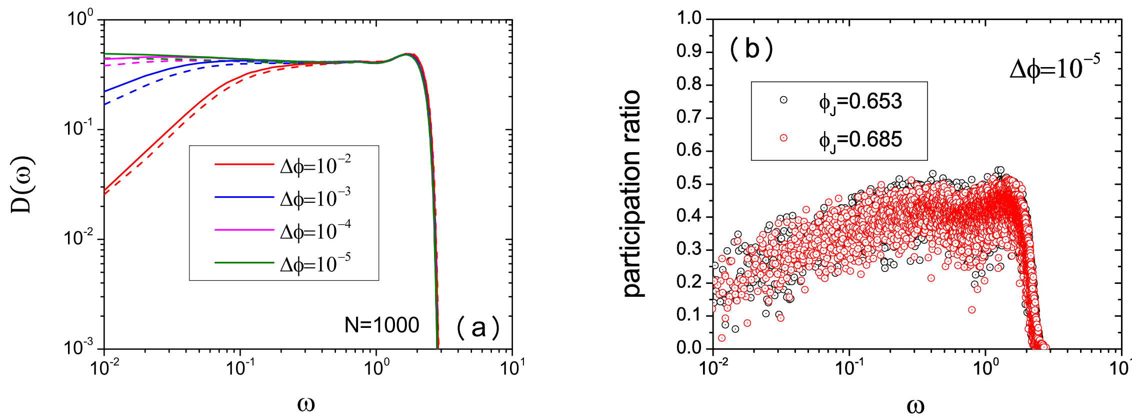

Figure R-1: Normal mode analysis of jammed sphere packings. Starting from a hard sphere packing at $\phi_{\rm J}$, we assign a soft harmonic sphere potential to particles, compress them by $\Delta\phi = \phi - \phi_{\rm J}$, and then minimize the energy. After removing of the rattlers from the configuration, we diagonalize the Hessian matrix of the obtained contact network and compute the density of states $D(\omega)$ and the participation ratio. (a): $D(\omega)$ of $\phi_{\rm J} = 0.653$ (dashed lines) and $\phi_{\rm J} = 0.685$ (solid lines) averaged over 500 samples for several $\Delta\phi$ from away from $\phi_{\rm J}$ ($\Delta\phi = 10^{-2}$) to close to the jamming ($\Delta\phi = 10^{-5}$). (b): Participation ratio computed from 3 samples for close to jamming, $\Delta\phi = 10^{-5}$.

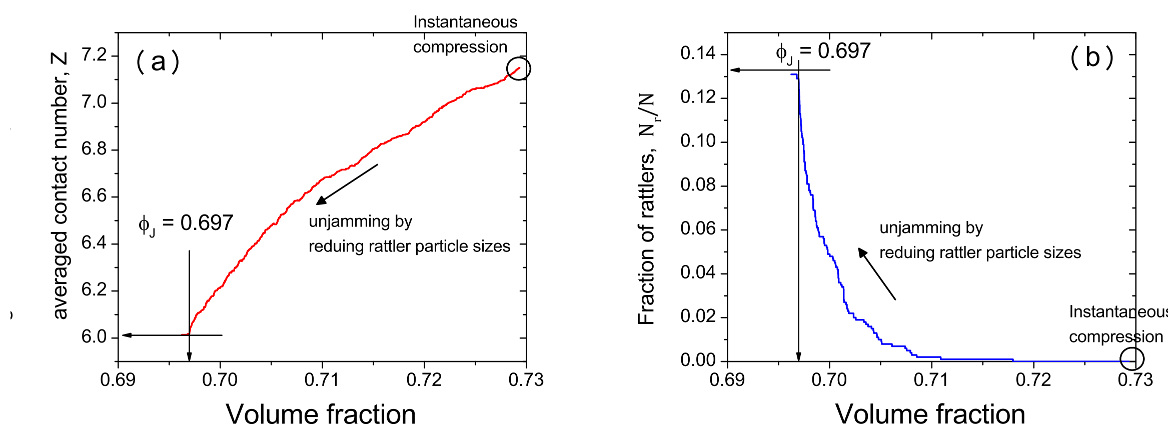

Figure R-2: An attempt to produce rattler-free isostatic packings by inflating and decompressing rattlers. Panel (a) and (b) depict the volume fraction evolution of the averaged contact number Z and fraction of rattlers $N_{\rm r}/N$, respectively. Starting from a hard sphere jammed packing at $\phi_{\rm J} = 0.697$ with $Z = 6.01$ and $N_{\rm r}/N = 0.133$, shown by the arrows, we instantaneously inflate the diameter of the rattlers up to $\sigma' = (1+\epsilon)\sigma$ and then minimize the energy of the rattlers only. We set $\epsilon = 0.2$ so that $N_{\rm r}/N \simeq 0$ as shown in the circle in (b), while Z is around 7.1 as seen in (a). Then, we repeatedly decompress the rattlers with $\Delta\epsilon = 10^{-5}$ and minimize the energy (unjamming process) of the rattlers, until a hard sphere packing is recovered. However, the system goes back to almost same state as the inital state: Z and $N_{\rm r}/N$ are essentially unchanged.