Comments on: ‘Generalized Spectral Form Factors and the Statistics of Heavy Operators’ (arXiv:2111.06373)

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In the preprint [1] the authors claim to confirm the OPE randomness hypothesis [2] proposed earlier by a subset of them. In the following we point out some logical flaws and shortcomings of the preprint.

1. One of the starting assumptions of the preprint is that random matrix universality applies to 2d CFTs. It is unclear where this assumption stems from. In the preprint, the notion of ‘a chaotic CFT’ is misleading as it has not been phrased appropriately in 2d CFT terms. Recent work on wormholes in AdS_3 gravity [3] reveal some similarities with RMT regarding level-repulsion. However, it is far from clear whether a conventional RMT description applies; the authors of [3] do mention what are the obstructions towards a RMT description. Furthermore, even if a RMT description applies, it is only for extremal microstates (*i.e.* in the JT gravity limit of low temperatures and fixed spin). The preprint [1] does not take these details into account. Hence, the assumption that RMT universality applies to 2d CFTs is naive.

2. The authors define a ‘simple probe’ OPE coefficients in equation (1.5), $Z(\tau_1, \tau_2, \tau_3)$. It is ambiguous what class of states (primaries or all states?) over which the summation acts in equation (1.5). It is to be noted that the matrix elements of descendant states (within a single Verma module) do not obey the standard ETH ansatz, as these are fixed by symmetries and they are not random – see [4].

   If at all the C_{123}'s are genuine OPE coefficients of a CFT, it is reasonable to view $Z(\tau_1, \tau_2, \tau_3)$ in equation (1.5) being defined as sum over primaries. However, such a definition is unusual from a CFT perspective and does not correspond to any standard object in 2d CFT. The reason is that a genuine genus-2 partition function should involve a sum over all states (including descendants). The genus-2 partition function can also be written as a sum over primaries but the contribution from the descendants is then encapsulated through the genus-2 conformal blocks – this significantly affects the dependence on the moduli of the Riemann surface. It is understandable that the form of the genus-2 block is intricate because of the infinite-dimensional Virasoro symmetry. However, there is no physical reason to remove the contribution from the descendants from CFT correlator or a higher genus partition function. The authors do not specify under what choice of CFT parameters can their object $Z(\tau_1, \tau_2, \tau_3)$ make sense. Since this is one of the starting points of the analysis presented in the paper, we think whatever conclusions the authors obtain apply to matrix elements of a toy model which bears similarity to a chaotic quantum mechanical system, not to OPE coefficients of a 2d CFT.

3. On a related point, it was shown in [5] by considering the trace-square distance that the reduced density matrices of a primary state do not agree with that of thermal state beyond the large central charge limit. Therefore, if the authors choose to focus on primaries alone they are restricting themselves strictly to the large central charge limit. Moreover, at finite central charge the number of descendants outnumber the number of primary states [6]. Therefore, regardless

of the fact that the OPE coefficients contain non-universal/chaotic information, taking into account the contribution from the descendant states is a necessity and not an option. Once the descendants enter the story, we also need to restrict to small windows of energies as well as KdV charges. Hence, the resulting description for thermalization/ETH will be given in terms of the generalized Gibbs ensemble of these KdV charges. The preprint [1] is oblivious to these subtleties.

4. It is unclear what the authors mean by claiming that their EFT description of OPE coefficients applies only to the 'ergodic regime/limit' – Sec. 2.4 of [1]. In this section, the authors declare that the task to justify EFT, which becomes a theory of random Hamiltonians, has not been carried out in CFTs and is beyond the scope of their work. Therefore, making statements about 2d CFTs which are based on random Hamiltonians is not sensible. Furthermore, it is a well known fact that in a CFT the spectrum of operator dimensions and OPE coefficients are a set of pure numbers without any time-dependence whatsoever. Therefore, any meaningful set of OPE coefficients should describe the physics across *all* time-scales regardless of whether it's the 'ergodic limit' or not.

5. A key object in this paper is the operator $\mathbb{O}$ – eqs. (2.10) and (3.2). It acts on the tripled tensor product of the state space and its matrix elements in the energy eigenbasis are in (2.10) defined to be the product of two OPE coefficients. The analysis of this operator begins with (3.2), where the operator is written in some basis of the tripled state space. However, we would like to point out that the choice is already quite restricted by writing the basis vectors as tensor products $|j_1 j_2 j_3\rangle$, which basically means that they choose a basis in the single state space and *not* in the triple. This becomes also clear from the unitary basis change which is then nothing but the tensor product of unitaries acting on the individual state spaces.

   In Section 3.1, the main goal is to compute the average of the matrix elements of $\mathbb{O}$ in the energy eigenbasis within a microcanonical window. Note that the tensors $\Omega_{j_1 j_2 j_3}$ in equation (3.2) are clearly dependent on the specific choice of the basis $|j_1 j_2 j_3\rangle$. Since the basis $\langle i_1 i_2 i_3 | j_1 j_2 j_3\rangle$ remains rather abstract and is not concretely spelled out, it is even more confusing what the overlaps with energy eigenstates, $\langle i_1 i_2 i_3 | j_1 j_2 j_3\rangle$, actually are. Further to exacerbate the situation, the authors readily take these overlaps to be Haar random unitaries, $U_{i_1 j_1}$, in equation (3.4). We do not see any justification for that other than the *assumption* that the theory behaves like a random matrix theory. The latter is what needs to be shown, but it rather is claimed to be confirmed by their results. This can only be true if RMT behaviour is assumed from the very start. Therefore, the claims about the confirmation of OPE randomness are based on a circular argument.

   The steps in equation (3.4) merely replicate what one does in RMT while deriving a version of the ETH ansatz (cf. Sec 2.2.2 of [7]) and these aspects have nothing to do with 2d CFTs.

6. While arriving at equation (3.10) from (3.8), the authors say that (3.10) is the right distribution of the OPE coefficients which has the variance in equation (3.8). It can be seen very easily that there can be numerous other distributions designed to produce the variance in (3.8). Therefore, while deducing equation (3.10) a highly non-unique *choice* is being made. This choice is valid if one is working within the RMT framework. However, as mentioned earlier there is no reason for this framework to apply to 2d CFT in the first place. Therefore, by no means does the analysis of the paper constitute a confirmation of the OPE randomness hypothesis.

   Furthermore, just mentioning that the $R_{EE'}$'s are 'random matrices' (e.g. in eq. (3.10)) does not convey much information. Do the authors mean that these are Gaussian ensembles? What about higher moments?

7. Finally, OPE coefficients in CFTs need to be consistent with the bootstrap conditions of crossing symmetry and modular covariance of correlators on Riemann surfaces of arbitrary genus. Although it is challenging to implement the bootstrap for 2d CFTs (owing to the intricate structure of the conformal blocks), it offers an infinite number of constraints which the OPE coefficients need to obey. As this verification has not been carried out for the hypothesis of [2], the validity of OPE randomness remains questionable. The same issue also exists with the previous work [8], by the same authors.

References

[1] A. Belin, J. de Boer, P. Nayak and J. Sonner, "Generalized Spectral Form Factors and the Statistics of Heavy Operators," [arXiv:2111.06373 [hep-th]].

[2] A. Belin and J. de Boer, "Random statistics of OPE coefficients and Euclidean wormholes," Class. Quant. Grav. **38** (2021) no.16, 164001 [arXiv:2006.05499 [hep-th]].

[3] J. Cotler and K. Jensen, "AdS_3 gravity and random CFT," JHEP **04** (2021), 033 [arXiv:2006.08648 [hep-th]].

[4] M. Beşken, S. Datta and P. Kraus, "Quantum thermalization and Virasoro symmetry," J. Stat. Mech. **2006** (2020), 063104 [arXiv:1907.06661 [hep-th]].

[5] P. Basu, D. Das, S. Datta and S. Pal, "Thermality of eigenstates in conformal field theories," Phys. Rev. E **96** (2017) no.2, 022149 [arXiv:1705.03001 [hep-th]].

[6] S. Datta, P. Kraus and B. Michel, "Typicality and thermality in 2d CFT," JHEP **07** (2019), 143 [arXiv:1904.00668 [hep-th]].

[7] L. D'Alessio, Y. Kafri, A. Polkovnikov and M. Rigol, "From quantum chaos and eigenstate thermalization to statistical mechanics and thermodynamics," Adv. Phys. **65** (2016) no.3, 239-362 [arXiv:1509.06411 [cond-mat.stat-mech]].

[8] A. Belin, J. De Boer, P. Nayak and J. Sonner, "Charged Eigenstate Thermalization, Euclidean Wormholes and Global Symmetries in Quantum Gravity," [arXiv:2012.07875 [hep-th]].