Reply to Report 3

This manuscript explores the detection of fractional Chern insulator (FCI) states in optical lattices through local density and current measurements, as provided by quantum gas microscopes. The model used for this study is the Bose-Hubbard Harper-Hofstadter model on the square lattice, focusing on a small magnetic flux window around pi/2 per plaquette. The Harper-Hofstadter model has been realized experimentally by several groups, and is thus a very common model for theoretical studies aiming to guide experiments towards the realization of FCIs. The authors focus mostly on hard-core bosons, but also include an estimation of finite U effects in the appendices. The authors have considered a system with edges and infinite wall confinement potential, which is consistent with some existing experiments. Their results are based on numerical calculations with DMRG. The authors have explored the local charge and current densities in the system, in the ground state as well as upon addition of one (or two oppositely) charged impurity potentials. Integrating the charge in a small region around the impurity yields a charge defect +/-e/2, as expected for Laughlin quasiparticles. Moreover, the authors have implemented a local (one plaquette) flux insertion numerically, and observed the expected quantized fractional charge pumping.

As pointed out by the authors themselves, these methods are not conceptually new, but have been proposed and even numerically explored before. The big strength of this manuscript is to thoroughly explore the finite-size effects (and robustness with respect to microscopic parameters) associated with each method. For every explored phenomenon, the authors provide numerical data for systems as large as can be simulated by state-of-the-art DMRG (which the data convinces us is large enough to enter a universal regime), as well as smaller systems, down to the small sizes where these universal phenomena break down. While the large size simulations are necessary to convince us of the universality of the results, the small size data is equally useful in view of experimental implementation, and the data in-between also provides useful information about the emergence of FCIs. In that sense, the results shown here nicely complement and confirm former studies (for example, the smallest sizes needed to observe universal FCI phenomena are in agreement with previous works). In conclusion, I recommend publication of this manuscript in Scipost Physics.

We thank the referee very much for the comments. Below we give a point-to-point response with the original comments cited in bold.

1- When discussing the addition of a QH-QP pair, the authors write 'The choice of such a pair of defects is motivated by the desire to keep the average filling away from the defects constant.' But later, they perform the same numerical experiment with just one hole. Can you please comment further on this? Clearly, adding just one QH (or QP) is favorable from the point of view of finite-size effects since the system only needs to accomodate one QH/QP instead of a pair (which additionally has to be sufficiently separated). Since the result seems to be the same (fractionally quantized charge), is there any real advantage to adding a QH-QP pair (vs just one QH or QP)? And in that case, would it be a good idea to place one defect delocalized around the edge and one defect in the center to maximize distance between the two defects for a given system size?

Thank the referee for the comments and good questions!
It has been a natural choice of investigating a pair of defects in previous literatures considering periodic boundary conditions. Nevertheless, when it comes to a finite system with open boundary condition, it is not obvious whether it is possible to host one single QP/QH excitation in the bulk. Motivated by the observation that the edge serves as a reservoir for excess particles, we extend the studies to the situation where only one single set of potential dips/bumps are applied in the bulk, leading to our interesting observation as presented in the last paragraph in page 8.
To avoid any possible confusion, we now add one more sentence after ‘The choice of such a pair…’ in the 2nd paragraph in page 8.

The main advantage to adding a QH-QP pair is that it offers a promising opportunity to perform braiding so as to confirm the fractional statistics. For example, since the QH and QP carry opposite charges, designing the same paths for QH and QP would cancel the Aharonov–Bohm phase, leaving the braiding phase to be easily determined.

In order to avoid mixing the bulk excitations with complex edge physics, in our present work we have assigned the QP/QH excitations as far as from the edges. It would be a very interesting question to investigate the fate of both bulk and edge excitations, as well as the possible coupling between them. We leave it as a future task and may not address them in the present manuscript.

2- fig. 3: QH and QP (upper and lower pannels of Fig b and e) should have same scale and y tics, so that it is easier to compare them to one another.

Thank the referee for this constructive comment. Figs. 3(b) and (e) have updated in the revised manuscript, and the caption has also been modified accordingly.

3- Inset of fig. 4a what are the two types of orange dots?

Thank the referee for pointing this out. The circles and squares respectively represent the charges in the left and right regions as defined in Fig. 3(c). Now we make it clearer in the caption of Fig. 4.

4- fig. 4 b 'The currents within the circle are zoomed in for clear visualization.' Please specify by how much the currents are enhanced (same thing for other zooms in other figures)

Thank the referee for the suggestion. The factor has been specified in the caption of Figs. 1, 4, 7 in the revised version.

5- p. 8, please fix the typo ' to estimate the the spatial extent of QHs' (two times 'the')

The typo has been fixed in the revised manuscript.