This manuscript contains some original ideas on the interplay between different types of vortices (generated by charge and spin U(1) topological defects) and their connection to both charge and spin transport n triplet superconductors. Many ideas and concepts in this manuscript follow from previous works by one of the authors with other authors as well as the current one (see Refs. 22, 28, and 45 in the current manuscript).

=> We thank the referee for evaluating our manuscript.

I have some concerns on the claimed phase diagram though. The effective vortex action of Eq. (2) contains two integer vortex fields, namely, m_i^c and m_i^{sp}, referring to charge and spin vortices, respectively. This effective action follows by straightforwardly integrating out the phases in the Villain action. However, these phase variables being decoupled [see, for instance, Eq. (5) in https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.99.197002 , which is one of the authors of the current submission (Chung) is the first author], and therefore this leads to two BKT transition temperatures, which are determined by the charge and spin stiffnesses.

=> We thank the referee for the critical comment. We respectfully disagree with the referee's statement "these phase variables being decoupled." Due to the U(1)×U(1) order parameter topology, the charge phase and the spin phase of the spin-triplet superconductors considered in our work are not decoupled. When the **d**-vector lies in the xy-plane, the order parameter is given by the pair of the superconducting gaps $(\Delta_{\uparrow\uparrow} = -|\Delta|e^{i(\theta-\alpha)}, \Delta_{\downarrow\downarrow} = |\Delta|e^{i(\theta+\alpha)})$, where θ and α are the charge phase and the spin phase, respectively. This U(1)×U(1) order parameter remains invariant under the simultaneous nontrivial transformations of the charge and the spin phases: $\theta \to \theta \pm \pi$ and $\alpha \to \alpha \pm \pi$, under which the charge state and the spin state change. Therefore, the charge and the phase are not decoupled at the order-parameter level. This has implications on vortices. The only requirement for the U(1)×U(1) order parameter to remain single-valued for vortices is $m^c \mp m^{sp} \in \mathbb{Z}$ where $m^c(m^{sp})$ is the winding number of charge(spin) phase. Therefore, in addition to vortices with integer winding numbers $m^c \in \mathbb{Z}$ and $m^{sp} \in \mathbb{Z}$, there are vortices with half-integer winding numbers such as $\left(m^c, m^{sp} = \pm\frac{1}{2}, \pm\frac{1}{2}\right)$, which are commonly referred to as half-quantum vortices. When tracing the charge phase θ and the spin phase α over a closed loop around half-quantum vortices, both change by $\pm\pi$, but the order-parameter is invariant.

Let us elaborate our response with the following points.

First, in Eq. (2) of our manuscript, the charge winding number and the spin winding number denoted by m_i^c and m_i^{sp} can be either integers or half-integers depending on the nature of the corresponding vortex. A full quantum vortex has $\left(m_i^c, m_i^{sp}\right) = (\pm 1, 0)$; a d-vector meron has $\left(m_i^c, m_i^{sp}\right) = (0, \pm 1)$; a half-quantum vortex has $\left(m_i^c, m_i^{sp}\right) = (\pm 1/2, \pm 1/2)$. We have added the following sentence after Eq. (2) in the revised manuscript to avoid a confusion:

> Here, the charge and the spin winding number, m_i^c and m_i^{sp}, can be either both integers or both half-integers depending on the nature of vortices.

Second, as the referee noted, the charge phase variable θ and the spin phase variable α appear decoupled in the free energy [Eq. (5) of the mentioned reference]. However, they are coupled intrinsically at the order-parameter level. The order-parameter matrix of spin-triplet superconductors is given by Eq. (1) of our main text. Note that it is invariant under simultaneous windings of the charge phase and the spin phase by $\theta \to \theta \pm \pi$ and $\alpha \to \alpha \pm \pi$, which reverses the sign of the orbital part of the wavefunction and the direction of the **d**-vector simultaneously. This intrinsic coupling of θ and α at the order-parameter level gives rise to the existence of half-quantum vortices with half-integral winding numbers $\left(m_i^c, m_i^{sp}\right) = (\pm 1/2, \pm 1/2)$ in addition to more conventional full-quantum vortices with $\left(m_i^c, m_i^{sp}\right) = (\pm 1, 0)$ and d-vector merons $\left(m_i^c, m_i^{sp}\right) = (0, \pm 1)$, which are associated with the invariance of order parameter under $\theta \to \theta \pm 2\pi$ and $\alpha \to \alpha \pm 2\pi$, respectively. These three types of vortices indicate three different types of BKT transitions. To avoid a confusion, we have added the following discussion between Eq. (1) and Eq. (2):

> The existence of half-quantum vortices in spin-triplet superconductors is due to the fact that the order-parameter matrix [Eq. (1)] remains single-valued by simultaneously rotating the d-vector by π (i.e., $\alpha \to \alpha \pm \pi$) while winding the overall phase by π (i.e., $\theta \to \theta \pm \pi$).

The caption of Fig. 1 has also been revised by adding the following discussion:

> A fqv, a dm, and a fqv have U(1) winding numbers $(m^c, m^{sp}) = (\pm 1, 0)$, $(0, \pm 1)$, and $(\pm 1/2, \pm 1/2)$, respectively, where m^c and m^{sp} are the winding numbers associated with θ and α.

The unnumbered equation after Eq. (2) for the BKT transition temperature is simply the sum of these just mentioned transition temperatures. This does not seem to naturally follow from Eq. (2). To add to the confusion, other BKT temperatures are referred to in the text (T_{BKT}^{dm}, T_{BKT}^{fqv}, and T_{BKT}^{hqv}). The authors have to clarify how these are obtained.

> => We thank the referee for the critical comment. It is well-known that the BKT transition temperature is determined by the interaction between vortices, which is captured by the second term of Eq. (2) of our manuscript. By adopting the known results for various types of superfluids and superconductors, the critical temperature for the BKT transitions driven by proliferations of vortices with charge and spin winding numbers (m^c, m^{sp}) is given by $T_{BKT}(|m^c|,|m^{sp}|) = \frac{\pi}{2k_B}[(m^c)^2K_c + (m^{sp})^2K_{sp}]$ as in our main text. The BKT transitions driven by full-quantum vortices, d-vector merons, and half-quantum vortices are described with (m^c,m^{sp}) = (±1,0), (m^c, m^{sp}) = (0, ±1), and (m^c, m^{sp}) = (±1/2, ±1/2), respectively. We would like to mention that the same approach has been used in exiting literature discussing the BKT transitions in superconductors with multi-component order parameter, e.g., in [Berg, Fradkin, and Kivelson, Nat. Phys. 5, 830 (2009)]. To avoid a confusion, we have revised the pertinent part as follows:
>
> > Since the BKT transition temperature is determined by the interaction between vortices, from the second term of Eq. (2), the BKT temperature for each vortex type can be determined by adopting the known results for superfluids and superconductors [30,41,50,53,54]:
> >
> > $$T_{BKT}(|m^c|,|m^{sp}|) = \frac{\pi}{2k_B}[(m^c)^2K_c + (m^{sp})^2K_{sp}]$$
> >
> > which yields
> >
> > $$T_{BKT}^{fqv} = \frac{\pi K_c}{2k_B}, T_{BKT}^{dm} = \frac{\pi K_{sp}}{2k_B}, T_{BKT}^{hqv} = \frac{\pi(K_c + K_{sp})}{8k_B},$$
> >
> > for the BKT transitions driven by proliferations of fqv with (m^c,m^{sp}) = (±1,0), dm with (m^c, m^{sp}) = (0, ±1), and hqv with (m^c, m^{sp}) = (±1/2, ±1/2), respectively.

It might also turn out that a coupling between the charge vortices and the magnetization has to be considered at the free energy level, leading to a reformulation of Eq. (2). In fact, I would expect the term $\sim (\nabla \times A)^2$ in the Ginzburg-Landau free energy to be replaced by something $\sim (\nabla \times \boldsymbol{A} - 4\,\pi \boldsymbol{M})^2$, where $\boldsymbol{M}$ magnetization. This would suggest a coupling between the charge vortex and s_z. Since s_z is conjugated to α, this would ultimately couple the charge and spin vortices.

=> We thank the referee for the valuable comment. We agree with the referee in that a charge vortex can have a finite spin polarization by the mentioned mechanism. In fact, the finite spin polarization of quasi-particles inside a charge vortex has been discussed in [Vargunin and Silaev, Sci. Rep. 9, 5914 (2019)] in the context of vortex-flux flow spin Hall effect in superconductors. However, the BKT transitions that are of interest to us are governed by long-range interactions between vortices, rather than properties of individual vortices. For this reason, we expect that while the local coupling between the charge vorticity and the spin density can change properties of individual vortices, it would not change the physics of the BKT transitions discussed in our work qualitatively. To clarify, we have added the following footnote in the main text with citation of the aforementioned reference:

> Vortices with finite charge-phase winding number can harbor a finite spin polarization, as discussed in Ref. [55] in the context of vortex-flux flow spin Hall effect in superconductors with spin-splitting field. However, the BKT transitions that are of interest to us in our work are governed by long-range interactions between vortices, rather than properties of individual vortices. For this reason, we expect that while the local coupling between the charge vorticity and the spin density can affect properties of individual vortices, it would not change the physics of the BKT transitions discussed in our work qualitatively.

I would like to ask the authors to clarify the points above before I can make a final assessment.

=> We hope our responses are satisfactory enough to let the referee evaluate our manuscript positively.