Reply to Report 1

$R_A.(1)$ *"The authors of this manuscript presented a systematic description of the recent well-established method to construct a complete family of many-body quantum Hamiltonians with ground-state of Jastrow form involving the pairwise product of a pair function in an arbitrary spatial dimension. In section 2, the parent Hamiltonians with 2-body & 3-body pairwise potentials in d-spatial dimensions are constructed. In section 3, the 1-body term serving as external potential in the Hamiltonian is introduced by adding one particle term to the Jastrow form of the wave function, Consequently, the long-range contributions are involved by mixing the 2- and 1-body couplings in the Hamiltonians. Some simple examples were mentioned in this section. The section 4 was devoted to construct 9 models by using the method discussed in previous sections, among which the first 4 have been well studied in literatures, for example, see PHYSICAL REVIEW RESEARCH 2, 043114 (2020), etc., and the last 5 ones are newly constructed. However, these are in fact not totally new. They can be a kind of combinations of previous 4 models. In the section 5, they illustrated how to construct the explicit Jastrow form wave function once the interaction is known. "*

Answer:
We thank the referee for the accurate summary of our contribution. The novel models we have introduced have no precedence in the literature to the best of our knowledge. We term the models according to names that allows to quickly appreciate the relation with some other models, but the additive character of the interactions cannot be taken for granted and generally mixing terms occur. Nonetheless, the strength of our contribution is in facilitating the systematic construction of new models: choose your favorite one- and and two-body functions in the Jastrow form and find the parent Hamiltonian through the equations we derive. For historical accidents, the generality of the formalism and the results we report have not been presented in spite of the various related attempts documented in the introduction starting in 1975 and sustained to this day. In this sense, we close this effort by providing the complete family, which is infinite. Our examples are chosen to show how instances of known models are included in our formalism and how to generate new models in these infinite family.

$R_A.(2)$ *"The weakest part of this paper is their method. It is not a new method and such kind of Hamiltonians have been studied before. In addition, the physical meaning of these models was not explained. "*

Answer:
It is important to appreciate our results in the light of the findings by Kane et al that show the universal low energy behavior of these models is unaffected by the three-body contributions. This makes the construction physically relevant, as the ground state wavefunction of a given

parent Hamiltonian in our formalism will capture the correct long-wavelength correlations of the a related Hamiltonian obtained by removing the three-body terms. What is crucial then is the form of the two-body interactions.

$R_A.(3)$ *"In view of the systematic construction of such kind of systems, I see that the manuscript was well organized and written. The key merit of this paper is that the parent Hamiltonian construction was systematically generalized to any spatial dimensions. In the 1d case, the newly constructed Hamiltonians involve 2-body interaction (Eq.11) and external potential (Eq.17). while the 2-body interaction and external potential are seen in higher dimensions ($d > 1$) case. I shall be happy to recommend their revised version of this submission for publication in SciPost. "*

Answer:
We thank the referee for acknowledging the strengths of our work and the recommendation for publication in SciPost.

$R_A.(4)$ *"At least, the authors should discuss physical understanding and a possible application of such constructed Hamiltonians in more details. They should also anticipate how such kind of wave functions can be used to calculate the correlation functions. "*

Answer:

We have already emphasized how such models capture the correct long-wavelength behavior of models lacking the three-body contributions, as already established by Kane et al.

We thank the referee for prompting us to indicate how correlations can be computed with these models. Anyone with a QMC code can determined at ease the numerically exact ground-state correlations of these models. But even those without such a code can resort to MC integration with software such as Mathematica for low number of particles. Furthermore, given the Jastrow structure there are strong analytical results at hand and these were already pointed out in the pioneering 1955 work by Jastrow. It is further possible to use efficient perturbative approximations of the kind worked out by Gaudin and coworkers. We have now added a discussion to clarify the various options. In short, computation of ground state correlations in these models is easier than in generic models thanks to the Jastrow structure.