Reply to the referee reports of the paper "Annealed averages in spin and matrix models"

We thank the referee for the appreciation of our work and the suggestions to improve the presentation of our results. Below you will find the reply to the referee's questions/remarks. We also added reference [21].

I. REPORT II

1. In the introduction the use of the dagger to note the dual vector to s is misleading as it suggest that the spin vector is complex.

   The notation have been corrected throughout the manuscript.

2. The notation $P(A_{ij})$ is misleading it should be $P_{ij}(a)$ as the probability distribution depends on the the pair ij.

   In the introduction and in the section applications we have changed the notation to $P_{A_{ii}}$ for diagonal matrix elements and $P_{A_{ij}}$ for off-diagonal. For a rotationally invariant model the distribution is independent of the specific choice of i and j. In the other sections we call P_r the probability distribution of a sub matrix of size r.

3. The notation Ds is a bit odd - why not simply d**s** as the integrals are finite dimensional and D is often used to functional integrals.

   The notation have been corrected throughout the manuscript.

4. The phase transition in the spherical SK model does not always occur, it does for the Wigner semi-circle law but for density of states which do not vanish at the edge of the support of the density of states there should be no transition. It is basically the same transition as Bose Einstein condensation. I wonder what happens to the phenomenology of eigenvalue detachment in this case, does the whole scenario remain the same ?

   We have added the sentence: "If the 'charge density' density instead falls at the edge sufficiently fast, then the 'electric field' does not reach a finite limit at distances $O(1)$ from the edge and there is no freezing mechanism, and hence no transition. An example of this is the case of a constant density of eigenvalues."

5. In the annealed case the discussion of the formation of a molecule seems to be closely related to Baik-Ben Arous-Peche (BBP) transition for spiked random matrices J. Baik, G. Ben Arous, and S. Peche, Phase transition of the largest eigenvalue for nonnull complex sample covariance matrices, Ann. Probab. 33, 1643 (2005), this describes how a rank one perturbation to a random matrix can shift the largest eigenvalue. It might be work discussing this given the papers overall link with RMT.

   We added the sentence: "In this context in fact, it is known that a rank one perturbation can shift the largest eigenvalue of the original matrix [baik2005]."

6. I found the section 2.8 a bit out of place and expected 2.9 to be a continuation of this discussion, it would be better to put this in the introduction or the conclusion and it doesn't really merit its own section.

   We think that in the introduction we don't enter in such details so we preferred leave this small section as it is.

7. The model 35 for $p = 2$ is the standard SK model no ? It might be helpful to say this. In figure 6 it would be useful to recall that it is $p = 2$ Ising. In general it might be useful to be more precise about which $p = 2$ is being discussed.

   We added this comment. In Fig. 6 actually we consider $p = 3$ (in the jargon p-spin is usually taken as $p = 3$) so we made it now explicit in the text. We also made explicit in Fig. 4 and 5 that there we consider $p = 2$.

8. Before equation 49 it would be clearer to call 49 the generating function of the probability distribution in equation 46.

   We added this remark.

9. The statement at the bottom of page 16 “Let us emphasize that this is a large N result, valid for r much smaller than N” is a bit vague. Should r be of order 1 or does $N^{1/2}$ work?

   we added “such that the repulsion between eigenvalues may be neglected”.