Dear Editor,

We resubmit our manuscript retitled *Unconventional Superconductivity arising from Multipolar Kondo Interactions* to SciPost with appropriate revisions addressing the comments raised by the referee. Details of our responses to the referee comments are provided below.

Sincerely yours,
Adarsh S. Patri
Yong Baek Kim

RESPONSE TO THE REFEREE 1

In the report, the referee requests answers to questions regarding the terminology and methodology of our work. We address the interesting questions and comments below.

(1) I am still not satisfied by the author's explanation of the 'mixing' of order parameters in different irreps. I understand that their Hamiltonian does not break symmetry. However, if converted to LG notation, their Hamiltonian states that OP1 couples to the "spatial gradient" of OP2 with the appropriate symmetry (and vice versa), since the scattering vertex is q dependent and symmetry breaking. This is not the same as mixing irreps, which would be q-independent. It does not mean that once OP1 exists, OP2 also exists, which is the common meaning of mixing/coexisting OPs. I assume this coupling, which always exists in a LG expansion, has a particularly large coefficient due to the unusual Kondo interaction, and so this spatial gradient is likely to be substantially present' Their language here is misleading and should be corrected.

We understand that the referee is concerned with the terminology of "mixing" of different order parameter irreps. To be precise, this terminology was employed due to the numerical results we obtained by solving the BCS gap equations, where we obtained non-vanishing order parameters for different irreps. To avoid potential confusion with the conventional meaning of mixing/coexisting order parameters, we revise the manuscript to remove these mentions, and simply state the existence of a variety of non-vanishing of multiple superconducting order parameters.

(2) I am also concerned by their derivation of this q-dependent interaction vertex. Typically, if I write down a pairing interaction with $f(q = k - k') = \cos q_x - \cos q_y$, then I separate it into even/odd parity components with $f(k - k') \pm f(k + k')$ and then separate these to write $f(k)f(k')$, where then the d-wave symmetry of f(q) is actually split between the two superconducting order parameters, as opposed to keeping it with the interaction vertex, as they do here. Now, given the symmetry of the Gamma's involved, they may be correct to do what they have done, however, it is at all not obvious. Given that what they are doing is so unusual, they must explain why their choice is the correct one, in detail, particularly why the momentum dependence is being kept with the vertex rather than going with the superconducting order parameter.

We thank the referee for this question, and we would like to provide some more details as to why our methodology reserved the momentum dependence with the vertex rather than attaching it with the superconducting order parameter. The primary difference between our interaction terms and the example presented by the referee is in the inability to easily separate the interaction $f(\mathbf{q} = \mathbf{k} - \mathbf{k}')$ into $f(\mathbf{k})f(\mathbf{k}')$. The reason for this inability is that the interaction potential we consider in our work is of the form of rational functions in $\mathbf{k}, \mathbf{k}'$. As an illustrative example, consider one of the pairing potentials,

$$f_{2\nu}(\mathbf{q} = \mathbf{k} - \mathbf{k}') \equiv \frac{a_2 q_\nu^2}{\left(a_0 \mathbf{q}^2 + m_{\mathcal{Q}}\right)^2 - a_2^2 \left(q_\mu^4 + q_\nu^4\right)}, \tag{1}$$

where $q_\nu^2 \equiv \frac{1}{2}(2q_z^2 - q_x^2 - q_y^2)$ and $q_\mu^2 \equiv \frac{\sqrt{3}}{2}(q_x^2 - q_y^2)$, and a_0 and $m_{\mathcal{Q}}$ are phenomenological Landau parameters characterizing the the quadrupolar fluctuations. Importantly, the pairing potential is in the form of a rational function (more specifically, composed of numerators and denominators of non-linear functions), where $\mathbf{k}$ and $\mathbf{k}'$ dependency cannot be easily separated. We contrast this with the pairing potential suggested by the referee above, $f(\mathbf{q} = \mathbf{k} - \mathbf{k}') = \cos q_x - \cos q_y$, which has an elegant trigonometric form that can be decoupled into $-\frac{3}{4}\left[(\cos k_x + \cos k_y)(\cos k'_x + \cos k'_y + (\cos k_x - \cos k_y)(\cos k'_x - \cos k'_y)\right]$ by employing standard trigonometric identities. Such elegant identities are not applicable for the complicated rational function we have in our case, and as such we leave the momentum dependence of the pairing potential with the vertex when numerically solving the gap equations. For the sake of completeness, we note that the cubic irrep label is used to classify the total angular momentum (J) of the Cooper pairs, and we presented the linear momentum ($\mathbf{k}$) dependence of the numerically solved for order parameters in Appendix I. We incorporate the reasons behind attaching the momentum dependence with the interaction vertex in the revised manuscript Sec. 3.