I. DERIVATION OF EQ. (33) OF THE MAIN PAPER

Thermal current in non-viscous fluid is given by :

$$Q_i = \gamma^2(\epsilon_0 + p_0)\delta v_i - \zeta_i \delta p - \mu_0 \delta J_i, \tag{1}$$

and we need to find δp in terms of our external perturbations δE_i and $\partial_i \delta T$. The energy conservation equation, relates δp to its gradients $\partial_i \delta p$ as follows :

$$\partial_t \Big[\gamma^2(\delta\epsilon + \zeta^2 \delta P) - 2\gamma^4(\epsilon_0 + p_0)\zeta_j \delta v^j\Big] - \zeta^i \partial_i \delta P = 0 \tag{2}$$

Integrating with respect to time gives,

$$\delta P(t) = \frac{1}{2+\zeta^2}\Big[2\gamma^2(\epsilon_0 + p_0)\zeta_j \delta v_j + \gamma^{-2}\zeta_j \int^t \mathrm{d}t' \partial_j \delta p(t')\Big]. \tag{3}$$

where we have used state equation $\epsilon = 2p$. The final step is to substitute for δp in Eq. (3) from the balance equation

$$-\partial_i \delta p = n_0 \delta E_i + s_0 \partial_i \delta T, \tag{4}$$

that leads to

$$Q^i(t) \supset \int^t dt' \frac{\gamma^{-2}}{2+\zeta^2} \zeta^i \zeta^j (n\delta E_j(t') + s\partial_j \delta T(t')). \tag{5}$$

In the d.c limit, the external perturbations δE_j and $\partial_i \delta T$ are time-independent. This is how the $t-$integration leads accumulative interpretation.