Reply to Second Report of Referee 2

R_2.(1) *"The authors find the family of many-body Hamiltonians with ground-state of Jastrow form in arbitrary spatial dimensions, finding that, in general, these Hamiltonians include two and three body interactions. This finding extends the Calogero-Marchioro result, which was valid only for 3D, to arbitrary dimensions. The authors also discuss the case of wave-function with one-particle terms, leading to long-range interactions in the parent Hamiltonian. Several models are discussed. Interestingly, they discuss the reverse-engineering of the Jastrow pair function corresponding to a parent Hamiltonian.*

The findings presented in this manuscript are interesting and sound. However, it is fair to say that they represent a generalization of previously know results. I find that the manuscript is definitely suitable for publication in SciPost Physics Core, provided that the comments reported in the "Requested Changes" are adequately addressed in a revised version. In order for me to give a strong recommendation for publication in the flagship journal SciPost Physics, the authors should better emphasize the relevance of the generalized results, and discuss more in depth the physics of at least one of the novel models introduced in this manuscript. Can the authors provide some interesting predictions for these models? Such predictions would highlight the relevance of the techniques discussed in the manuscript."

Answer:
We thank the referee for the accurate summary of our contribution.

We wish to draw the attention of the referee to the work by Kane et al. which established that parent Hamiltonians of Jastrow wavefunctiosn share the same long-wavelength behavior than that of the same Hamiltonian in the absence of three-body interactions. That makes the resulting models physically appealing: the physics is set by the two-body interactions. Further, the family is infinite ans instances can be found by reverse engineering for given interactions. In view of these results and the long-time impact of the preceding related works on the topic, it seems clear that our work is an important contribution meriting publication in SciPost Physics. In some sense, it closes a search started in 1975 by Calogero by identifying the complete family of parent Hamiltonians of Jastrow wavefunctions in any spatial dimension. To put in perspective our results, it is important to notice that the more limited Calogero construction is actually a reference result and in a sense a textbook result (see [13] and [29] among many others). In addition, as we mention in the discussion, our work has a plethora of ramifications.

Taking into account the feedback by the referee, we have revised the manuscript emphasizing its strengths and relevance.

R_2.(2) *"1) In the introduction, the authors state that the Jastrow wave-function with only two-body terms is suitable to describe quantum solids. It is my understanding that, in fact, this wave-function only captures the properties of fluid states. As mentioned by the authors in the*

conclusions, the Nosanov-Jastrow wave-function is instead suitable to describe the solid state. "

Answer:
The referee is completely right. The sentence was not precise enough and we have rewritten it for clarity.

R_2.(3) *"2) In the introduction, the authors write "Slater determinants of such Jastrow functions are also widely used in and quantum chemistry." First, there is perhaps a typo ("...in and..."). More importantly, this statement is not clear. In fact, electronic systems are often described via products of Jastrow functions and Slater determinants of single-particle wave-functions. Fermionic (i.e., antisymmetric) wave-functions can also be built starting from pair orbitals, but using, in general, Pfaffian wave-functions (PRL 96, 130201 (2006), J. Chem. Theory Comput. 16.10, 6114-6131 (2020)). "*

Answer:
We thank the referee for pointing out the typo, that has been corrected.

We acknowledged that we had previously only touched on this important aspect only in passing. In this revision, we have expanded the discussion along the suggestion and references indicated by the referee.

R_2.(4) *"3) In the conclusion, the authors mention the Nosanov-Jastrow wave-function for bosonic solid states. However, it is worth mentioning that the original model does not satisfy the bosonic symmetry. In order to account for Bose-Einstein statistics, various approaches have been introduced, including symmetrized wave-functions (J. Stat. Mech. P07003 (2005), NJP 11 013047 (2009)), shadow wave-functions (PRB 38, 4516 (1988), PRL 60, 1970 (1988), PRB 71 140506 (2005)) and permutation-sampling methods (PRB 17 1070 (1978), PRL 108, 155301 (2012)). "*

Answer:
We fully agreed with the referee. In the earlier version we mentioned this only briefly but we have expanded the discussion in this revision to reflect the broader efforts in quantum solids.

R_2.(5) *"4) The authors use the term quasi-exactly solvable models. The meaning is elucidated only at the end of the manuscript, and it refers to models for which only part of the spectrum is obtained. This definition should be given earlier. Also, it is not clear if, in fact, in most cases only the ground-state energy is known, and if its evaluation requires additional computations (e.g., Monte Carlo sampling of the Jastrow wave-function.)"*

Answer:
We thank the referee for suggesting this change that has now been implemented to facilitate

the reading of the text. The ground-state energy follows directly from our construction. It is formally zero for the the parent Hamiltonian acting on the Jastrow wavefunction. In practice, the two body and three body term generally give rise to constant terms which are then interpreted as the ground-state energy. This is in full correspondence with the 1D scenario (see e.g. Ref[16]) and the few instances discussed in 2D in the literature (e.g. [32,33]). For instance, in 1D, it is known that the familiar expressions for the zero-point energy of the Calogero-Sutherland model and the attractive Lieb-Liniger model (each with ground states of Jastrow form) arise exclusively from constant contributions of the two-body and three-body potentials.

Overall, we are indebted to the referee for the insightful comments and the help to put our work in a broader context emphasizing its physical relevance and pointing out to key references, some of which we were unfamiliar with. We hope that with this revision, the referee finds this current manuscript suitable for publication in SciPost Physics.