Response to Referee #1

We would like to thank the referee for her or his detailed and critical reading of our manuscript, and for all of the insight in the comments this raised. We respond following the numbering scheme of the referee:

1) The referee is correct that the specific heat will scale asymptotically as $L^{\gamma/\nu}$ for all cases with explicit breaking of the Z_2 symmetry (i.e. with a nontrivial rotation of the Ising model). In Fig. R1 of this reply, we show finite-size scaling data for the rotated Ising model that confirm this point. As the referee anticipated, there is a crossover lengthscale to this asymptotic behaviour that becomes larger for smaller mixing angles (ϕ). The important point is that the divergence of the correlation length is independent of ϕ: in the regime where it dominates over the microscopic lengthscales, the model has a universal description in terms of the expected scaling exponents. The "mixing crossover" is a secondary effect resulting from the competition between the different types of scaling behaviour.

   In our studies of the FFTL model, limited computing time gives us somewhat limited resolution in the critical coupling and temperature. This prevents us from observing a true asymptotic $L^{\gamma/\nu}$ power law, in either the susceptibility or the specific heat. We have stated this more clearly in the revised manuscript. Nevertheless, we are able to show that the rotated Ising model describes our data by the following procedure.

   First, as the referee suggested, in Fig. R2, we provide data for the scaling of the quartet correlation length shown in Fig. 8 of the manuscript. Here one observes that, within the error bars, the maximal correlation length does rise linearly with the system size for both $J_2/J_1 = 1$ and $J_2/J_1 = 0$. This indicates that we are still close to the critical regime. The data for both cases also agree rather closely with each other, which can be explained by the correlation length remaining independent of the mixing.

   Second, we have performed a direct comparison of Fig. 8(c) to its equivalent in the rotated Ising model, which is now presented in Fig. 8(d) of the revised manuscript. To mimic the uncertainty in the critical coupling and temperature, we also included a deliberate (but small) detuning of the Ising model from its critical point. The result provides a very close match with the observations contained in Fig. 8(c). We have added this discussion to the text of the revised manuscript.

   Finally, we also note that the rotated Ising model analysis is relevant not only at the critical point (Fig. 8) but also in the region around it, as discussed in the original manuscript by comparing the features of Fig. 7, most notably the asymmetry of the specific-heat maxima, with those of the data presented in Fig. 6. We trust that these comments (both above and in the revised manuscript) do help to satisfy the referee of the utility of the rotated Ising model analysis, and hence do justify its inclusion in the manuscript.

2) We thank the referee for making this point. We wished to communicate to the reader that the numerical method we developed is an important part of our study, rather than being a barely relevant technical add-on. The referee's comment tells us that we did not succeed in our aim. She or he is correct that in a qualitative analysis the abstract-update approach we develop is not required to obtain the physical results we present. However, by taking this approach we did benefit significantly in coding efficiency and physical interpretation.

   Thus, rather than move Sec. 3 to an appendix, we have altered the text to make its role and embedding clearer to the reader (or at minimum to the non-specialist reader). For this we have added three paragraphs at the beginning of the section to explain in full the use of arbitrary bases and the consequences these bases have for the SSE loop-update scheme. These paragraphs make the roles of Subsecs. 3.1 and 3.2 explicit, and we have made alterations in Secs. 1, 2, 4 and the abstract to stress the connectivity through Sec. 3. We hope that the referee will find these alterations sufficient to communicate an accurate message concerning the role of our method within the study and the field.

3) In Appendix A we provide the value of c.

We would like to thank the referee once again for enabling us to improve the quality of discussion and presentation in our manuscript.

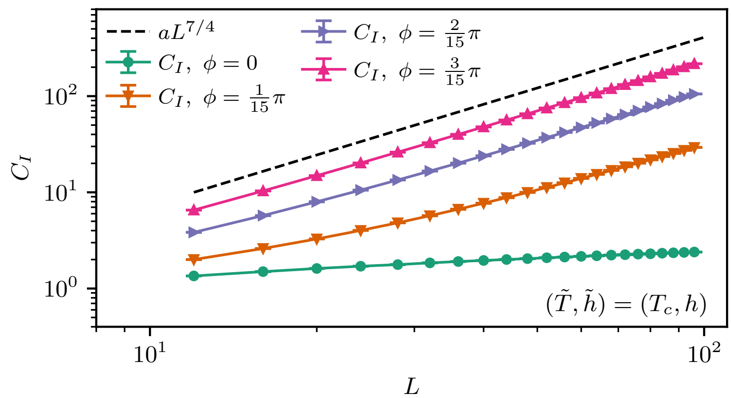

Figure R1: Finite-size scaling of the specific heat, C_I, of the rotated Ising model at the critical point. The mixing angles correspond to the panels of Fig. 7 of the manuscript. For comparison, the expected asymptotic scaling for $\phi \notin \{0, \pi\}$, $L^{\gamma/\nu} = L^{7/4}$, is shown, scaled by a nonuniversal constant a.

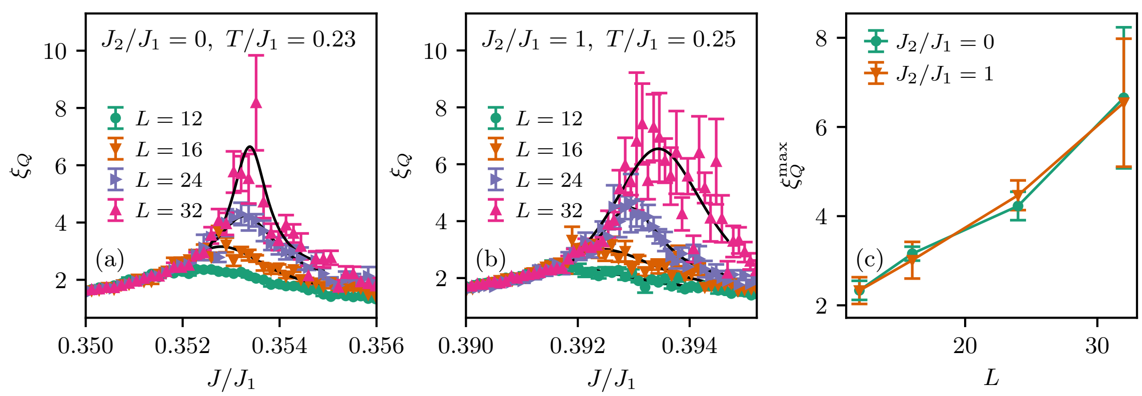

Figure R2: Finite-size scaling of the quartet correlation length, ξ_Q, in the FFTL. Constant-T slices close to (a) the $J_2/J_1 = 0$ and (b) the $J_2/J_1 = 1$ critical point. (c) Scaling of the maxima extracted from panels (a) and (b).