

$$H_c = \frac{\dot{A}}{A} = \frac{d\ln A}{d\eta}, \tag{9}$$

where the dot reveals differentiation corresponding to η. Consequently, one can define

$$\eta = \int \frac{dt}{A} = \int \frac{d\ln A}{H_c}. \tag{10}$$

Following Refs.[18, 29, 30], we can construct rainbow metrics by replacing $d\eta \rightarrow \frac{d\eta}{f(\varepsilon)}$ for the conformal time coordinate and $dx^i \rightarrow \frac{dx^i}{g(\varepsilon)}$ for all spatial coordinates. Thence, the metric tensor (8), in the rainbow formalism, takes the form

$$g_{\mu\nu} = A^2(\eta)\left[\frac{\delta^1_\mu\delta^1_\nu + r^2\delta^2_\mu\delta^2_\nu + (r\sin\theta)^2\delta^3_\mu\delta^3_\nu}{g^2} - \frac{\delta^0_\mu\delta^0_\nu}{f^2}\right]. \tag{11}$$

In order to investigate the propagation of PGWs in the gravity's rainbow framework, we can use the ReggeWheeler scheme[41]. In a recent paper, Rostworowski[53] has discussed linear perturbations of the FLRW model by employing results presented in Ref.[41] and concluded that the set of field equations can be abated in the case of solving a single master scalar wave equation. Similar to the black hole perturbation schemes, two copies of master equation identify axial and polar parts of perturbations[53]. In accordance with the ReggeWheeler formalism[41], it is written that

$$\breve{g}_{\mu\nu} = g_{\mu\nu} + eh_{\mu\nu} + O(e^2), \tag{12}$$

where $\breve{g}_{\mu\nu}$ denotes the perturbed metric while the corresponding perturbations are represented by $h_{\mu\nu}$ and the parameter e measures the oscillation strength. It is significant to note here that higher-order terms of the strength parameter e will be ignored in our investigation. According to the polar perturbation gauge proposed by Clarkson et al. in Ref.[54], we can write

$$h_{\mu\nu} = Y\left[(\psi_1+\psi_2)(\frac{\delta^0_\mu\delta^0_\nu}{f^2}+\frac{\delta^1_\mu\delta^1_\nu}{g^2})+\frac{r^2\psi_2}{g^2}(\delta^2_\mu\delta^2_\nu+\sin^2\theta\delta^3_\mu\delta^3_\nu)+\frac{\psi_3}{fg}(\delta^0_\mu\delta^1_\nu+\delta^1_\mu\delta^0_\nu)\right] \quad (13)$$

with[55]

$$\frac{d^2Y}{d\theta^2} = -l(l+1)Y - \cot\theta\frac{dY}{d\theta}. \quad (14)$$

Here, $\psi_1 = \psi_1(\eta, r)$, $\psi_2 = \psi_2(\eta, r)$, $\psi_3 = \psi_3(\eta, r)$ and $Y = Y(\theta) := Y_l^m(\theta)$ is the standard spherical harmonics, where l corresponds to the angular momentum and m implies its projection on z-axis[41]. Here, we assume $m = 0$ and $l = 2, 3, 4, ...$ to study a wave-like solution[41]. This assumption means that ϕ will be vanished in our results. As a result, the non-zero components of perturbed metric tensor are given as follows:

$$\breve{g}_{\mu\nu} = \frac{e(\psi_1+\psi_2)Y - A^2}{f^2}\delta^0_\mu\delta^0_\nu + \frac{e(\psi_1+\psi_2)Y + A^2}{g^2}\delta^1_\mu\delta^1_\nu + \frac{r^2}{g^2}[e\psi_2 Y + A^2]\delta^2_\mu\delta^2_\nu + \left(\frac{r\sin\theta}{g}\right)^2[e\psi_2 Y + A^2]\delta^3_\mu\delta^3_\nu + \frac{e\psi_3 Y}{fg}(\delta^0_\mu\delta^1_\nu+\delta^1_\mu\delta^0_\nu) + O(e^2). \quad (15)$$

Stress-energy tensor describing the background matter is generally written as

$$T^\mu_\nu = (\rho+p)V^\mu V_\nu + pg^\mu_\nu, \quad (16)$$

where ρ, p and V_μ denote the energy density, pressure and the components of four-velocity, respectively. Here, we can assume also that[55]

$$\breve{\rho} = \rho + e\varphi_1(\eta, r)Y\rho + O(e^2), \quad (17)$$

$$\breve{p} = p + e\varphi_2(\eta, r)Yp + O(e^2), \quad (18)$$

where $e\varphi_1(\eta, r)Y$ and $e\varphi_2(\eta, r)Y$ indicate the corresponding contrasts. In such a perturbed scheme, the fluid also feels distortions and it is not necessarily comoving. Moreover, the appropriate velocity in the rainbow framework is defined as[56]

$$\Xi_i \equiv \frac{\frac{dx^i}{g}}{\frac{dx^0}{f}} = \frac{V^i}{V^0}, \quad (19)$$

where the perturbed four-velocity reads[55]

$$V_\mu \Rightarrow \begin{cases} V_0 = -\frac{A}{f} + \frac{e(\psi_1+\psi_2)Y}{2Af} + O(e^2) \\ V_1 = \frac{eA\xi_1(\eta,r)Y}{g} + O(e^2) \\ V_2 = \frac{e\xi_2(\eta,r)}{g}\frac{dY}{d\theta} + O(e^2) \\ V_3 = \frac{e\sin\theta\xi_3(\eta,r)}{g}\frac{dY}{d\theta} + O(e^2) \end{cases} \tag{20}$$

and satisfies the normalization condition $V_\mu V^\mu = -1 + O(e^2)$ with the help of auxiliary functions ξ_1, ξ_2 and ξ_3.

III. COSMOLOGY VIA ZEROTH-ORDER FIELD EQUATIONS

Substituting the metric (11) into the field equations $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}$ leads us to the subsequent Friedmann equations

$$3H_c^2 = \frac{A^2}{f^2}\rho, \tag{21}$$

$$H_c^2 + 2\dot{H}_c = -\frac{A^2}{f^2}p. \tag{22}$$

Also, the conservation formula $\nabla_\mu T^\mu_\nu = 0$ can be written explicitly as[4] $\dot{\rho}+3H_c(\rho+p) = 0$. It is noteworthy to remind here that equations (21)-(22) and the conservation relation are dependent on each other. We can easily see that the stress-energy conservation relation can be obtained also by making use of equations (21) and (22).

Now, with the help of equations (21) and (22), we obtain

$$\dot{H}_c = -\sigma H_c^2. \tag{23}$$

with

$$\sigma = \frac{1+3\omega}{2}, \tag{24}$$

where $\omega = \frac{p}{\rho}$ is the Equation-of-State (EoS) parameter of background material. The cold dark matter ($\omega = 0$), dust matter ($\omega = 0$), radiation ($\omega = \frac{1}{3}$), ultra-relativistic particle ($\omega = \frac{1}{2}$), sub-relativistic matter ($\omega = \frac{1}{4}$), and the stiff fluid ($\omega = 1$) dominated epochs lead to the deceleration behavior (solid orange line in FIG. 1) of the cosmos while the phantom dark energy ($\omega < -1$), cosmological constant ($\omega = -1$), incompressible fluid ($\omega = -1$) and the quintessence dark energy ($-1 < \omega < -\frac{1}{3}$) dominated phases indicate the speedy expansion state (dashed line in FIG. 1). According to the data obtained from the Planck Telescope[57], it is now known that there is a limit on the value of EoS parameter:

$$\omega > \begin{cases} -1.56^{-0.48}_{+0.60} & \text{(Planck) TT+lowE,} \\ -1.58^{-0.41}_{+0.52} & \text{(Planck) TT, TE, EE+lowE,} \\ -1.57^{-0.40}_{+0.50} & \text{(Planck) TT, TE, EE+lowE+lensing.} \end{cases} \tag{25}$$

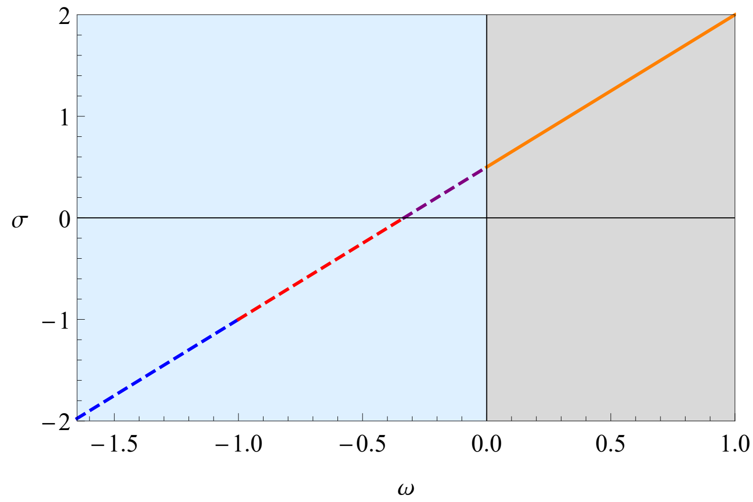

FIG. 1: Illustration of σ versus ω.

Integrating equation (23), as a result, we achieve that

$$\frac{H_c(\eta)}{H_0} = \frac{1}{1 + \sigma H_0(\eta - \eta_0)}, \tag{26}$$

where $H_0 = H_c(\eta_0)$ is the Hubble constant. Thus, one can say that the big bang happens at the following critical time

$$\eta_{cri} = \eta_0 - \frac{1}{\sigma H_0}. \tag{27}$$

In addition to this, the cosmic scale factor is given by

$$\frac{A(\eta)}{A_0} = \left[1 + \sigma H_0(\eta - \eta_0)\right]^{\frac{1}{\sigma}}, \tag{28}$$

where $A_0 = A(\eta_0)$ denotes the current value of the parameter. According to the recent astrophysical data observed via the LIGO and VIRGO Interferometers[58], $H_0 = 70.00^{+12.00}_{-8.00}$ km Sec^{-1} Mpc^{-1}. Finally, substituting expressions (26) and (28) into equation (21) leads us to a relation for the background material distribution:

$$\rho = \frac{3f^2 H_0^2}{A_0^2} \left[1 + \sigma H_0(\eta - \eta_0)\right]^{2\left(1+\frac{1}{\sigma}\right)}. \tag{29}$$

IV. FIRST-ORDER PERTURBATIONS AND THEIR EFFECTS

Each non-zero component of the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, up to the first-order in perturbations, are obtained as follows:

$$G_{00} = \frac{eY}{2f^2r^2A^2}\Big\{\Big(g^2[l(l+1)+2] - 4f^2r^2H_c^2\Big)\psi_1 + 2r^2f^2H_c\dot{\psi}_1 + 2g^2r\psi_1' \\ +2\Big(g^2l(l+1) - 6f^2r^2H_c^2\Big)\psi_2 + 6f^2r^2H_c\dot{\psi}_2 - 4rg^2\psi_2' \\ -2r^2g^2\psi_2'' - 4rfgH_c\left(2\psi_3 + r\psi_3'\right)\Big\} + 3H_c^2, \tag{30}$$

$$G_{01} = G_{10} = \frac{eY}{2fgr^2A^2}\Big\{\Big(g^2l(l+1) - 4f^2r^2\dot{H}_c - 2f^2r^2H_c^2\Big)\psi_3 \\ -2fgrH_c(2\psi_1 + r\psi_1' - r\psi_2') + 2fgr(\dot{\psi}_1 - r\dot{\psi}_2')\Big\}, \tag{31}$$

$$G_{02} = G_{20} = \frac{e}{2fA^2}\frac{dY}{d\theta}\Big\{2f(H_c\psi_2 - \dot{\psi}_2) + g\psi_3' - f\dot{\psi}_1\Big\}, \tag{32}$$

$$G_{11} = -\frac{f^2}{g^2}\Big[H_c^2 + 2\dot{H}_c\Big] + \frac{eY}{2g^2r^2A^2}\Big\{\Big(g^2[l(l+1)-2] - 8f^2r^2\dot{H}_c\Big)\psi_1 - 2rg^2\psi_1' \\ +4rfg\dot{\psi}_3 - 4f^2r^2\dot{H}_c\psi_2 - 2f^2r^2H_c(\dot{\psi}_1 - \dot{\psi}_2) - 2f^2r^2\ddot{\psi}_2\Big\}, \tag{33}$$

$$G_{12} = G_{21} = \frac{e}{2gA^2}\frac{dY}{d\theta}\Big[g\psi_1' - f\dot{\psi}_3\Big], \tag{34}$$

$$G_{22} = \frac{G_{33}}{\sin^2\theta} = -\frac{f^2r^2}{g^2}\Big[H_c^2 + 2\dot{H}_c\Big] - \frac{erY}{2g^2A^2}\Big\{-2f^2r(H_c^2 - \dot{H}_c)\psi_1 + 2g^2\psi_1' + g^2r\psi_1'' \\ +f^2r\ddot{\psi}_1 + 4f^2r\dot{H}_c\psi_2 - 2f^2rH_c\dot{\psi}_2 + 2f^2r\ddot{\psi}_2 - 2fg\dot{\psi}_3 - 2fgr\dot{\psi}_3'\Big\}, \tag{35}$$

where the prime represents the derivative with respect to r. On the other hand, the surviving components of $T_{\mu\nu} = g_{\mu\alpha}T^{\alpha}_{\nu}$, up to first order in perturbations, are calculated as

$$T_{00} = \frac{eY\rho}{f^2}\Big[A^2\varphi_1 - \psi_1 - \psi_2\Big] + \frac{A^2}{f^2}\rho, \tag{36}$$

$$T_{01} = T_{10} = -\frac{eY}{fg}\Big[\xi_1A^2(\rho+p) - p\psi_3\Big], \tag{37}$$

$$T_{02} = T_{20} = -\frac{eA\xi_2}{fg}(\rho+p)\frac{dY}{d\theta}, \tag{38}$$

$$T_{03} = T_{30} = -\frac{eA\xi_3\sin\theta}{fg}(\rho+p)\frac{dY}{d\theta}, \tag{39}$$

$$T_{11}=\frac{A^2p}{g^2}+\frac{epY}{g^2}\left[A^2\varphi_2+\psi_1+\psi_2\right], \tag{40}$$

$$T_{22}=\frac{T_{33}}{\sin^2\theta}=\frac{A^2r^2p}{g^2}+\frac{epr^2Y}{g^2}\left[A^2\varphi_2+\psi_2\right]. \tag{41}$$

So, the perturbed field equations are written as follows

$$\begin{aligned}&\frac{eY}{2f^2r^2A^2}\left\{\left(g^2[l(l+1)+2]-4f^2r^2H_c^2\right)\psi_1+2r^2f^2H_c\dot{\psi}_1+2g^2r\psi_1'\right.\\&\qquad+2\left(g^2l(l+1)-6f^2r^2H_c^2\right)\psi_2+6f^2r^2H_c\dot{\psi}_2-4rg^2\psi_2'\\&\left.-2r^2g^2\psi_2''-4rfgH_c\left(2\psi_3+r\psi_3'\right)\right\}+3H_c^2=\frac{eY\rho}{f^2}\left[A^2\varphi_1-\psi_1-\psi_2\right]+\frac{A^2}{f^2}\rho,\end{aligned} \tag{42}$$

$$\begin{aligned}&\left[g^2l(l+1)-4f^2r^2\dot{H}_c-2f^2r^2H_c^2\right]\psi_3-2fgrH_c(2\psi_1+r\psi_1'-r\psi_2')\\&\qquad+2fgr(\dot{\psi}_1-r\dot{\psi}_2')=-2r^2A^2\left[\xi_1A^2(\rho+p)-p\psi_3\right],\end{aligned} \tag{43}$$

$$2f(H_c\psi_2-\dot{\psi}_2)+g\psi_3'-f\dot{\psi}_1=-\frac{2A^3\xi_2}{g}(\rho+p), \tag{44}$$

$$0=\xi_3(\rho+p) \tag{45}$$

$$\begin{aligned}&-\frac{f^2}{g^2}\left[H_c^2+2\dot{H}_c\right]+\frac{eY}{2g^2r^2A^2}\left\{\left(g^2[l(l+1)-2]-8f^2r^2\dot{H}_c\right)\psi_1-2rg^2\psi_1'+4rfg\dot{\psi}_3\right.\\&\left.-4f^2r^2\dot{H}_c\psi_2-2f^2r^2H_c(\dot{\psi}_1-\dot{\psi}_2)-2f^2r^2\ddot{\psi}_2\right\}=\frac{A^2p}{g^2}+\frac{epY}{g^2}\left[A^2\varphi_2+\psi_1+\psi_2\right],\end{aligned} \tag{46}$$

$$g\psi_1'-f\dot{\psi}_3=0, \tag{47}$$

$$\begin{aligned}&-\frac{erY}{2g^2A^2}\left\{-2f^2r(H_c^2+\dot{H}_c)\psi_1+2g^2\psi_1'+g^2r\psi_1''+f^2r\ddot{\psi}_1+4f^2r\dot{H}_c\psi_2-2f^2rH_c\dot{\psi}_2\right.\\&\left.+2f^2r\ddot{\psi}_2-2fg\dot{\psi}_3-2fgr\dot{\psi}_3'\right\}-\frac{f^2r^2}{g^2}\left[H_c^2+2\dot{H}_c\right]=\frac{A^2r^2p}{g^2}+\frac{epr^2Y}{g^2}\left[A^2\varphi_2+\psi_2\right].\end{aligned} \tag{48}$$

Now, we are in a stage to investigate effects of the polar perturbations. To achieve this goal, we can discuss behavior of the unknown perturbation functions. Using the zeroth-order extended Friedmann equations (21) and (22) in the perturbed field equations (42) and (46), we get the following expressions for the material distribution functions φ_1 and φ_2

$$\begin{aligned}\varphi_1=&\frac{1}{6r^2f^2A^2H_c^2}\left\{\left(g^2[l(l+1)+2]+2f^2r^2H_c^2\right)\psi_1+2r^2f^2H_c\dot{\psi}_1+2g^2r\psi_1'\right.\\&\qquad+2\left(g^2l(l+1)-3f^2r^2H_c^2\right)\psi_2+6f^2r^2H_c\dot{\psi}_2-4rg^2\psi_2'\\&\qquad\left.-2r^2g^2\psi_2''-4rfgH_c\left(2\psi_3+r\psi_3'\right)\right\},\end{aligned} \tag{49}$$

$$\varphi_2 = \frac{-1}{2r^2 f^2 A^2 (H_c^2 + 2\dot{H}_c)} \left\{ \left(g^2[l(l+1) - 2] + 2f^2 r^2 H_c^2 - 4f^2 r^2 \dot{H}_c \right) \psi_1 - 2rg^2 \psi_1' \right.$$
$$\left. + 4rfg\dot{\psi}_3 + 2f^2 r^2 H_c^2 \psi_2 - 2f^2 r^2 H_c (\dot{\psi}_1 - \dot{\psi}_2) - 2f^2 r^2 \ddot{\psi}_2 \right\}. \quad (50)$$

We see that the four-velocity perturbation functions ξ_1, ξ_2 and ξ_3 do not appear in the above expressions. However, equations (43)-(45) describe deformation of the four-velocity components. It is easy to conclude that equation (45) immediately indicates that $\xi_3 = 0$. From the modified zeroth-order Friedmann equations (21) and (22), we can write

$$\rho + p = \frac{2f^2}{A^2}(H_c^2 - \dot{H}_c), \quad (51)$$

$$p = -\frac{f^2}{A^2}(H_c^2 + 2\dot{H}_c). \quad (52)$$

Subsequently, inserting the above expressions into equations (43) and (44) leads us to

$$\xi_1 = \frac{-g}{4r^2 f^2 A^2 (H_c^2 - \dot{H}_c)} \left[gl(l+1)\psi_3 - 2frH_c(2\psi_1 + r\psi_1' - r\psi_2') + 2fr(\dot{\psi}_1 - r\dot{\psi}_2') \right], \quad (53)$$

$$\xi_2 = \frac{g}{4fA(H_c^2 - \dot{H}_c)} \left[\dot{\psi}_1 - 2(H_c\psi_2 - \dot{\psi}_2) - \frac{g}{f}\psi_3' \right]. \quad (54)$$

It seems that equation (47) does not depend on matter density or pressure explicitly and yields

$$g\psi_1' = f\dot{\psi}_3. \quad (55)$$

On the other hand, using equation (46) together with (48) leads to the following differential equation

$$\left\{ g^2 [l(l+1) - 2] - 2f^2 r^2 \dot{H}_c \right\} \psi_1 + 2rfg\dot{\psi}_3 - 2r^2 f^2 H_c \dot{\psi}_1 + f^2 r^2 \ddot{\psi}_1 - fgr^2 \dot{\psi}_3' = 0. \quad (56)$$

Thus, making use of the relation (55) in the above differential equation leads to

$$f^2 \ddot{\psi}_1 - g^2 \psi_1'' - 2f^2 H_c \dot{\psi}_1 + \frac{2g^2}{r}\psi_1' + \left\{ \frac{g^2}{r^2}[l(l+1) - 2] - 2f^2 \dot{H}_c \right\} \psi_1 = 0. \quad (57)$$

To solve this equation, we can assume that

$$\psi_1 = rA\Phi(\eta, r), \quad (58)$$

which yields

$$f^2 \ddot{\Phi} - g^2 \Phi'' + \left\{ \frac{g^2 l(l+1)}{r^2} - f^2 \dot{H}_c - f^2 H_c^2 \right\} \Phi = 0. \quad (59)$$

The exact solution of this differential equation can be obtained via separation of the variables method. From this point of view, we can define $\Phi(\eta, r) = T(\eta)R(r)$ and assume a separation constant $-\lambda^2$. So, using equations (23) and (26) and defining a new variable

$$u = \frac{1}{H_0} + \sigma(\eta - \eta_0), \tag{60}$$

we find

$$\frac{d^2T}{du^2} + \left\{ \frac{\lambda^2}{\sigma^2 f^2} - \frac{1-\sigma}{\sigma^2 u^2} \right\} T = 0, \tag{61}$$

$$R'' + \left\{ \frac{\lambda^2}{g^2} - \frac{l(l+1)}{2r^2} \right\} R = 0. \tag{62}$$

Thus, the solutions are

$$T(\eta) = \sqrt{\frac{1}{H_0} + \sigma(\eta - \eta_0)} \left[c_1 J_\mu(z) + c_2 Y_\mu(z) \right], \tag{63}$$

$$R(r) = \sqrt{r} \left[c_3 J_\nu(\frac{\lambda r}{g}) + c_4 Y_\nu(\frac{\lambda r}{g}) \right], \tag{64}$$

with

$$\mu = \frac{1}{2} - \frac{1}{\sigma}, \tag{65}$$

$$z = \frac{\lambda}{f\sigma H_0} + \frac{\lambda}{f}(\eta - \eta_0), \tag{66}$$

$$\nu = \frac{\sqrt{1 + 2l + 2l^2}}{2}. \tag{67}$$

Here, c_1, c_2, c_3 and c_4 stand for the integration constants, $J_a(x)$ is the first kind Bessel function and $Y_a(x)$ represents the Bessel function of the second kind:

$$J_a(x) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x}{2}\right)^{a+2n}}{n!(n+a)!}, \tag{68}$$

$$Y_a(x) = \frac{J_a(x)\cos[a\pi] - J_{-a}(x)}{\sin[a\pi]}. \tag{69}$$

Now, we substitute equations (28), (63) and (64) into $\psi_1(\eta, r) = rA(\eta)T(\eta)R(r)$, it leads to

$$\begin{aligned} \psi_1(\eta, r) &= r^{\frac{3}{2}} \left[1 + \sigma H_0(\eta - \eta_0)\right]^{\frac{1}{2}+\frac{1}{\sigma}} \left[c_3 J_{\frac{\sqrt{1+2l+2l^2}}{2}} \left(\frac{\lambda r}{g}\right) + c_4 Y_{\frac{\sqrt{1+2l+2l^2}}{2}} \left(\frac{\lambda r}{g}\right) \right] \\ &\times \left[c_1 J_{\frac{1}{2}-\frac{1}{\sigma}} \left(\frac{\lambda}{f\sigma H_0} + \frac{\lambda}{f}(\eta - \eta_0) \right) + c_2 Y_{\frac{1}{2}-\frac{1}{\sigma}} \left(\frac{\lambda}{f\sigma H_0} + \frac{\lambda}{f}(\eta - \eta_0) \right) \right]. \end{aligned} \tag{70}$$

To investigate the coupling impact on the propagation of PGWs, we focus on the radiation ($\sigma = 1$) dominated stage as an example. On this purpose, in FIG. 2, we illustrate the metric perturbation function $\psi_1(\eta, r)$ versus η and r. Here, we consider four different sets of the rainbow parameters: $(f, g) = (2, 1)$ (upper-left), $(f, g) = (5, 1)$ (upper-right), $(f, g) = (1, 2)$ (bottom-left) and $(f, g) = (1, 5)$ (bottom-right). One can easily see that the increase in the value of the rainbow parameter f causes an increase in the maximal amplitude of wave while the increase in the value of g leads to an increase in the maximal amplitude.

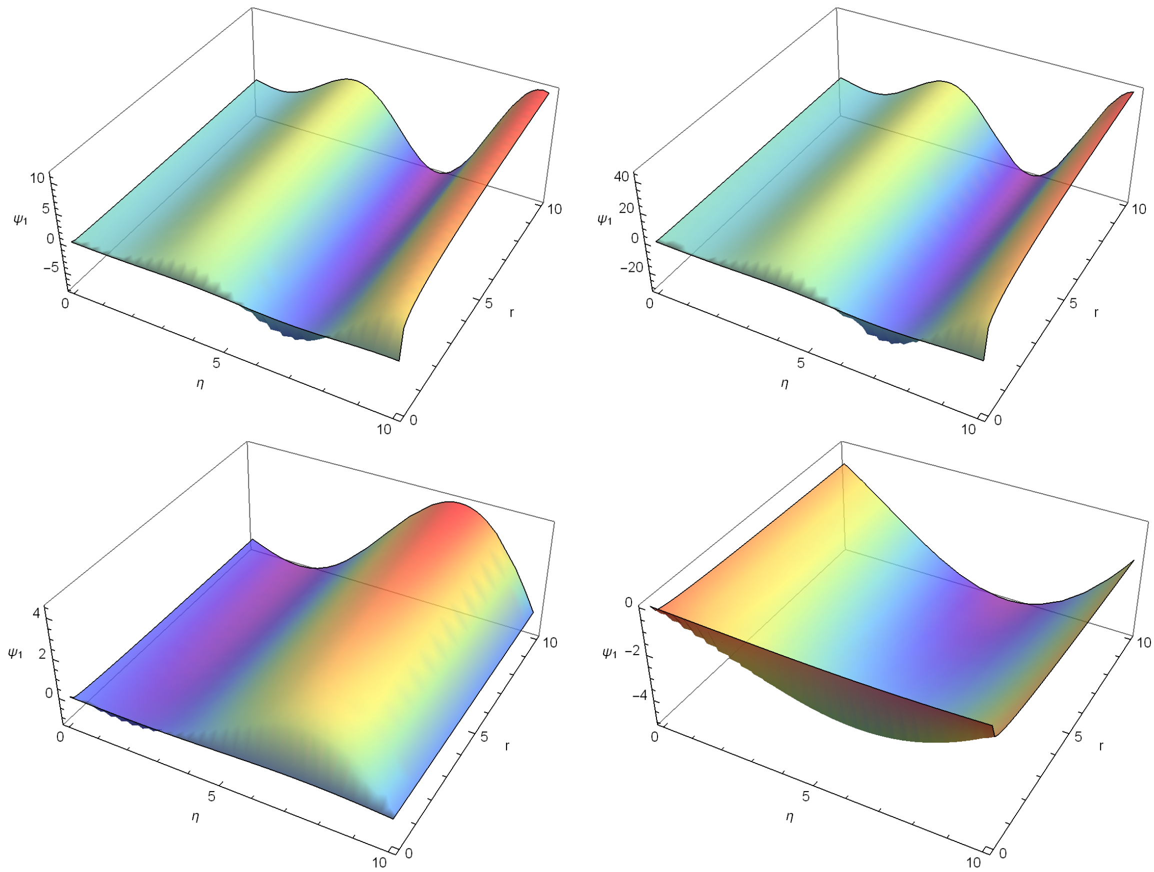

FIG. 2: $\psi_1(\eta, r)$ versus η and r for the radiation stage ($\sigma = 1$) with $H_0 = 70.00$, $\eta_0 = 0$, $l = 2$, $c_1 = c_2 = c_3 = c_4 = 0.01$, $(f, g) = (2, 1)$ (upper-left), $(f, g) = (5, 1)$ (upper-right), $(f, g) = (1, 2)$ (bottom-left) and $(f, g) = (1, 5)$ (bottom-right).

We can now find out the exact expression of metric perturbation coefficient $\psi_3(\eta, r)$. To reach this goal, we use the new variable

$$y(\eta) = H_0 u = 1 + \sigma H_0(\eta - \eta_0) \tag{71}$$

in equation (55), thus it is found that

$$\frac{d\psi_3}{dy} - \frac{g}{f\sigma H_0}\psi_1' = 0. \tag{72}$$

As a result, inserting the solution (70) in the above differential equation yields the subsequent solution

$$\begin{aligned}
\psi_3(y(\eta), r) \;=\;& c_1 r + \sqrt{\frac{r}{2y}}\frac{(\frac{\lambda}{f\sigma})^{-\frac{1}{\sigma}}}{4\sigma H_0\lambda^2\Gamma(\mu+1)}\left\{c_1(2-\sigma)\left[\sqrt{2}\lambda y\left(\frac{\lambda y}{f\sigma}\right)^{\frac{1}{\sigma}}\Gamma(\mu)J_\mu\left(\frac{\lambda y}{f\sigma}\right)\right.\right.\\
&\left.-2^{\frac{3}{2}-\mu}\sqrt{f\sigma\lambda y}\right] - c_2\sec\left(\frac{\pi}{\sigma}\right)\left[2\sqrt{2}\lambda\sigma y\left(\frac{\lambda y}{f\sigma}\right)^{\frac{1}{\sigma}}\Gamma(\mu+1)J_{1-\mu}\left(\frac{\lambda y}{f\sigma}\right)\right.\\
&\left.\left.+(\sigma-2)\sin\left(\frac{\pi}{\sigma}\right)\left(\sqrt{2}\lambda y\left(\frac{\lambda y}{f\sigma}\right)^{\frac{1}{\sigma}}\Gamma(\mu)J_\mu\left(\frac{\lambda y}{f\sigma}\right) - 2^{\frac{3}{2}-\mu}\sqrt{f\sigma\lambda y}\right)\right]\right\}\\
&\times\left\{c_3\left[3gJ_\mu\left(\frac{\lambda r}{g}\right) - r\lambda\left(J_{\mu+1}\left(\frac{\lambda r}{g}\right) - J_\mu\left(\frac{\lambda r}{g}\right)\right)\right]\right.\\
&\left.+\times\, c_4\left[3gY_\mu\left(\frac{\lambda r}{g}\right) - r\lambda\left(Y_{\mu+1}\left(\frac{\lambda r}{g}\right) - Y_\mu\left(\frac{\lambda r}{g}\right)\right)\right]\right\},
\end{aligned} \tag{73}$$

where Γ indicates the gamma function.

To get the remaining perturbation coefficient $\psi_2(\eta, r)$, we can focus on the squared sound speed formula $\upsilon_s^2 = \frac{\partial p}{\partial \rho}$. It is generally expected that a physically acceptable model should satisfy Herreras cracking concept[59], i.e. $0 \leq \upsilon_s^2 \leq 1$. On the other hand, fundamental stability and causality principles indicate two significant physical bounds on the speed of sound[60]:

- The squared sound speed should be positive ($\upsilon_s^2 \geq 0$),
- Small perturbations of the background should emit at the local speed of light at most ($\upsilon_s^2 \leq 1$).

Then, it is written that

$$\upsilon_s^2 = \frac{\delta p}{\delta \rho} = \frac{\varphi_2 p}{\varphi_1 \rho}, \tag{74}$$

which means

$$3H_c^2\varphi_1 + (H_c^2 + 2\dot{H}_c)\varphi_2 = 0 \tag{75}$$

with $\upsilon_s^2 = 1$. So, replacing the expressions of φ_1 and φ_2 presented in equations (49) and (50), one can reach an evolution equation for the perturbation function $\psi_2(\eta, r)$:

$$2f^2\ddot{\psi}_2 - 3g^2\psi_2'' + 4f^2\dot{H}_c\dot{\psi}_2 - \frac{6g^2}{r}\psi_2' - \left[10f^2H_c^2 - 3g^2 l(l+1)\right]\psi_2 - \frac{\Omega(\eta, r) + \tilde{\Omega}(\eta, r)}{r^2} = 0, \tag{76}$$

where

$$\Omega(\eta, r) = \left[g^2 \left\{2 - l(l+1)\right\} + 4r^2 f^2 H_c^2 - 2r^2 f^2 \dot{H}_c\right] \psi_1 - 4r^2 f^2 H_c \dot{\psi}_1 - 4rg^2 \psi_1' \tag{77}$$

$$\tilde{\Omega}(\eta, r) = 2rfg(6H_c + 1)\psi_3 + 6r^2 fgH_c\psi_3'. \tag{78}$$

Moreover, inserting equations (70), (73) and the solution of (76) in equation (49), (50), (53) and (54), the other perturbation functions φ_1, φ_2, ξ_1 and ξ_2 can also be evaluated exactly. Additionally, the non-zero components of the four-velocity vector become

$$V_0 = -\frac{A_0}{f}[1 + \sigma H_0(\eta - \eta_0)]^{\frac{1}{\sigma}} + \frac{e(\psi_1 + \psi_2)Y}{2A_0 f}[1 + \sigma H_0(\eta - \eta_0)]^{-\frac{1}{\sigma}}, \tag{79}$$

$$V_1 = \frac{eA_0\xi_1 Y}{g}[1 + \sigma H_0(\eta - \eta_0)]^{\frac{1}{\sigma}}, \tag{80}$$

$$V_2 = \frac{e\xi_2}{g}\frac{dY}{d\theta}, \tag{81}$$

with

$$Y(\theta) = Y_2^0(\theta) = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2\theta - 1), \tag{82}$$

$$\frac{dY(\theta)}{d\theta} = \frac{dY_2^0(\theta)}{d\theta} = \frac{1}{8}\sqrt{\frac{5}{\pi}}\sin 2\theta. \tag{83}$$

for $l = 2$.

V. THE HUYGENS PRINCIPLE

Making use of the definition (7), Malec et al.[61] investigated the relation between the areal and luminosity distances from the perspective of classical Maxwell theory and proved that the master equation identifying the propagation of electromagnetic waves is given by

$$\left[-\frac{\partial^2}{\partial\eta^2} + \frac{\partial^2}{\partial r^2}\right]\phi_l(\eta, r) = \frac{l(l+1)}{\varrho^2}\phi_l(\eta, r), \tag{84}$$

where $\varrho = \sinh r$ for the open universe, $\varrho = r$ for the flat universe and $\varrho = \sin r$ for the closed universe. The authors also concluded that the solution of equation (84) in a general form can be written as[61]

$$\phi_l(\eta, r) = \varrho^l \underbrace{\frac{\partial}{\partial r}\frac{1}{\varrho}\frac{\partial}{\partial r}\frac{1}{\varrho}\cdots\frac{\partial}{\partial r}\frac{\Sigma_1 + \Sigma_2}{\varrho}}_{l}, \tag{85}$$

with $\Sigma_1(\eta, r) = \Sigma_1(r - \eta)$ and $\Sigma_2(\eta, r) = \Sigma_2(r + \eta)$ and then they showed that the above solutions obey Huygens' principle. Furthermore, making use of the ReggeWheeler gauge[41] in the GR theory, Malec and Wylężek[45] studied the propagation of axial GWs and Kulczycki and Malec[55] investigated axial and polar modes of GWs. In both of these papers[45, 55], it was concluded that the Huygens principle is satisfied only in the radiation era and the master wave equation is exactly the same as equation (84). In our investigation, defining new variables $\eta = f\tau$ and $r = g\varsigma$ transforms equation (59) into

$$-\frac{d^2\Phi}{d\tau^2} + \frac{d^2\Phi}{d\varsigma^2} = \left\{\frac{l(l+1)}{2\varsigma^2} + Q\right\}\Phi, \tag{86}$$

where

$$Q = f^2(\sigma - 1)H_c^2 = \frac{f^2(\sigma - 1)H_0^2}{[1 + f\sigma H_0(\tau - \tau_0)]^2}. \tag{87}$$

For the radiation dominated phase ($\omega = \frac{1}{3}$ or $\sigma = 1$), we have

$$\lim_{\sigma \to 1} f^2(\sigma - 1)H_c^2 = 0, \tag{88}$$

which means equation (86) now becomes

$$\left[-\frac{d^2}{d\tau^2} + \frac{d^2}{d\varsigma^2}\right]\Phi(\tau, \varsigma) = \frac{l(l+1)}{2\varsigma^2}\Phi(\tau, \varsigma), \tag{89}$$

So, it is clearly seen that equations (84) and (89) have the same form, thus we can say that the Huygens principle is satisfied also in the rainbow gravity formalism for the radiation dominated case. In general, it is concluded that the additional term Q is the mainspring of the violation of the Huygens principle in the gravity's rainbow formalism. From equation (87), we can say that Huygens principle does not hold in the cold dark matter ($\omega = 0$ or $\sigma = \frac{1}{2}$), ultra-relativistic particle ($\omega = \frac{1}{2}$ or $\sigma = \frac{5}{4}$), sub-relativistic matter ($\omega = \frac{1}{4}$ or $\sigma = \frac{7}{8}$), stiff fluid ($\omega = 1$ or $\sigma = 2$), phantom dark energy ($\omega < -1$ or $\sigma < -1$), incompressible fluid ($\omega = -1$ or $\sigma = -1$) and the quintessence dark energy ($-1 < \omega < -\frac{1}{3}$ or $-1 < \sigma < 0$) dominated stages of the cosmos. In FIG. 3, we analyze the behavior of Q for the dark energy (left plot) and ultra-relativistic matter (right plot) dominated phases to give examples. Since GWs can backscatter on the curvature of the geometry and a part of the radiation would come with a delay, they do not satisfy the Huygens principle[45]. On the other hand, since electromagnetic waves propagate along with null cones, such waves obey the principle[62].

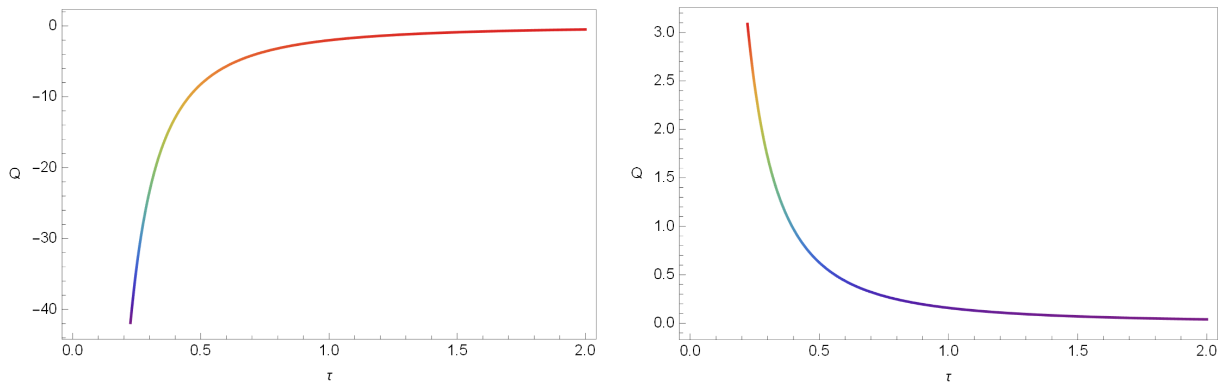

FIG. 3: The behavior of $Q(\eta)$ for the dark energy ($\sigma = -1$, left plot) and ultra-relativistic matter ($\sigma = \frac{5}{4}$, right plot) stages with $H_0 = 70.00$, $\tau_0 = 0$, $l = 2$, $c_1 = c_2 = c_3 = c_4 = 0.01$ and $f = 2$.

VI. SUMMARY

It is now known that some of the violent (or most energetic) processes in the universe produce GWs. The existence of a such phenomenon was predicted firstly by Einstein in his GR theory. The corresponding mathematical ground in the GR implied that massive accelerating cosmic objects (for instance black holes or neutron stars orbiting around each other) would warp the structure of the cosmos in such a way that cosmological radiation of fluctuating geometry would emit in all directions away from the source. Nowadays, it is commonly believed that these interesting ripples may include information about the nature of gravity. The direct observations of a GW signal by kilometric-size improved gravity-wave interferometers started a new era in modern astronomy. In addition to this, this mysterious window composes a validation of gravity models, because they are taken into account in all calculations of gravitational radiation.

In the light of above information, we think that discussing how accurately such observations could bound alternative gravity theories would be interesting and may yield significant conclusions. In this article, we have studied the effects produced by polar GWs via the framework of gravity's rainbow. In the first step, we have presented perturbation assumptions for the conformally flat rainbow metric and the background material content. Also, the four-velocity is assumed to be non-commoving. It is important to remind at this point that all field equations are reduced to the GR case[55] for the limiting condition $f = g = 1$. Our main conclusions can be specified as follows:

- In the second part of the investigation, we have obtained exact forms of the unknown perturbation functions by making use of the zeroth-order and the first-order background equations. We have seen that the increase in the value of the rainbow parameter f causes an increase in the maximal amplitude of gravitational wave while the increase in the value of g leads to an increase in the maximal amplitude.

- We have found that the factor $\xi_3(\eta, r)$ appearing in V_3 components of the four-velocity vector vanishes, which means polar GWs do not effect this component.

- In the third step, we have investigated the Huygens principle in the RG framework and found that the principle is strictly obeyed only in the radiation era. The additional term $Q = f^2(\sigma - 1)H_c^2$ written in the master equation explaining the propagation of polar GWs is the main cause of the violation of the Huygens principle in the gravity's rainbow formalism.

- Currently, we know that the Universe is in an accelerated expansion era. Thus, investigating the propagation of GWs in this speedy enlarging universe would be crucial. In this regard, one can easily expand our investigation by assuming $-1.5 < \sigma < 0$ for expanding matter and discuss how such forms of GWs can effect the spacetime tissue in the recent epoch.

DATA AVAILABILITY

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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