Reply to Report 2

The paper studies signatures of fractional Chern insulators (FCI) which could be detected using optical lattice microscopy. There are continueing experimental efforts to realize such a phase with cold atoms in optical lattices, but it remains challenging to reach larger system sizes (i.e. of a few rather than two atoms). The present manuscript focuses on systems of a scale (10-20 atoms) which seems to be realistic in the near future, but for which it is not a priori clear how well the system would actually exhibit the expected bulk behavior. At the same time, such a system size is already challenging for classical simulations, which the authors manage by using DMRG. The obtained results demonstrate very convincingly that, even for those modest sizes, the expected bulk behavior (flat density, fractionally charged excitations, etc) could be observed. These are relevant and solid findings which I recommend to publish on SciPost.

Thank the referee very much for the comments. Below we give a point-to-point response with the original comments cited in bold.

1) The definition of "filling" (nu) is only given in Section 3, but the term is already used in abstract and introduction. Since there are two different types of fillings (nu and rho), there might be some confusion. In fact, a precise definition of nu is never given (only: nu APPROX N/N_Phi). Of course, there are finite-size effects, but in the thermodynamic limit nu should be sharply defined.

We thank the referee for pointing out this potential confusion. In the revised manuscript, we use the words ‘filling’ per flux quantum for $\nu=N/N_plaq$ and ‘density’ for $\rho=N/N_site$ to avoid any possible confusion.
Yes. The definition should be precise. Now we revise the approximate definition as the definite one in the 2nd paragraph of Section 3.

2) Section 2 mentions that results are for hard-core bosons, but only hold for finite U. Here the authors should refer to their supplemental material (Part B) which discusses finite U. The statement in Sec. 2 reads a bit as if all results would hold for essentially arbitrary values of U -- which of course is not the case.

Thank the referee for the suggestion and comment. In the revised version, we modify our statement a bit and refer to the Appendix B in Section 2.

3) What motivates the choice of alpha=1/4? The authors should explain that this is not a random choice.

Yes. We add the motivation in Section 2 after introducing the flux per plaquette.

4) In Fig. 1d, the value I_n is claimed to be consistent with the current at "short times" after the quench. What does the authors mean by "short times"? The n=2 curve oscillates between something negative (consistent) and zero (not consistent). So maybe the statement that the time-

averaged I_n is consistent with the sign of the current would be more accurate? Apart from the sign, is it possible to extract the quantitative value of the currents? Can the quantization of currents be measured in that way?

We thank the referee for these good questions!

By ‘short times’ we mean the time regime where the negative imbalance I_2 appears (i.e. t< 4), which is now further clarified in page 5 of the revised manuscript.

Yes. The imbalance I_2 oscillates after some time (t>4). This may be related to the equilibration of the edge currents belonging to different rows. Without mentioning any time-averaged behavior, here we would like to emphasize the nice correspondence between the counter-propagating probability currents of the ground state and the negative imbalance near the beginning of quench dynamics.

We did not see a quantized feature from the currents, which will be another very interesting question to be investigated further. We postpone it as a future task and do not address them in the present work.

5) In Part B, the authors say that "converging" behavior has been observed by choosing larger simulation parameters. Does it mean that the shown results have not yet converged? Why don't they show the results for larger xi?

By ‘converging behavior’, we mean that we have checked our simulation parameters that are large enough to have converging results. The results for larger bond dimension (\chi=600) are the same as what have been shown (\chi=400). To avoid misunderstanding, we rephrase a bit our statement in the revised manuscript.

6) Typos: second paragraph introductions FCT -> FCI
page 5: currents ... "are" plotted versus the y-coordinate

Thank the referee for careful reading. The typos have been fixed in the revised version.

7) The authors may consider adding these references:
- Eliot Kapit, Paul Ginsparg, and Erich Mueller, Phys. Rev. Lett. 108, 066802 (2012)
- Tobias Graß, Bruno Juliá-Díaz, and Maciej Lewenstein, Phys. Rev. A 89, 013623 (2014)

Thank the referee for informing the references, which are relevant and have been cited in the revised version. Several more related references [42,44,97,98] are also included.