Dear Editor and Referee,

Thank you for sending us the correspondence on our manuscript "Stroboscopic aliasing in long-range-interacting quantum systems" **scipost_202103_00022v1**. We thank the referee for their positive comments and constructive suggestions. In addition to the responses to the referee's comments below, we have made all the suggested changes and marked them in blue on the manuscript. We hope that, in the current form, our work will be found suitable for publication in Scipost.

Yours Sincerely,

Shane P. Kelly, Eddy Timmermans, Jamir Marino, Shan-Wen Tsai

RESPONSE TO INDIVIDUAL COMMENTS OF THE REFEREE

> *I am puzzled by the results in Fig.4 and by the conclusions the authors draw from them. In particular the fact that at finite alpha it seems that increasing N the stability of the n=2 aliasing increases, leading to the conclusion in Sec. 3.2 that:*
>
> *"The stroboscopic aliasing is therefore not a fine tuned point in parameter space, rather, it shows robustness to the inclusion of long-range spin-spin interactions....In this respect, it survives a purely mean-field description of dynamics."*

We understand the concerns of the referee regarding our presentation in Section 3.2, and we have added an additional paragraph to address those concerns and to better describe our results. Below, we respond to the questions raised by the referee.

> *Overall I am rather skeptical about the stability of the aliasing to many-body quantum fluctuations since, as the authors state at the beginning of Sec.3 .3, it requires the existence of a well defined single-frequency collective mode, which is probably not the case in a generic many-body setting. Can the authors comment on this point?*

The referee is correct that it requires a well defined single-frequency collective mode, and that such a mode is not present in a generic many-body setting. For short range interactions, and for $\alpha > 2$, finite momentum excitations can indeed mix with the collective mode and will generically lead to a short lifetime of collective oscillations. However, for longer range interactions, $\alpha < 1$, previous studies have shown that the excitation of finite momentum spin-waves are suppressed[1–11] by a factor of $N^{\alpha-1}$. Thus, for $\alpha < 1$, only the zero-momentum collective mode is left in the thermodynamic limit. Our results on the stability of aliasing to many-body quantum fluctuations, and similar results in driven long range interacting systems[9–13], can thus be understood as a consequence of the suppression of spin-waves[1–11].

> *In this light, the fact that increasing the system size makes the aliasing more and more stable, as shown in Fig.5 is, according to me, a demonstration of the fact that the method chosen by the author (DTWA) is not able to actually capture thermalization. It seems instead that the fluctuations included by DTWA become less and less important in the thermodynamic limit. Can the authors comment on this point?*

The fluctuations included by DTWA become less important in the thermodynamic limit because all fluctuations become less relevant in the thermodynamic limit when $\alpha < 1$. This follows from the discussion above, and is explained in more detail by other works and methods[1–11]. One such method, developed by one of the authors and others, presents a particularly clear picture[9–13]. In that method, a time dependent Holstein-Primakov expansion is made around the time evolving collective mode and yields predictions for the production of finite-momentum spin-waves. The suppression of spin-waves by a factor of $N^{\alpha-1}$ is then derived explicitly and becomes exact in the limit of small spin-wave density. Thus the expectation from that method, and other works[1–11], is that production of finite momentum excitations is suppressed until a late time large in $N^{1-\alpha}$ (see in particular ref [7] which has an exact calculation). This explains our observation that, for $\alpha < 1$, fluctuations become less important when N is increased.

> *I would have expected that for $\alpha \neq 0$, when the model is non-integrable, the system would heat up towards infinite temperature in the thermodynamic limit as expected from a periodically driven system. Could the authors comment on this expectation and whether it should be met by their model at finite alpha?*

This expectation is reasonable, and it is what we find for a sufficiently large $\alpha > 0.6$. Below that α, long range interactions suppress the excitation of spin-waves and heating as discussed above. Thus we find that thermalization to an infinite temperature state is prevented by the long range interactions on the time scales large in $N^{1-\alpha}$.

> *Can they quantify this heating?*

As discussed above, entropy production, and the excitation of finite momentum excitations, are suppressed for times large in $N^{1-\alpha}$. Such results generically apply to heating as would be captured by $Q = \langle H_1(t)\rangle - \langle H_1(0)\rangle$. Unfortunately studying such a quantity involves a few subtleties which prevent us from drawing simple conclusions. The main issue is that the energy in the initial state $\langle H_1(0)\rangle = 0$ is already close to the infinite temperature value: at $t = 0$ we start in a state polarized in the $\hat{y}$ direction resulting in

$$\langle H_a \rangle = \left\langle -\sum_{k=1}^{N} \sigma_k^x + \frac{\Lambda_a}{2N^{1-\alpha}} \sum_{k,j=1}^{N} \frac{\sigma_k^z \sigma_j^z}{|k-j|^\alpha} \right\rangle = 0, \tag{1}$$

while at infinite temperature, the system is completely depolarized and one again finds $\langle H_a \rangle = 0$. Therefore, Q would be approximately 0, even in the case of large entropy production. We therefore leave a more detailed analysis of these problems for a future work.

> *The authors could improve the presentation of this section, including (i)clarifying which case they are considering in Fig 5 (fully connected alpha=0 or not?Finite N or large N), (ii) writing down the Lindblad master equation they consider and (iii) mention in few words how they actually solve it.*
>
> *Minor Issue: References - some of the articles seem misplaced or not properly cited. For example Ref 39 has, from what I can see, nothing to do with Discrete Time Crystals and should not be cited together with 37-38.*

We thank the referee for these suggestions, and have included them in revised manuscript:

- α and N added explicitly in the caption of Fig. 5 and in the main text.
- To address (ii) and (iii) we have added a paragraph at the end of page 8 and beginning of page 9.
- Citation 39 is now referred to at the appropriate place.

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[3] Bojan Žunkovič, Markus Heyl, Michael Knap, and Alessandro Silva, "Dynamical quantum phase transitions in spin chains with long-range interactions: Merging different concepts of nonequilibrium criticality," Phys. Rev. Lett. **120**, 130601 (2018).

[4] Amit Dutta, Gabriel Aeppli, Bikas K Chakrabarti, Uma Divakaran, Thomas F Rosenbaum, and Diptiman Sen, *Quantum phase transitions in transverse field spin models: from statistical physics to quantum information* (Cambridge University Press, 2015).

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[6] Romain Bachelard and Michael Kastner, "Universal Threshold for the Dynamical Behavior of Lattice Systems with Long-Range Interactions," Phys. Rev. Lett. **110**, 170603 (2013).

[7] Michael Kastner, "Diverging Equilibration Times in Long-Range Quantum Spin Models," Phys. Rev. Lett. **106**, 130601 (2011).

[8] Mauritz van den Worm, Brian C. Sawyer, John J. Bollinger, and Michael Kastner, "Relaxation timescales and decay of correlations in a long-range interacting quantum simulator," New J. Phys. **15**, 083007 (2013).

[9] Alessio Lerose, Jamir Marino, Andrea Gambassi, and Alessandro Silva, "Prethermal quantum many-body kapitza phases of periodically driven spin systems," Physical Review B **100**, 104306 (2019).

[10] Alessio Lerose, Bojan Žunkovič, Jamir Marino, Andrea Gambassi, and Alessandro Silva, "Impact of nonequilibrium fluctuations on prethermal dynamical phase transitions in long-range interacting spin chains," Physical Review B **99**, 045128 (2019).

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[12] Andrea Pizzi, Johannes Knolle, and Andreas Nunnenkamp, "Higher-order and fractional discrete time crystals in clean long-range interacting systems," arXiv preprint arXiv:1910.07539 (2019).

[13] Bihui Zhu, Jamir Marino, Norman Y. Yao, Mikhail D. Lukin, and Eugene A. Demler, "Dicke time crystals in driven-dissipative quantum many-body systems," New J. Phys. **21**, 073028 (2019).