Response to Referee

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August 15, 2021

Manuscript Title : Quantum aspects of chaos and complexity from bouncing cosmology: A study with two-mode single field squeezed state formalism

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Dear Editor,

We thank the referee for his/her report and insightful comments on our work. We also thank the Editor for the correspondence. Based on the comments and suggestions made by the referee we have modified the draft considerably and provided clarifications wherever required. Based on the changes made in the revised version of the draft we request the Editor to reconsidering this paper for publication in the SciPost Physics Core. Our responses to the referees' comments are appended below point-wise:

I. Response to Referee

1. **Referee's Comment :**
   The notion of circuit complexity should be expanded upon.

   Authors' Response:
   We thank the referee for asking an expansion on the notion of circuit complexity.

   The concept of circuit complexity was primarily used in the field of Computer Science to know the depth of different circuits. It is basically defined as the effort required to carry out a given task or the difficulty in implementing a given task. In QFT, the task to carry out is to prepare a given target state from a certain reference state.

   This concept mainly came into the picture to understand the bulk geometry from information of the boundary field theory in the context of AdS/CFT correspondence. It becomes more important in the context of black hole physics. For a black hole the search for an observable to probe physics behind the horizon remains continued. It was found that the entanglement entropy couldn't probe the bulk geometry behind the horizon of black holes and this prompted Susskind and collaborators to study new observables of the bulk which they conjectured should be dual to the circuit complexity of the boundary field theory. These led to the famous "Complexity=Volume (CV)" and "Complexity=Action (CA)" conjectures. Volume of the CV conjecture refers to the volume of an extremal co-dimension one surface extending the boundary time slice into the bulk whereas the Action of the CA conjecture refers to the gravitational action of the Wheeler-De-Witt patch.

Thus, to develop a proper understanding of the bulk geometry one needs to understand the complexity of the boundary field theory. However, the development of circuit complexity for field theories is still at its infancy.

We have added this discussion in the introduction and on the first paragraph of Pg-6.

2. **Referee's Comment :**
   Use is made here of the linear cost functional and the quadratic cost functional. What would be the pros and cons of handling other types of cost functionals? A comment would be beneficial.

   Authors' Response :
   A reasonable cost functional has to satisfy the following features [1].

   - Smoothness
   - Positivity
   - Triangle inequality
   - Positive Homogeneity

   The main reason for using the linear and the quadratic cost functional was because apart from satisfying all the above conditions, the two has a beautiful physical interpretation. To be precise, the linear cost function has a direct connection with counting the number of gates of the quantum circuit whereas the use of quadratic cost function allows to interpret the problem as finding the shortest distance between the reference and the target states.

   The higher order κ family of cost functionals can be considered as a generalization of the linear cost functional. The complexities calculated from these cost functionals agrees with the results of holographic complexities but these cost functionals do not satisfy the homogeneity property i.e the cost function calculated from these cost functionals $\mathcal{D}(U(\sigma)) = \int_0^1 ds F(U(s), Y^I(s))$ are not invariant under the reparametrization of s. We have added this discussion on Pg-7 in the paragraph below equation 2.9 in the modified version of the draft.

   However, both the general κ family of cost functions including the linear and the quadratic ones depend on the choice of the basis [2] in terms of which the operators $\mathcal{M}_I$ of the Hamiltonian expansion is expressed as given in eqn (2.3) of the draft. This problem can be resolved by constructing the cost functions in terms of the Schatten norm as defined in eqn (2.11) of the draft [2]. This discussion can be found in the first line of Pg-8 in the modified version of the draft.

3. **Referee's Comment :**
   The notion of the squeezed state formalism, central to the presentation, could benefit from further discussion.

Authors' Response :

We thank the referee for asking to add some more discussion on squeezed state formalism.

In this paper, we have applied the concept of circuit complexity in the field of Cosmology. Scalar perturbations on an expanding background can naturally be described with the formalism of squeezed quantum states. We will choose the ground state while the mode is inside the horizon as our reference state, and a target state consisting of the time-evolved cosmological perturbation on the expanding background to compute the quantum complexities. Thus squeezed state formalism gives an elegant way of defining the reference and the target state between which we are going to calculate the circuit complexity.

Above all, the whole idea of squeezed state formalism can be easily understood using the well known model of inverted harmonic oscillator. In the squeezed state formalism, the wave function is squeezed with a large uncertainty in one direction and with a small uncertainty in another direction. Similar observations can be found if one looks into the phase space trajectories of a inverted harmonic oscillator. The presence of one growing and one decaying solution produces a squeezing effect even in the classical level.

The main idea behind the squeezed states is to re-parametrize the unitary operator as the product of a squeezed and a rotation operator. The squeezed and the rotation operator can further be expressed entirely in terms of the creation and the annihilation operators. The significance of the rotation operator is less as it mainly produces a phase factor. However, the squeezing operator is of prime significance as the entire problem and all the important observables can eventually be expressed in terms of two quantities, the squeezing parameter and the squeezing angle.

Thus, the squeezed state formalism not only gives an elegant way of finding the target and the reference state but also helps to express all the important observables in terms of only two quantities.

We have added this discussion on Pg-8 in the paragraph below eqn 2.14.

Finally, we are thankful to the referee as well as the Editor for providing insightful comments and the correspondence, which helped us to improve this paper. We believe that our justifications to the referee's report along with the necessary changes made in the modified version of our manuscript has addressed all the questions and comments raised by the referee. Based on the mentioned changes performed in the draft we, therefore, request the Editor to reconsider this revised draft for publication in the SciPost Physics Core.

References

[1] M. A. Nielsen, A geometric approach to quantum circuit lower bounds, Quantum Info. Comput. 6 (May, 2006) 213–262.

[2] M. Guo, J. Hernandez, R. C. Myers and S. M. Ruan, "Circuit Complexity for Coherent States," JHEP **10**, 011 (2018) [arXiv:1807.07677 [hep-th]].