Reply to Report 1 (29.11.2021)

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We thank the Referee for raising some important remarks on the references and the notation. We make here a point by point summary of the changes we made in order to address the requests:

1. We have added two recent reviews about the use of machine learning techniques in many-body physics [1,2] in the first paragraph, each of them containing a lot of references. We left the already present more specific ones regarding the detection of phase transitions in the second paragraph.

2. The repeated full names together with the definitions of the abbreviation were adjusted according to the suggestions.

3. We fixed the sentence.

4. We adjusted the notation coherently using only λ_i to indicate the components of the input vectors.

5. As pointed out by the Referee, the skip connections were introduced in Residual Networks particularly for solving the problem of gradient vanishing that prevents the proper optimization of a deep network's parameters. Moreover – as well elucidated in [30] (previously [22]) – the presence of the skip connections allows better performances both for the sake of classification and reconstruction as the network becomes deeper and deeper because leads to the learning of good abstract representations in the bottleneck, rather than local details of the data. We added a brief explanation in the main text on page 6.

6. According to our knowledge about the study of the latent space of the AEs, the linking of the latent variables' behaviour to some physical meaning is quite complex and usually has to do with the order parameter if the aim is the phase recognition. We cited the work by S. J. Wetzel [6] and recently we found this work [Niklas Käming, *et al*, 2021 Mach. Learn.: Sci. Technol. 2 035037] that we have added among the references as [12].

7. We agree with the Referee about the missing references. Actually we already cited two of the mentioned papers in Section 4 but it is perfectly reasonable to introduce them before while first naming the Entanglement Spectrum. We updated the text accordingly.

8. The Eq. (14) from the paper mentioned by the Referee [Yang Zhao, *et al*, Phys. Rev. B 86, 224406 (2012)] is the version of the celebrated Calabrese-Cardy [Entanglement entropy and quantum field theory, Pasquale Calabrese and John Cardy J. Stat. Mech. (2004)] for periodic boundary conditions. It links the scaling of the entanglement entropy as a function of the bipartition length to the central charge of the effective conformal field theory description of

the system. Instead, in our case we use the entanglement spectrum for only one bipartition – splitting the system in two equal subportions – in order to feed the GAN. The consequences of the CFT description on the whole spectrum at a fixed bipartition can be observed in the equally-spaced parabolic structures as explained in [21] but a direct connection with the correspondent central charge is possible only looking at their degeneracies, as it is done in the latter paper. Since the description of the CFT features in the ES is treated very accurately in [21] and it is rather technical, we think this is beyond the scope of this manuscripts and could deflect the attention of the reader from the actual message.

Additional corrections

During the Review process of our manuscript, we became aware of some typos that we corrected in the Resubmission phase. We indicate them below, together with some new references, in order to keep track of the changes:

1. There was a missing overall minus sign in the right hand side term of Eq. (3)
2. We added Refs.[19,43] as well as a note added for Ref.[52]